

The Russell–Dummett Correspondence on Frege and His *Nachlaß*

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Sir Michael Dummett (1925–2011) was a noted philosopher, logician, political activist, and tarot card expert. Apart from this last description, as well as his devout Catholicism, he and Russell had a lot in common. When he was knighted in 1999, it was for “services to philosophy and to racial justice”; he was someone who combined an interest in abstract philosophical issues with a passionate interest in improving the political situation around him. He was also one of the first researchers to take analytic philosophy seriously as a topic for historical study. In particular, he was one of the first serious scholars of Frege’s philosophy, and remains perhaps the best known and most influential, at least in English, and quite possibly in any language. Before his death, he published several books on Frege’s philosophy, the first of which, a 708-page tome entitled *Frege: Philosophy of Language* appeared in 1973 and was influential not only in historical circles, but then-contemporary philosophy of language circles as well. Dummett’s researches on this book began two decades before it was published and it was in those initial researches that he came to correspond with Russell, primarily through his interest in the Frege–Russell correspondence.

Dummett first wrote to Russell on 6 October 1953, introducing himself, claiming to be writing a book on Frege, and asking to see their correspondence. Frege’s *Nachlaß* had not yet been published at that time, and Dummett was particularly interested in what light their correspondence might shed on Frege’s reaction to Russell’s paradox, which Frege admitted rendered the logical system of his *Grundgesetze der Arithmetik* inconsistent in an Appendix hastily prepared and added to the 1902 second volume of *Grundgesetze*. Dummett conjectured that Frege’s “virtual silence between 1904 and 1917” was likely due to his having been preoccupied by attempts to solve Russell’s paradox.

By the end of 1953, Russell had not only shown Frege’s letters to him to Dummett, but had provided him with photostats. However, in his first letter of 10 October 1953, he cautioned Dummett not to get his hopes up:

But I am afraid they do not throw much light on Frege’s attitude to the paradox. They are mainly concerned in refuting suggestions of my own, which turned out to be inadequate.

While it is true that the correspondence deals more with suggestions made by Russell than suggestions made by Frege, Frege’s letter to Russell of 20 October 1902 does contain a summary of the proposed solution Frege would make in the Appendix, according to which two non-coextensive functions F and G may nonetheless have the same extension if they differ from each other only with regard to whether that extension itself falls under them (Frege, 1980).

In a reply of 12 January 1954, Dummett graciously replied that Russell did himself an “injustice” in describing Frege’s letters as nothing but refutations, and points to their discussion of a certain paradox of relations. (This is the third contradiction Russell discusses at the opening of his 1908 paper “Mathematical Logic as Based on the Theory of Types.”) Consider the relation T which holds between relations R and S just in case R does not hold between itself and S . Does T hold between T and T , i.e., between itself and itself? It does just in case it does not. Frege at first thought that this paradox could not be formulated given his function/object distinction. A relation, for Frege, is understood as a first-level function with two argument places whose value is always a truth-value. A first-level function can only take objects as argument, and thus cannot take itself as argument. In the correspondence, however, Russell points out that Frege is committed to a “double value-range” $\hat{\alpha}\hat{\epsilon}R(\alpha, \epsilon)$, for every relation $R(,)$, and these double-value ranges can play the role of relations thought of as objects. A version of the paradox of relations is then formulable in Frege’s system, as Russell eventually got Frege to admit.

Dummett goes on to ask Russell if he ever studied in any detail Frege’s proposed “solution” to Russell’s paradox put forth in the Appendix mentioned above. He mentions an article by Boleslaw Sobociński 1949–1950 in which it is proven that Frege’s solution does not work: it simply leads to a more complicated contradiction. (For more recent discussion of Frege’s “way out”, see Landini, 2006.) Sobociński credits this discovery to Stanisław Leśniewski, but Dummett wonders whether or not anyone prior had known about the failure

of Frege's approach, including Frege himself. He adds, "...if so, he must surely have worked on another solution. I think he must have done, because of those years from 1904 to 1918 when he produced practically no work at all". Dummett credits Russell for relating the contradiction to Cantor's diagonal proof of his powerclass theorem, adding in a P.S.:

P.S. I have thought for some time that your original intuition that the contradiction is closely connected with Cantor's proof that is the root of the whole matter. I think that if he had realised this, Frege would never had produced his bogus solution. But in the letters he brushes your suggestion aside on what seem to me flimsy grounds.

I have argued elsewhere that the simple class form of Russell's paradox is not the only Cantorian problem discussed in their correspondence about which Frege should have concerned (Klement 2001, Klement 2002, chap. 6).

Russell's reply (14 January 1954) was relatively short:

I do not remember Frege's solution of the contradiction, but must at the time have examined it and thought it inadequate. ... I note with pleasure what you say about I note with pleasure what you say about $2^N > N$.

In fact, Russell for awhile specifically endorsed a version of Frege's theory as evinced by the note added to the end of Appendix A of *The Principles of Mathematics* (p. 522), but then later, as he told Jourdain in 1906 came to the conclusion that it "wouldn't do" (Grattan-Guinness, 1977, p. 78). Although manuscripts have survived in which Russell explores Frege's proposed solution (see especially Russell, 1994), it is not known whether Russell discovered its formal inconsistency, or just found it philosophically inadequate or technically cumbersome.

In the summer of 1954, Dummett travelled to Münster, Germany, Heinrich Scholz had kept those copies of portions of Frege's *Nachlaß* which had not been destroyed in World War II. (For the history of Frege's *Nachlaß*, see Frege, 1979, pp. ix–xiii.) There he was also able to acquire copies of Russell's side of the Frege-Russell correspondence. After he returned, he sent Russell two reports on what he had found there, a shorter one and then a longer, much more detailed one. Already in the first short report (30 July 1954) he mentions having discovered regrettable features of Frege's political opinions:

There was also a copy of diary Frege kept in the last year of his life, mostly about politics. His political opinions were – at least at time – very distasteful; he was a strong nationalist, a Bismarckian conservative who believed that Bismarck's one mistake was the introduction of parliamentarianism into Germany, and worst, of all, an anti-Semite. There is a hint that he had held more liberal views at an earlier period of his life.

Russell wrote in reply (3 August 1954) that he was "Sorry to learn what you tell me about Frege's political opinions".

On 2 September 1954, Dummett followed with a wonderfully detailed and quite generous letter, five-typed pages, in which Dummett goes into detail about the contents of Frege's *Nachlaß*, both in terms of its philosophical content, as well as Frege's distasteful political opinions. Although Frege's *Nachlaß* has now been published (in both German and English), and any reader can make up his or her own mind about its contents, I hope that Dummett's summary will itself one day be published. If nothing else, it illuminates the development of Dummett's own scholarship, crucial to the historiography of Frege studies in English. Here is a crude summary of what Dummett put in the letter:

1. He reported that W. V. Quine had accompanied him for a few days on his visit, and that the two of them had different impressions about whether Scholz was showing them all there was to see.
2. He noted Frege's career-long intention to write a book on the philosophy of logic, which never came to pass.
3. He noted that Frege maintained the sense/reference (*Sinn/Bedeutung*) distinction till the end of his career, and applied it not just to proper names of objects, but concept and other function expressions

as well.

4. He claims that Frege came close to “Wittgenstein’s *Tractatus* idea that to know the sense of a sentence is to know in what circumstances it is to be called true”. (Presumably the basis for this claim is merely Frege’s claim that the sense of a complete sentence is a sense which picks out the truth-value of the sentence as reference.)

5. He notes that Frege thought that “the whole of logic is a development of the concept “true”” and the distinction between truth and falsity.

6. He notes that Frege did not think that the reference of part of an expression is a part of the reference of the whole expression, giving as an example that the reference of “Denmark” is not a part of the reference of “the capital of Denmark”.

7. Dummett claims, rather boldly, and no doubt controversially, that “in his last two or three years, he [Frege] appears to have been very much under Wittgenstein’s influence”; as grounds for this he notes:

- Frege no longer believed that there were logical objects, and thought there were no logical grounds for believing in an infinity of them; cf. *Tractatus* §§4.441, 5.535.
- Frege thought philosophy should involve a “struggle with language”; cf. *Tractatus* §4.0031.
- He claimed that set theory was a spurious subject; cf. *Tractatus* §6.031.

In his book, Dummett was much more reserved, claiming only that these things “may indicate the influence of Wittgenstein on Frege” (Dummett, 1973, p. 663).

8. He noted that in the last few years of his life, Frege ceased to think arithmetic could be grounded in logic, and sought a geometric ground instead, with Kantian intuitions of time and space playing a central role.

9. He claims that Frege was “scornful” of elementary arithmetic, claiming that it deals with “children’s numbers”, now believing that geometry was the more central mathematical discipline.

10. He notes that nothing survived after his correspondence with Russell having to do with Russell’s paradox or the inconsistency of his logical system; Dummett reports Scholz as having claimed that there were a few documents dealing with the topic which did not survive, and that although Frege never found the formal defect with the solution given in the Appendix, he became unsatisfied with it because of its lack of “intuitive evidence”.

11. He reports that no additional draft material of the planned third volume of *Grundgesetze* can be found.

12. He claims that Frege claimed that there is a single true geometry, and that it is either Euclidean or non-Euclidean, and notes that in his correspondence with Hilbert, Frege seems to reject independence proofs for the parallel axiom. (This issue has been the subject of much debate in the recent secondary literature; Blanchette 2012 is perhaps the most up-to-date resource.)

13. He reports a number of biographical facts about Frege, his marriage and children.

14. He notes that Frege had been “a supporter of something called the Deutschvölkische Freiheitspartei, which I gather was a precursor of the Nazi party”; he also notes that Frege “thought that Jews ought to be expelled from Germany, or at least deprived of political rights” but that Frege’s anti-Semitism seems to have been limited to the final few years of his life.

15. Dummett writes “I find it very disheartening that a man for whom I have always felt such admiration and respect could at any time have held such opinions”, a sentiment he repeated years later when his first book on Frege was finally published (Dummett, 1973, p. xii).

16. He makes note of a “touching letter” Frege wrote to his adopted son when bequeathing him his unpublished manuscripts, in which he claims that there was gold in them still to be found.

17. He expresses his disappointment that Frege’s correspondence with Wittgenstein was not found with the materials he found, but claims that it is “clearly not the case” that Frege was completely unable to understand Wittgenstein. (Their correspondence has since been found, but the extent to which they

truly achieved any understanding, or influence, remains controversial; see, e.g., Floyd 2011.)

Compared to this long, generous letter, Russell's reply, on 16 September 1954, was rather brief. After thanking Dummett for the letter, he wrote that "I am pained by his political opinions but still more by his wishing to base mathematics upon geometry," and goes on to express surprise that Frege rejected alternative geometries. After Dummett confirms Frege's attitude about geometries, Russell wrote again, on 17 December 1954 with a similar message, going on to tell a story about the Pythagoreans.

What interested me most was what you have already told me of, the geometric foundations of mathematics. This pained me much more than his anti-Semitism. It was evidently the result of despairing reflections on the contradictions. It's a curious and rather close parallel to the history of the Pythagoreans, the arithmetical philosophy of Pythagoras, having apparently been refuted by irrationals, the Pythagoreans developed the geometrical theory of proportion which is set forth in Euclid books V–VI. But I think they were more excusable than Frege.

Russell relates this story about the Pythagoreans in more detail in notes made for *History of Western Philosophy*—on this, see Vianelli 2001.

Their correspondence trails off for a few years, and picks up again when Dummett wrote to Russell on 6 October 1957 reporting that most of Frege's *Nachlaß* and correspondence was being prepared for publication, first in German, then in English. He asks Russell if he would be willing to translate his letters to Frege for the project. In reply (9 October 1957), Russell at first agrees, though cannot remember what language the originals were in, and is unsure whether he would be translating from German to English for the English publication—or English to German for the German publication. On 12 October, Dummett clarifies that it is the first of these, and assures him there is no hurry, as the German edition is due first.

Indeed, it was not until 1963 that Dummett sent Russell copies of his letters to translate. By this time, now in his 90s, Russell felt unable to undertake the translation himself, citing among other reasons his ongoing work for nuclear disarmament. He also despairs at their contents, writing on 4 August 1963:

In any case, they seem to be very confused and unsatisfactory. I was bewildered by the contradiction and floundered about like a man who is out of his depth and cannot swim.

In response to this (on 7 September 1953), Dummett confesses that he too, ever since the bombing of Hiroshima, had come to regard "the use and even the conditional intention to make use of nuclear weapons as absolutely indefensible". He also claims that the interest in the correspondence with Frege is "now largely historical", but wisely adds that "one can never be certain that there is not to be found an idea which can still be made fruitful". (And indeed, there are thoughts in the correspondence which have been made use of in later researches; to give just one example, Russell's "zig-zag theory" of 1903–04, mentioned in these letters among other places, has been reconstructed into a number of interesting logical theories in Cocchiarella 1987.)

However he claims that the "chief reason" for pressing Russell into his own translation is the following.

... it would be absurd to make any selection of Frege's *Nachlaß* which did not include that correspondence [given] its very great historical interest. ... I should not ask you if it were not that your English is so celebrated. I could, indeed, easily produce a faithful version of the letters; but I think it would be absurd to publish anything by you in English which did not display that stylistic elegance which your other writings possess and which I could not achieve.

Dummett then asked if Russell were still unwilling to undertake the translation himself, whether he would be willing to look at his translation, to which Russell agreed on 11 September 1963. To my knowledge, however, Dummett never produced his own translation of the Frege–Russell correspondence, which was only published in English in 1980, translated by Hans Kaal (in Frege 1980).

There is much here about which more could be said. The Frege–Russell Correspondence is indeed one of the most historically interesting correspondences in the history of philosophy, and it is a shame Russell was unable to do the translations himself. Much more can be said about Russell's reaction to Frege's attempted solution to the paradox. We might also puzzle over Russell's priorities in finding Frege's change of



heart over the foundations of arithmetic more troubling than his anti-Semitism—but at least he was troubled. Part of what I like most about the Dummett–Russell correspondence is how Dummett assumes that Russell will be most interested in the details of Frege’s views in philosophical logic, 30 and more years after Russell had ceased contributing to that field, and that Russell in no way proved this untrue.

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“ALL RIGHT! FOR THE LAST TIME. WHO’S THE BRAINS BEHIND THIS?”

This cartoon from the *Evening Standard* refers to the week-long prison sentence served by Russell in September 1961, following his conviction on public order charges brought after a large central London peace demonstration in commemoration of Hiroshima Day (6 August). Our Russell—the notorious crime Lord.