Unit 6
The Bernoulli and Binomial Distributions
Practice Quiz
SOLUTIONS

1. According to a recent poll, 41% of US citizens approved of the job the President was doing at the time. Assume that this proportion was actually true for the whole of the US population.

   (a) Suppose a simple random sample of 10 citizens was selected. What is the probability that a majority (more than 5) approved of the job that the President was doing?

   Answer: .18

   Solution:

   Define X is the number of approve. We need to find P(X> 5).

   Recall that binomial distribution is given by

   $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, where p is the population parameter for the probability of success, n is the number of trials and x is the number of successes.

   We have, p=0.41, n=10 and x>5.

   P(X> 5) = P(x=6) + P(X=7) + P(x=8) + P(X=9) +P(x=10)

   $$= \binom{10}{6}(0.41)^6(1-0.41)^{10-6} + \binom{10}{7}(0.41)^7(1-0.41)^{10-7} + \binom{10}{8}(0.41)^8(1-0.41)^{10-8} +$$
   $$+ \binom{10}{9}(0.41)^9(1-0.41)^{10-9} + \binom{10}{10}(0.41)^{10}(1-0.41)^{10-10}$$

   =0.1834 **Yikes. That was a lot of hand calculation. Fortunately, we can let the computer do the work**

(b) Suppose a simple random sample of 50 citizens was selected. What is the probability that a majority (more than 25) approved of the job that the President was doing?

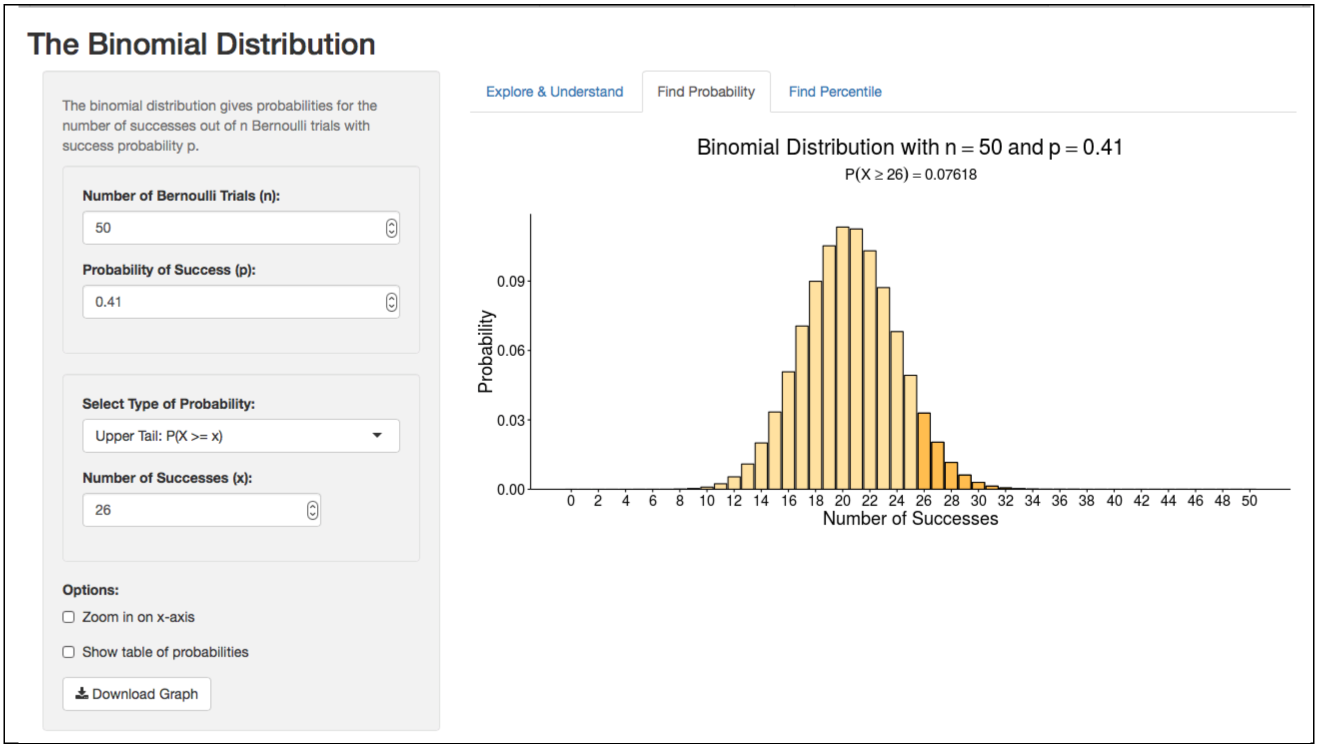
Answer: .08

Solution:

$P(X > 25)$ **= $P(X \geq 26)$**= 0.0761 **Note: X>25 is the same as $X \geq 26$. This is handy on next page**

This result can be obtained as in part (a). A more practical approach would be to use a probability calculator applet on http://www.artofstat.com/webapps.html

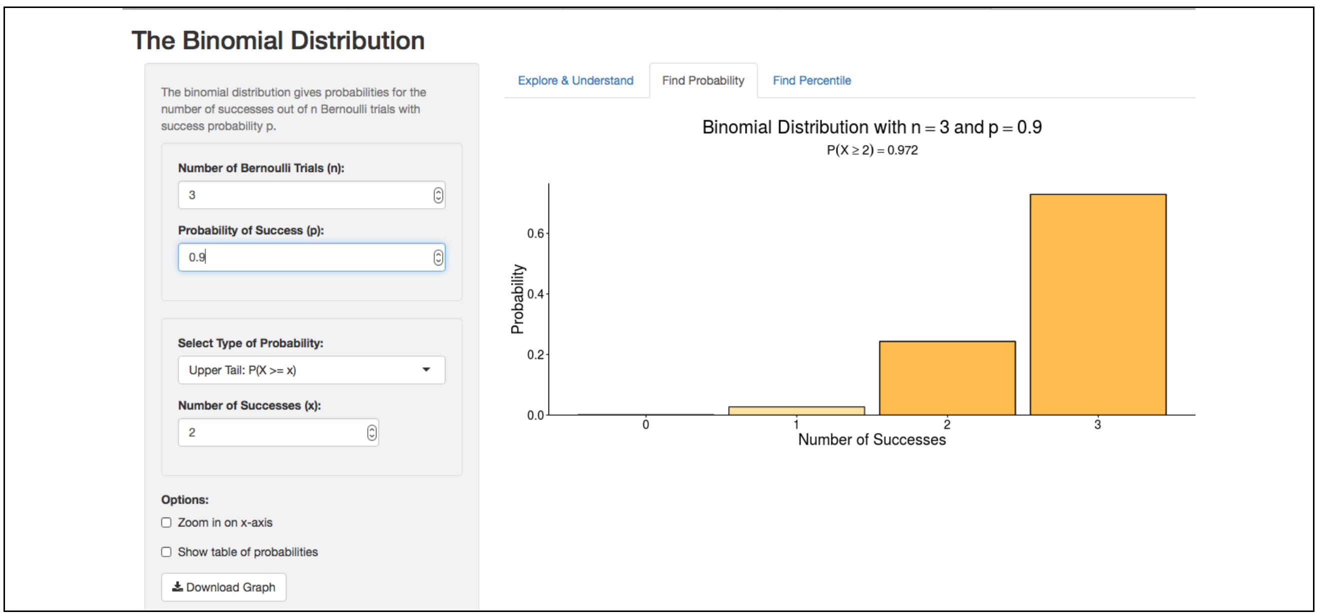
Click on the tab **"Find Probability"**, then choose as your inputs the following: 1) Number of Bernoulli trials = **50**, 2) Probability of success, p = **0.41** 3) Select type of probability **Upper tail P(X >= x)** and 4) Number of successes (x) = **26** You will then see:



2. The probability that a person suffering from a migraine headache will obtain relief from a particular drug is 0.9. Three randomly selected sufferers from migraine headache are given the drug. Find the probability that the number obtaining relief will be 2 or 3.

Answer: .97
Solution:
P(X>=2) = P(x=2) + P(x=3) = 0.9720

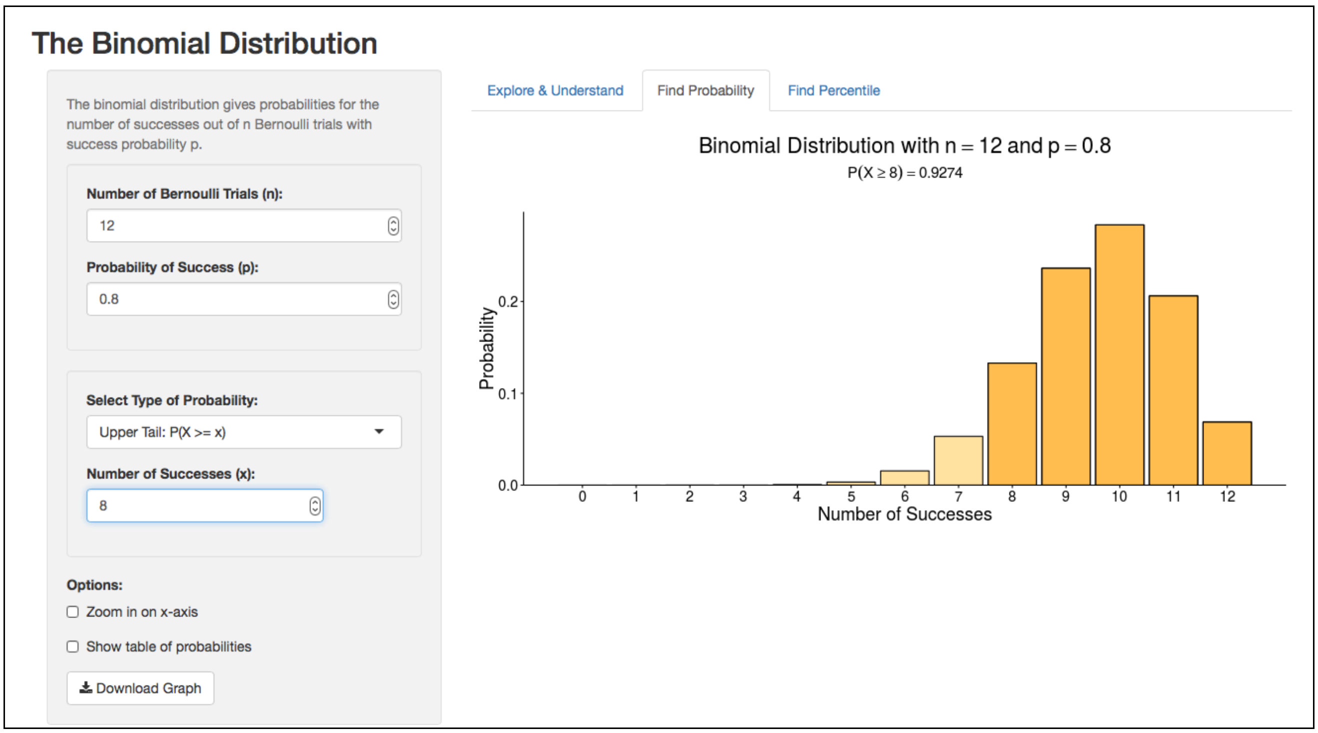
$$P(X >= 2) = \binom{3}{2}(0.9)^2(1-0.9)^{3-2} + \binom{3}{3}(0.9)^3(1-0.9)^{3-3} = 0.9720$$



3. A home security system has a 80% reliability rate, meaning that it goes off 80% of the time when there is a burglary. Suppose that 12 homes equipped with this system experience an attempted burglary. What is the probability that more than 7 alarms go off?

Answer: .93
Solution:
$P(X>7) = 0.9274$

$P(X>7) = \mathbf{P(X\geq 8)} = P(X=8)+ P(X=9)+ P(X=10)+ P(X=11)+ P(X=12)$



4. ***(Source: Virtual Lab) .*** The common form of hemophilia is due to a defect on the X chromosome (one of the two chromosomes that determine gender). We will let "h" denote the defective gene, linked to hemophilia, and H the corresponding normal gene. Women have two X chromosomes, and "h" is recessive. Thus, a woman with gene type HH is normal, a woman with gene type "hH" or "Hh" is free of disease but is a carrier; and a woman with gene type "hh" has the disease. A man has only one X chromosome (the other sex chromosome, the Y chromosome, plays no role in the disease. A man with gene type h has hemophilia and a man with gene type H is healthy.

Mother XX	**Father X**
HH – Normal Hh – Carrier hH – Carrier hh - Hemophilia	H - Normal h - Hemophilia

(a) Suppose that a mother is a carrier and the father is healthy. They have a son. What is the probability that the son will have hemophilia? Will be healthy?

Answer: Pr[son is hemophiliac] = .5 Pr[son is healthy] = .5
Solution:
If child is a son, then father contributed a Y chromosome. The Y plays no role in disease. Thus son's chances of H and h are both 0.5

(b) Suppose that a mother is a carrier and the father has hemophilia. They have a daughter. What is the probability that the daughter will have hemophilia? Will be a carrier?

Answer: Pr[daughter is hemophiliac] = .5 Pr[daughter is carrier] = .5
Solution:
If child is a daughter, then father contributed an X chromosome. If it is known that the father is a hemophiliac, then we know he contributed an h with probability 1. If the mother is a carrier, then she is hH (or other way around) plays no role in disease. Thus daughter is hh or hH, each with probability .5