Unit 7 – Hypothesis Testing
Homework #12 (Unit 7 – Hypothesis Testing, part 2 of 2)

SOLUTIONS

1. Halcion is a sleeping pill that is relatively rapidly metabolized by the body and therefore having fewer hangover effects the next morning, compared to other sleeping pills. Opponents of Halcion argue that, because this agent is so rapidly metabolized by the body, patients do not sleep as long with this drug as with Dalmane. Data on 10 insomniacs, each of whom took Dalmane on one occasion and Halcion on a second, is collected. The variable measured is number of hours of sleep:

Patient	Number of Hours Sleep with Dalmane	Halcion
1	4.58	3.97
2	5.19	4.88
3	3.94	4.09
4	6.32	5.87
5	7.68	6.93
6	3.48	4.00
7	5.72	5.08
8	7.04	6.95
9	5.27	4.96
10	5.84	5.13

Do these data suggest that Halcion is not as effective as Dalmane with respect to number of hours of sleep? Carry out an appropriate statistical test and interpret your findings. You may assume that the measurements of sleep are continuous, distributed normal.

ANSWER

These data are paired measurements of an outcome measured on a continuum in a single sample. It is of interest to compare the responses of the paired measurements. The correct test is therefore a paired t-test.

For these data, a paired t-test suggests that the average difference in hours slept (Dalmane – Halcion) = 0.32 is statistically significant (one sided p-value = .018).

SOLUTION

This question is asking for a hypothesis test of the equality of two means in the setting of paired data. The data are paired because each participant was measured on two occasions, once on Dalmane and once on Halcion.

Research Question. Are sleep durations shorter on Dalmane than on Halcion?

Assumptions.

$\bar{d}$ is distributed Normal $(\mu_d\ ,\ \sigma_d^2/10)$

Differences are calculated as (Dalmane – Halcion)

For these 10 paired measurements, we have

Obs	dalmane	halcion	diff
1	4.58	3.97	0.61
2	5.19	4.88	0.31
3	3.94	4.09	-0.15
4	6.32	5.87	0.45
5	7.68	6.93	0.75
6	3.48	4.00	-0.52
7	5.72	5.08	0.64
8	7.04	6.95	0.09
9	5.27	4.96	0.31
10	5.84	5.13	0.71

H_O and H_A.

$H_O : \mu_d = 0$

$H_A : \mu_d > 0$ ("Dalmane is better than Halcion") – one sided

Test statistic is a t-score.

$$t_{score} = \left[\frac{(\bar{d}) - E[\bar{d})|H_O \text{true}]}{\hat{SE}[(\bar{d})|H_O \text{true}]} \right]$$

Obtain sample mean of the differences, $\bar{d}$

$$\bar{d} = \frac{\sum_{i=1}^{10} d_i}{10} = \left[\frac{0.61 + 0.31 + \ldots + 0.71}{10} \right] = 0.32$$

Preliminary – Obtain sample variance of the differences, S_d^2

$$S_d^2 = \frac{\sum_{i=1}^{10}\left(d_i - \bar{d}\right)^2}{(n-1)} = \frac{\sum_{i=1}^{10}\left(d_i - 0.32\right)^2}{9} = \frac{(0.61-0.32)^2 + \ldots + (0.71-0.32)^2}{9} = 0.1688889$$

Obtain the estimated standard error, $\hat{SE}[\ \bar{d}\ |\ H_O \text{ true}]$

$$\hat{SE}[\ \bar{d}\ |\ H_O \text{ true}] = \sqrt{\frac{S_d^2}{n}} = \sqrt{\frac{0.1688889}{10}} = 0.1299573$$

Putting these all together, the solution for the test statistic is

$$t_{score} = \left[\frac{(\bar{d})\text{-}E[\bar{d})|H_O\text{true}]}{\hat{SE}[(\bar{d})|H_O\text{true}]}\right] = \left[\frac{0.32\ \text{-}\ 0}{0.1299573}\right] = 2.4623$$

Degrees of freedom = (n-1) = (10-1) = 9.

<u>*"Evaluation" rule*</u>.

The likelihood of these findings or ones more extreme if H_O is true is
$\text{p-value} = \Pr\left[\left(\bar{d}\right) \geq 0.32\ |\ H_O \text{ true}\right]$.

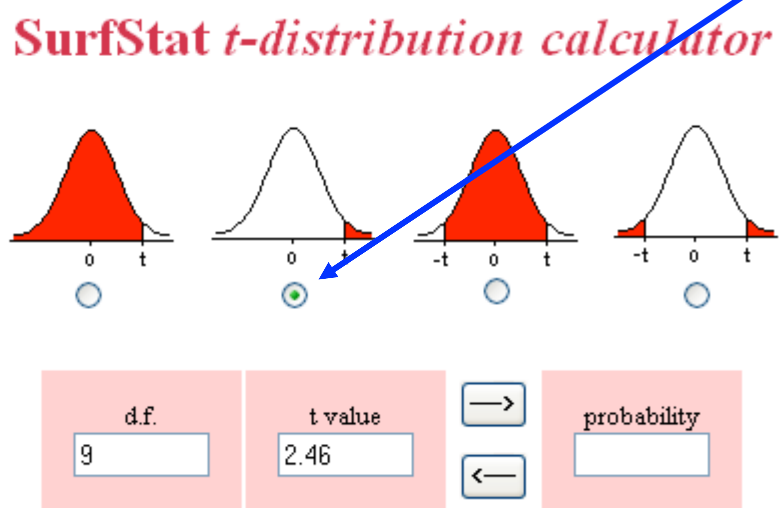
<u>*Calculations*</u>.

$\text{p-value} = \Pr\left[t_{score} \geq 2.46\right]$ where degrees of freedom = 9

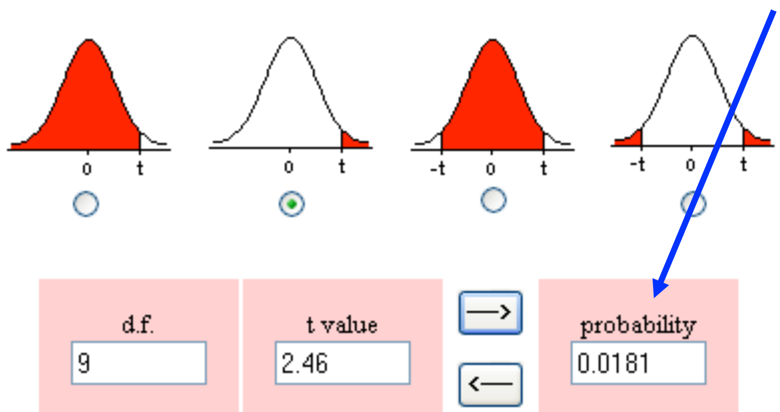
=.018

SurfStat t-distribution calculator to obtain the achieved level of significance (p-value)
http://www.anu.edu.au/nceph/surfstat/surfstat-home/tables/t.php

Enter the following, being sure to click on the radio dial for a RIGHT TAIL



After pressing the RIGHT ARROW, you should obtain 0.0181 in the probability box.



"Evaluate".
The assumption of the null hypothesis H_O (duration of sleep is the same with both drugs) has led to an unlikely result. Under the null hypothesis the chance that the difference in average hours slept is as great or greater than 0.32 hours is about 2 in 100. "2 chances in 100" is "unlikely" enough that I warrants rejecting the null hypothesis. Put another way, in this sample, the average difference of 0.32 hours is statistically significant.

2. For the Halcion versus Dalmane data in Exercise 1, construct a 99% confidence interval estimate of discrepancy in the efficacies of the two drugs. Compare this to the acceptance region that would have been obtained had you constructed a statistical test with type I error pre-specified at 0.01.

ANSWER The 99% confidence interval is (-0.10, 0.74).
The acceptance region is $\bar{d} < 0.3666$.

SOLUTION

Solution for the 99% CI is as follows

$$\bar{d} = 0.32 \qquad \hat{SE}(\bar{d}) = \frac{S_d}{\sqrt{n}} = 0.1299573$$

$df=(n-1)=9$ $\qquad t_{1-\alpha/2;df} = t_{.995;9} = 3.25$ from the calculator on the web above.

$$99\% \text{ CI for } \mu_d = \bar{d} \pm (t_{.995;DF=9}) \hat{SE}(\bar{d})$$

$$= (0.32) \pm (3.25)(0.1299573) = (-0.1024, +0.7424)$$

Solution for the acceptance region of a one sided test with alpha = .01 is obtained by reasoning as follows

Rejection occurs for t-score $\geq$ the 99th percentile of a student's t on df=9 →

Rejection occurs for t-score $\geq t_{.99;\, df=9}$ →

Acceptance occurs for t-score $< t_{.99;\, df=9}$ →

Substituting in the definition of a t-score allows us to write this equivalently as

Acceptance occurs for $\dfrac{\bar{d}-0}{\hat{SE}(\bar{d})} < t_{.99;df=9}$ By plugging in numbers for the SE and the percentile →

Acceptance occurs for $\dfrac{\bar{d}}{0.1299573} < 2.821$ where I used the calculator on the web as before →

Acceptance occurs for $\bar{d} < 0.3666$

Comparison
These two regions overlap but are not identical. They are not identical because the confidence interval is two sided whereas the acceptance region is one sided.