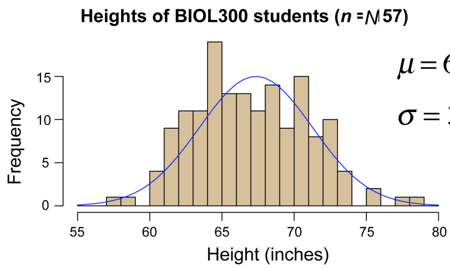
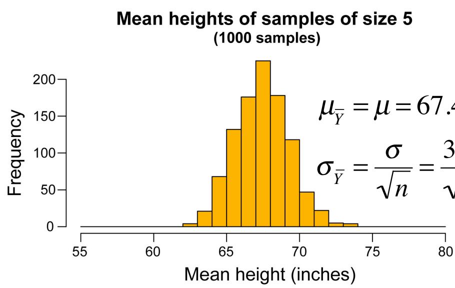


$$\mu = 67.4$$
$$\sigma = 3.9$$



$$\mu_{\bar{Y}} = \mu = 67.4$$
$$\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{3.9}{\sqrt{5}} = 1.7$$

$\bar{Y}$ is normally distributed whenever:
Y is normally distributed
or
n is large

Inference about means

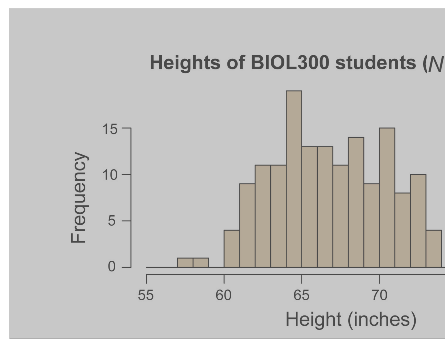
Because $\bar{Y}$ is normally distributed, we can convert its distribution to a standard normal distribution:

$$Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}} = \frac{\bar{Y} - \mu}{\sigma / \sqrt{n}}$$

This would give a probability distribution of the difference between a sample mean and the population mean.

But... We don't know σ...

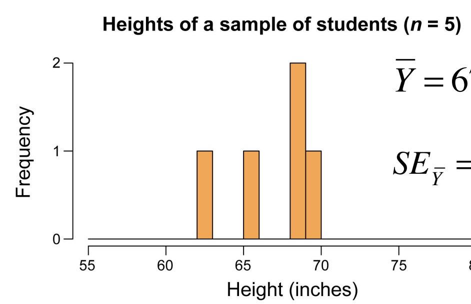
However, we do know s, the standard deviation of our sample. We can use that as an estimate of σ.



$$\mu = 67.4$$
$$\sigma = 3.9$$

In most cases, we don't know the real population distribution.

We only have a sample.



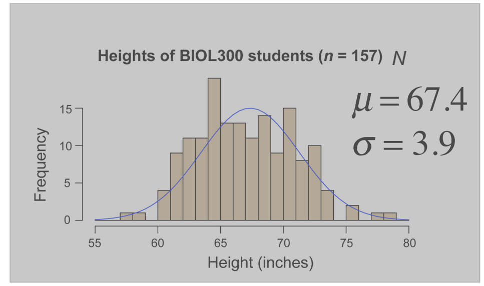
$$\bar{Y} = 67.1 \qquad s = 3.1$$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} = \frac{3.1}{\sqrt{5}} = 1.4$$

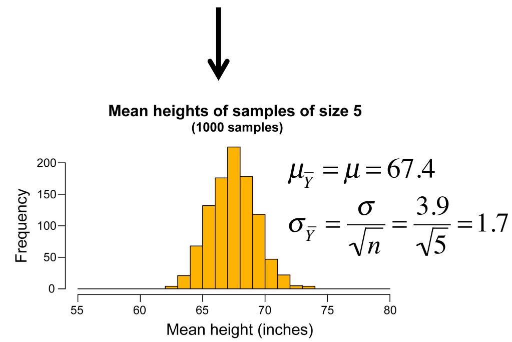
We use this as an estimate of $\sigma_{\bar{Y}}$

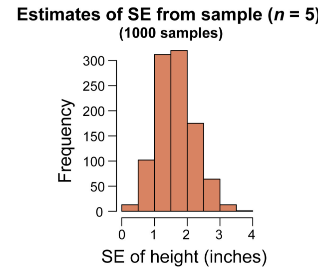
A good approximation to the standard normal is then:

$$t = \frac{\bar{Y} - \mu}{SE_{\bar{Y}}} = \frac{\bar{Y} - \mu}{s/\sqrt{n}}$$

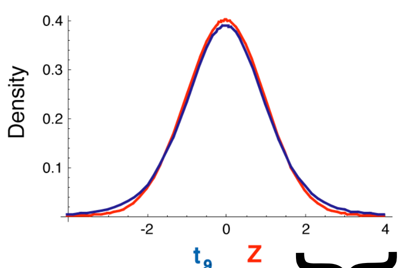


When we take a random sample, we are drawing a sample mean and a sample variance from probability distributions.





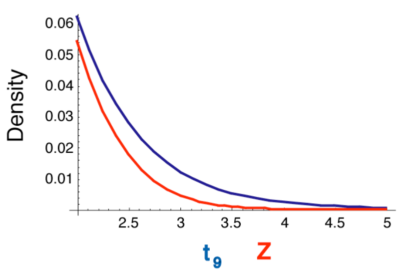
t has a Student's *t* distribution



$$\boxed{Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}}}$$

$$\boxed{t = \frac{\bar{Y} - \mu}{SE_{\bar{Y}}}}$$

Discovered by William Gossett, of the Guinness Brewing Company



Degrees of freedom

df = *n* - 1

We use the *t*-distribution to calculate a confidence interval of the mean

$$-t_{\alpha(2),df} < \frac{\bar{Y}-\mu}{SE_{\bar{Y}}} < t_{\alpha(2),df}$$

We rearrange the above to generate:

$$\bar{Y} - t_{\alpha(2),df} SE_{\bar{Y}} < \mu < \bar{Y} + t_{\alpha(2),df} SE_{\bar{Y}}$$

Another way to express this is: $\bar{Y} \pm SE_{\bar{Y}}\, t_{\alpha(2),df}$

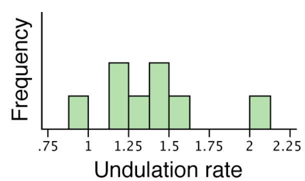
95% confidence interval for a mean

Example:
Paradise flying snakes

Undulation rates (in Hz)

0.9, 1.4, 1.2, 1.2, 1.3, 2.0, 1.4, 1.6

Estimate the mean and standard deviation



$$\bar{Y} = 1.375$$

$$s = 0.324$$

$$n = 8$$

Find the standard error

$$\bar{Y} \pm SE_{\bar{Y}}\, t_{\alpha(2),df}$$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} = \frac{0.324}{\sqrt{8}} = 0.115$$

Find the critical value of *t*

$$df = n - 1 = 7$$

$$t_{\alpha(2),df} = t_{0.05(2),7}$$
$$= 2.36$$

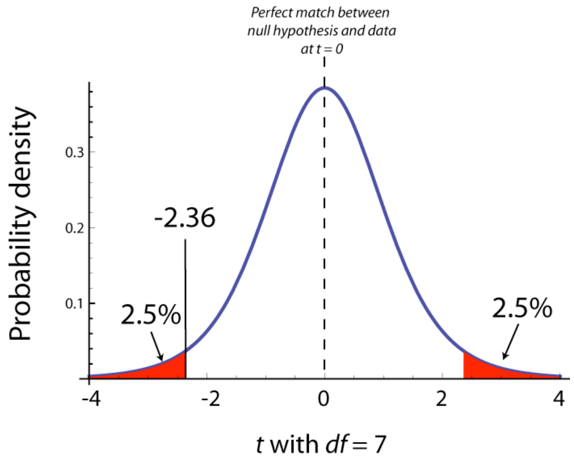


Table C: Student's *t* distribution

df								
	$\alpha(2)$:	**0.2**	**0.10**	**0.05**	**0.02**	**0.01**	**0.001**	**0.0001**
df	$\alpha(1)$:	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**	**0.0005**	**0.00005**
1		3.08	6.31	12.71	31.82	63.66	636.62	6366.20
2		1.89	2.92	4.30	6.96	9.92	31.60	99.99
3		1.64	2.35	3.18	4.54	5.84	12.92	28.00
4		1.53	2.13	2.78	3.75	4.60	8.61	15.54
5		1.48	2.02	2.57	3.36	4.03	6.87	11.18
6		1.44	1.94	2.45	3.14	3.71	5.96	9.08
7		1.41	1.89	$\boxed{2.36}$	3.00	3.50	5.41	7.88
8		1.40	1.86	2.31	2.90	3.36	5.04	7.12
9		1.38	1.83	2.26	2.82	3.25	4.78	6.59

Putting it all together...

$$\bar{Y} \pm SE_{\bar{Y}}\, t_{\alpha(2),df} = 1.375 \pm 0.115\ (2.36)$$
$$= 1.375 \pm 0.271$$

$$1.10 < \mu < 1.65$$

(95% confidence interval)

99% confidence interval

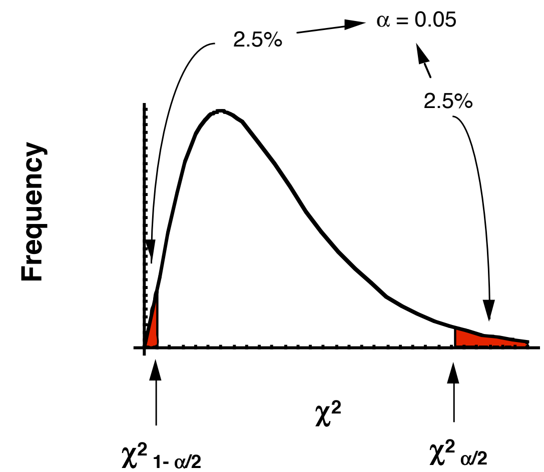
$$t_{\alpha(2),df} = t_{0.01(2),7} = 3.50$$

$$\bar{Y} \pm SE_{\bar{Y}}\ \ t_{\alpha(2),df} = 1.375 \pm 0.115\ (3.50)$$
$$= 1.375 \pm 0.403$$

$$0.97 < \mu < 1.78$$

Confidence interval for the variance

$$\frac{df \quad s^2}{\chi^2_{\frac{\alpha}{2},df}} \le \sigma^2 \le \frac{df \quad s^2}{\chi^2_{1-\frac{\alpha}{2},df}}$$



95% confidence interval for the variance of flying snake undulation rate

$$\frac{df \quad s^2}{\chi^2_{\frac{\alpha}{2},df}} \le \sigma^2 \le \frac{df \quad s^2}{\chi^2_{1-\frac{\alpha}{2},df}}$$

df = n - 1 = 7

$s^2 = (0.324)^2 = 0.105$

$$\chi^2_{\frac{\alpha}{2},df} = \chi^2_{0.025,7} = 16.01$$

$$\chi^2_{1-\frac{\alpha}{2},df} = \chi^2_{0.975,7} = 1.69$$

Table A

df / *X*	0.999	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.001
1	1.6 E-6	3.9E-5	0.00016	0.00098	0.00393	3.84	5.02	6.63	7.88	10.83
2	0	0.01	0.02	0.05	0.1	5.99	7.38	9.21	10.6	13.82
3	0.02	0.07	0.11	0.22	0.35	7.81	9.35	11.34	12.84	16.27
4	0.09	0.21	0.3	0.48	0.71	9.49	11.14	13.28	14.86	18.47
5	0.21	0.41	0.55	0.83	1.15	11.07	12.83	15.09	16.75	20.52
6	0.38	0.68	0.87	1.24	1.64	12.59	14.45	16.81	18.55	22.46
7	0.6	0.99	1.24	$\boxed{1.69}$	2.17	14.07	$\boxed{16.01}$	18.48	20.28	24.32
8	0.86	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.95	26.12

95% confidence interval for the variance of flying snake undulation rate

$$\frac{df\ s^2}{\chi^2_{\frac{\alpha}{2},df}} \le \sigma^2 \le \frac{df\ s^2}{\chi^2_{1-\frac{\alpha}{2},df}}$$

$$\frac{7\ (0.324)^2}{16.01} \le \sigma^2 \le \frac{7\ (0.324)^2}{1.69}$$

$$0.0459 \le \sigma^2 \le 0.435$$

95% confidence interval for the *standard deviation* of flying snake undulation rate

$$\sqrt{\frac{df\ s^2}{\chi^2_{\frac{\alpha}{2},df}}} \le \sigma \le \sqrt{\frac{df\ s^2}{\chi^2_{1-\frac{\alpha}{2},df}}}$$

$$\sqrt{0.0459} \le \sigma \le \sqrt{0.435}$$

$$0.21 \le \sigma \le 0.66$$

One-sample *t*-test

The *one-sample* t-*test* compares the mean of a random sample from a normal population with the population mean proposed in a null hypothesis.

Test statistic for one-sample *t*-test

$$t = \frac{\bar{Y} - \mu_0}{s / \sqrt{n}}$$

μ_0 is the mean value proposed by H_0

Hypotheses for one-sample *t*-tests

H_0 : The mean of the population is μ_0.

H_A: The mean of the population is not μ_0.

Example: Human body temperature

H_0 : Mean healthy human body temperature is 98.6ºF.

H_A: Mean healthy human body temperature is not 98.6ºF.

Human body temperature

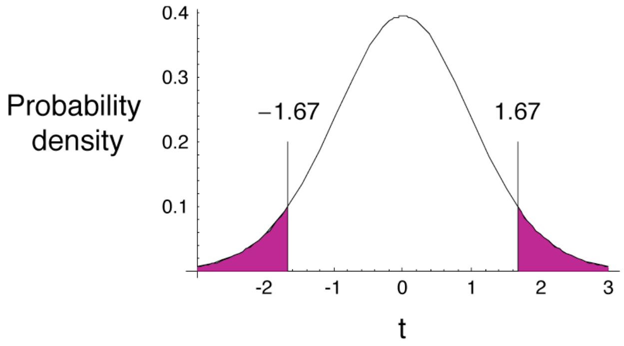
$$n = 24$$
$$\bar{Y} = 98.28$$
$$s = 0.940$$

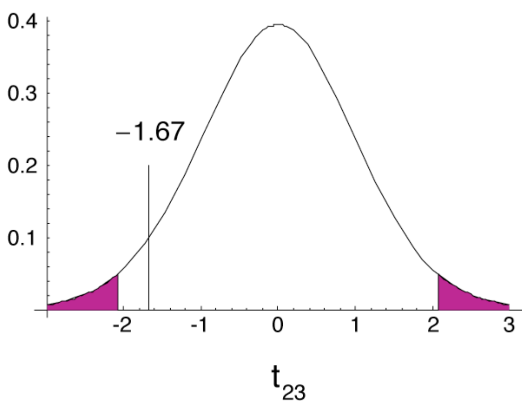
$$t = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}} = \frac{98.28 - 98.6}{0.940/\sqrt{24}} = -1.67$$

Degrees of freedom

df = n-1 = 23

Comparing *t* to its distribution to find the *P*-value





A portion of the *t* table

df	α(1) =0.1 α(2)=0.2	α(1) =0.05 α(2)=0.10	α(1) =0.025 α(2)=0.05	α(1) =0.01 α(2)=0.02	α(1) =0.005 α(2)=0.01
...	...	...	...	...	...
20	1.33	1.72	2.09	2.53	2.85
21	1.32	1.72	2.08	2.52	2.83
22	1.32	1.72	2.07	2.51	2.82
23	1.32	1.71	$\boxed{2.07}$	2.5	2.81
24	1.32	1.71	2.06	2.49	2.8
25	1.32	1.71	2.06	2.49	2.79

-1.67 is closer to 0 than -2.07, so $P > 0.05$.

With these data, we cannot reject the null hypothesis that the mean human body temperature is 98.6.

Body temperature revisited: *n* = 130

$$n = 130$$
$$\bar{Y} = 98.25$$
$$s = 0.733$$

$$t = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}} = \frac{98.25 - 98.6}{0.733/\sqrt{130}} = -5.44$$

Body temperature revisited: *n* = 130

$$t = -5.44$$

$$t_{0.05(2),129} = \pm 1.98$$

t is further out in the tail than the critical value, so we could reject the null hypothesis. Human body temperature is *not* 98.6ºF.

One-sample *t*-test: Assumptions

- The variable is normally distributed.
- The sample is a random sample.