## Introduction

This is a supplement to help you with three of the most common challenges in understanding and performing statistical hypothesis tests:

1. **Translation**
**"I don't know how to translate the wording of the question into knowing what test I should be performing."**

2. Writing down the **null ($H_0$)** and **alternative ($H_A$)** hypotheses.
**"I'm not sure which is the null and which is the alternative and I don't know whether to do a one sided or a two sided p-value calculation."**

3. Knowing which **probability distribution calculator** to use.
**"I can't keep track of this."**

## 1. Tips for Translation

BIOSTATS 540 (Introductory Biostatistics) provides an introduction to just a limited number of hypothesis testing scenarios. So no worries. The sample is either from one sample or two, the data are either paired or independent, and the distribution of the variable is either normal or binomial.

***TIP #1 -*** As you read each statement of the problem, think systematically and identify the following:

| | Question | Answer |
|---|---|---|
| 1 | What is the **design**?<br>ONE group<br>PAIRED<br>TWO independent groups | |
| 2 | What **type of outcome data** is this?<br>CONTINUOUS(normally distributed) with:<br>* Population variance(s) known<br>* Population variance(s) NOT known<br>DISCRETE (binomial outcome data) | |
| 3 | What is the **focus**?<br>MEAN (one or two)<br>VARIANCE (one or two)<br>BINOMIAL EVENT PROB (one or two) | |
| 4 | **One** sided or **Two** sided? | |
| | | |

## 2. Writing down the null ($H_O$) and alternative ($H_A$) hypotheses.

Remember. Most of the time *(there are exceptions, of course, but we'll get to these)*, the investigator is seeking to advance the alternative hypothesis (e.g., - the new treatment is great). The null hypothesis is the "nothing is going on" hypothesis (as the term "null" suggests!)

***TIP #2 -*** Write down the **alternative hypothesis first**. Take care to indicate whether this is one sided or two sided (see tip #3 next).

***TIP #3 -*** To discern one versus two sided, re-read each statement of the problem, looking for and then **highlighting, the words** that tell you whether the test to do should be **one sided** or **two sided**.

| Words that Suggest Test is ONE sided | Words that Suggest Test is TWO sided |
|---|---|
| less than<br>reduced<br>greater<br>improved | different<br>has changed |
| | |

## 3. Knowing Which Probability Calculator to Use.

*In a nutshell*: If the test is of a mean or difference of means, the correct calculator is the Normal(0,1) or Student t. If the test is of a variance, the correct calculator is chi square. If the test is of a ratio of variances, the correct calculator is the F distribution. If you go back to the table of contents, you could write these in. Here is what you would get, highlighted in ***bold green italics***.

### 1 Group, Outcome Continuous and Distributed Normal

6. **Test for $\mu$, $\sigma^2$ Known** ***calculator: Normal(0,1)***
7. **Test for $\mu$, $\sigma^2$ Known – Critical Region Approach** ***calculator: Normal(0,1)***
8. **Test for $\mu$, $\sigma^2$ Unknown** ***calculator: Student t***
9. **Test for $\sigma^2$** ***calculator: Chi Square***

### Paired Data, Outcome Continuous and Distributed Normal

10. **Test for $\mu_{DIFFERENCE}$ – Paired Data Setting** ***CHOOSE ONE***
    ***calculator: Normal(0,1) if population variance known***
    ***calculator: Student t if population variance is NOT known***

### 2 Independent Groups, Outcome Continuous and Distributed Normal

11. **Test for $[\mu_1 - \mu_2]$** ***CHOOSE ONE***
    ***calculator: Normal(0,1) if population variances are known***
    ***calculator: Student t if population variances are NOT known***

12. **Test for Equality of Two Variances ($\sigma_1^2 / \sigma_2^2$)** ***calculator: F distribution***

### 1 Group, Outcome Discrete and Distributed Binomial

13. **Test for Proportion $\pi$** ***CHOOSE ONE***
    - **13.1 Exact Test** ***calculator: Exact Binomial***
    - **13.2 Normal Approximation Test** ***calculator: Normal(0,1)*.**

### 2 Independent Groups, Outcome Discrete and Distributed Binomial

14. **Test for $[\pi_1 - \pi_2] = 0$** ***calculator: Normal(0,1)***

## Examples

### Example #1

The personnel department of a large company would like to determine if the amount of time it takes for employees to arrive at work. A random sample of 12 employees is selected and the time in minutes is recorded, with the following results:

15 30 50 60 25 65 45 90 75 50 50 20

At the .01 level of significance, is there evidence that the average travel time of employees is less than 60 minutes?

***Solution:***

| | Question | Answer |
|---|---|---|
| 1 | What is the **design**?<br>ONE group<br>PAIRED<br>TWO independent groups | ***ONE group*** |
| 2 | What **kind** of data is this?<br>CONTINUOUS(normally distributed) with:<br>* Population variance(s) known<br>* Population variance(s) NOT known<br>DISCRETE (binomial outcome data) | ***CONTINUOUS, normal with variance NOT known.*** |
| 3 | What is the **focus**?<br>MEAN (one or two)<br>VARIANCE (one or two)<br>BINOMIAL EVENT PROB (one or two) | ***MEAN of one population*** |
| 4 | One sided or Two sided? | ***ONE sided because of wording less than 60 minutes*** |
| | | |

**Correct Test to Use:** **One Sample t test for μ, where $\sigma^2$ is not known.**

**Null:** $\mu = 60$

**Alternative:** $\mu < 60$, **one sided**

**Notes 9. ONE Sample:** page 19

## Example #2

An auditor for the Department of Energy wishes to study the price of unleaded gasoline per gallon in NY city. A random sample of 50 gas stations was selected with the following results: $\bar{X} = \$1.36$ and $S = \$0.07$ Is there evidence that the average price of unleaded gasoline is different from \$1.30 per gallon?

***Solution:***

| | Question | Answer |
|---|---|---|
| 1 | What is the **design**?<br>ONE group<br>PAIRED<br>TWO independent groups | ***ONE group*** |
| 2 | What **kind** of data is this?<br>CONTINUOUS(normally distributed) with:<br>* Population variance(s) known<br>* Population variance(s) NOT known<br>DISCRETE (binomial outcome data) | ***CONTINUOUS, normal with variance NOT known.*** |
| 3 | What is the **focus**?<br>MEAN (one or two)<br>VARIANCE (one or two)<br>BINOMIAL EVENT PROB (one or two) | ***MEAN of one population*** |
| 4 | One sided or Two sided? | ***TWO sided because of wording different from \$1.30 per gallon*** |
| | | |

**Correct Test to Use:** **One Sample t test for $\mu$, where $\sigma^2$ is not known.**
**Null:** $\mu = \$1.30$
**Alternative:** $\mu \neq \$1.30$ **two sided**
**Notes 9. ONE Sample:** **page 19**

## Example #3

A consumer reporting agency wished to determine whether an "unknown brand" calculator sells at a lower price than the "famous brand" calculator of the same type. A random sample of eight stores was selected and the prices (at the stores) of each of the two calculators were recorded with the following results.

| Store | Unknown Brand | Famous Brand |
|---|---|---|
| 1 | 10 | 11 |
| 2 | 8 | 11 |
| 3 | 7 | 10 |
| 4 | 9 | 12 |
| 5 | 11 | 11 |
| 6 | 10 | 13 |
| 7 | 9 | 12 |
| 8 | 8 | 10 |

At the .01 level of significance, is there evidence that the unknown brand sells for a lower price?

***Solution:***

| | Question | Answer |
|---|---|---|
| 1 | What is the **design**?<br>ONE group<br>PAIRED<br>TWO independent groups | ***PAIRED*** |
| 2 | What **kind** of data is this?<br>CONTINUOUS(normally distributed) with:<br>* Population variance(s) known<br>* Population variance(s) NOT known<br>DISCRETE (binomial outcome data) | ***CONTINUOUS, normal with variance NOT known.*** |
| 3 | What is the **focus**?<br>MEAN (one or two)<br>VARIANCE (one or two)<br>BINOMIAL EVENT PROB (one or two) | ***MEAN of one population*** |
| 4 | One sided or Two sided? | ***ONE sided because of wording sells for a lower price*** |
| | | |

**Correct Test to Use:** **Paired data one Sample t test for $\mu_{DIFFERENCE}$ where $\sigma^2_{DIFFERENCE}$ is not known.**

**Null:** $\mu_{DIFFERENCE} = 0$ **for difference defined [Unknown – Famous]**

**Alternative:** $\mu_{DIFFERENCE} < 0$ **one-sided**

**Notes 9. ONE Sample:** **page 39**

## Example #4

Olmstead [1953] conducted a study of cases with convulsive disorders. Among the cases there were 82 males and 118 females. At the 5% significance level, test the hypothesis that a case is equally likely to be of either sex.

***Solution:***

| | Question | Answer |
|---|---|---|
| 1 | What is the **design**?<br>ONE group<br>PAIRED<br>TWO independent groups | ***ONE group*** |
| 2 | What **kind** of data is this?<br>CONTINUOUS(normally distributed) with:<br>* Population variance(s) known<br>* Population variance(s) NOT known<br>DISCRETE (binomial outcome data) | ***DISCRETE, Binomial.*** |
| 3 | What is the **focus**?<br>MEAN (one or two)<br>VARIANCE (one or two)<br>BINOMIAL EVENT PROB (one or two) | ***Binomial event probability*** |
| 4 | One sided or Two sided? | ***TWO sided because of wording equally likely*** |
| | | |

**Correct Test to Use:** **One Sample Test for Proportion π.**
**Null:** **$\pi = 0.50$**
**Alternative:** **$\pi \neq 0.50$ two sided**
**Notes 9. ONE Sample:** **page 51**

## Example #5

Holtzman et al [1975] conducted a study of a diet designed to reduce the phenylalanine level in children with phenylketonuria (PKU). After obtaining informed consent, eligible children of 4 years of age were randomly divided into two groups. Children in one group received the new, experimental, diet. Children in the other group followed their usual diet. Study investigators measured the phenylalanine levels (mg/dl) at the end of the study with the following results.

| | Experimental Diet | Usual Diet |
|---|---|---|
| Number in group, n | 4 | 5 |
| Mean phenylalanine (mg/dl) | 16.7 | 26.9 |
| Standard deviation | 7.3 | 4.1 |

***Solution:***

| | Question | Answer |
|---|---|---|
| 1 | What is the **design**?<br>ONE group<br>PAIRED<br>TWO independent groups | ***TWO independent groups*** |
| 2 | What **kind** of data is this?<br>CONTINUOUS(normally distributed) with:<br>* Population variance(s) known<br>* Population variance(s) NOT known<br>DISCRETE (binomial outcome data) | ***CONTINUOUS, normal with variances NOT known.*** |
| 3 | What is the **focus**?<br>MEAN (one or two)<br>VARIANCE (one or two)<br>BINOMIAL EVENT PROB (one or two) | ***MEANS of two populations*** |
| 4 | One sided or Two sided? | ***ONE sided because investigators hope that diet will reduce phenylalaninee levels*** |
| | | |

**Correct Test to Use:** **Two Sample t test for equality of $\mu_{EXPERIMENTAL}$ and $\mu_{USUAL}$, where $\sigma^2_{EXPERIMENTAL}$ and $\sigma^2_{USUAL}$ are not known.**

**Null:** $\mu_{EXPERIMENTAL} = \mu_{USUAL}$ which is the same as $[\mu_{USUAL} - \mu_{EXPERIMENTAL}] = 0$

**Alternative:** $\mu_{EXPERIMENTAL} < \mu_{USUAL}$ or $[\mu_{USUAL} - \mu_{EXPERIMENTAL}] > 0$ **one sided**

**Notes 10. TWO Samples:** page 7