1. Let $\vec{a} = \langle -3, 1, -2 \rangle$, $\vec{b} = 2\vec{i} + 4\vec{j} - \vec{k}$, $\vec{u} = \langle 1, -1 \rangle$, and $\vec{v}$ have representation $\vec{AB}$ where $A(4, 1)$ and $B(0, 1)$ are points in $\mathbb{R}^2$.

$\vec{v} = \langle 0-4, 1-1 \rangle = \langle -4, 0 \rangle$

(a) Find the scalar and vector projections of $\vec{v}$ onto $\vec{u}$.

$$\text{proj}_{\vec{u}} \vec{v} = \left(\frac{\vec{v} \cdot \vec{u}}{|\vec{u}|} \right) \frac{\vec{u}}{|\vec{u}|} = \underbrace{\left(\frac{-4}{\sqrt{2}} \right)}_{\text{comp}_{\vec{u}} \vec{v}} \frac{\langle 1, -1 \rangle}{\sqrt{2}} = \langle -2, 2 \rangle$$

(b) Find the vector projection of $\vec{a}$ onto $\vec{b}$.

$$\text{proj}_{\vec{b}} \vec{a} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right) \frac{\vec{b}}{|\vec{b}|} = \left(\frac{\overbrace{-3 \cdot 2 + 1 \cdot 4 - 2 \cdot -1}^{=0}}{\sqrt{2^2 + 4^2 + (-1)^2}} \right) \left(\frac{\vec{b}}{\sim} \right) = \vec{0}$$

i.e. $\langle 0, 0, 0 \rangle$

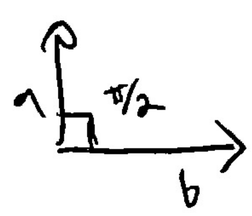
(c) Geometrically speaking, what must be true of $\vec{a}$ and $\vec{b}$? Justify your answer.

$\vec{a}$ and $\vec{b}$ are perpendicular, since $0 = \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

means $\cos\theta = 0$

$\Downarrow$

$\theta = \pi/2$



(d) Find the (smaller) angle between $\vec{u}$ and $\vec{v}$. (Please simplify your answer, when possible.)

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta \Rightarrow -4 = (\sqrt{2})(4) \cos\theta$$

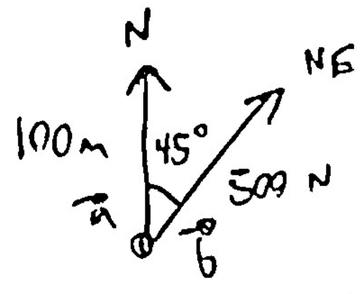
$$-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} = \cos\theta$$

So $\theta = 3\pi/4$

(e) How does the sign of the dot product relate to the (smaller) angle between the dotted vector?

Corresponds to sign of $\cos\theta$, so a positive $\cos\theta$ means an angle less than $\pi/2$ while a negative $\cos\theta$ means an angle greater than $\pi/2$.

2. A boat travels 100 meters due north ($\vec{a}$) while the wind exerts a force of 500 newtons ($\vec{b}$) toward the northeast. How much work does the wind do?



The work done by the wind is $|\vec{a}\cdot\vec{b}|$ Joules (Newton-meters)

$$|\vec{a}\cdot\vec{b}| = \left| |\vec{a}||\vec{b}| \cos\theta_{\vec{a},\vec{b}} \right|$$

$$= \left| 100\cdot 500\cdot \frac{\sqrt{2}}{2} \right| = 25000\sqrt{2} \text{ Joules}$$

3. Compute the cross product $\vec{a}\times\vec{b}$ of the two vectors $\vec{a} = \langle 1,2,3\rangle$ and $\vec{b} = \langle 2,1,2\rangle$. Then find *two unit vectors* orthogonal to both $\vec{a}$ and $\vec{b}$.

$$\langle 1,2,3\rangle \times \langle 2,1,2\rangle = \begin{matrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 2 & 3 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 \end{matrix} = (2\cdot 2 - 3\cdot 1)\hat{i} + (3\cdot 2 - 1\cdot 2)\hat{j} + (1\cdot 1 - 2\cdot 2)\hat{k}$$

$$= \langle 1,4,-3\rangle$$

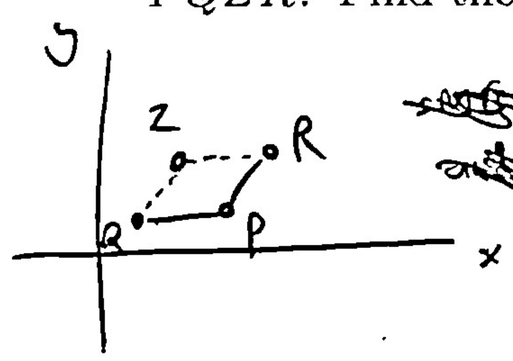
so two unit vectors orthogonal to $\vec{a}$ and $\vec{b}$ are

$$\frac{1}{\sqrt{26}}\langle 1,4,-3\rangle \text{ and } \frac{-1}{\sqrt{26}}\langle 1,4,-3\rangle$$

now $|\langle 1,4,-3\rangle| = \sqrt{(1)^2+(4)^2+(-3)^2} = \sqrt{26}$

4. Consider the points $P(3,1,0)$, $Q(1,1,0)$, and $R(4,3,0)$ in the xy-plane.

(a) What are the coordinates of the (unique) point Z that completes a parallelogram $PQZR$? Find the area of this parallelogram.



~~we need~~

In order to be a parallelogram, we need

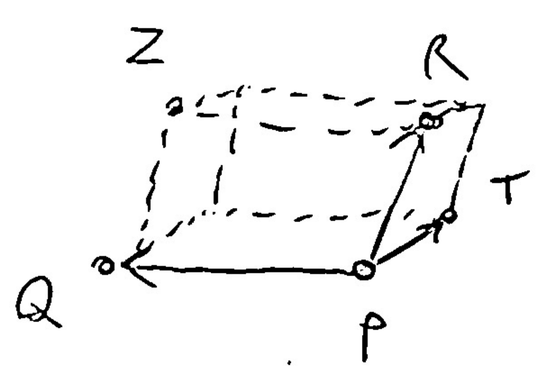
$$P_x - Q_x = 3 - 1 = 2 = \overset{4}{R_x} - Z_x \Rightarrow \underline{Z_x = 2}$$

$$R_y - P_y = 3 - 1 = 2 = Z_y - Q_y \Rightarrow \underline{Z_y = 3}$$

so $Z(2,3,0)$ is pt.

area is 4 m^2 (base · height = 2·2)

(b) Consider the additional point $T(3,2,2)$. What is the volume of the parallelepiped with adjacent edges PQ, PT, and PR?



$$\vec{PQ} = \langle 1-3, 1-1, 0-0\rangle = \langle -2,0,0\rangle$$

$$\vec{PT} = \langle 3-3, 2-1, 2-0\rangle = \langle 0,1,2\rangle$$

$$\vec{PR} = \langle 4-3, 3-1, 0-0\rangle = \langle 1,2,0\rangle$$

volume is e.g. $\left|\vec{PR}\cdot(\vec{PQ}\times\vec{PT})\right|$

$$= \left|\langle 1,2,0\rangle\cdot\langle 0,4,-2\rangle\right| = 8 \text{ un}^3$$

5. Let θ be the angle between the vectors $\vec{u} = 2\vec{i} + 3\vec{j} - 6\vec{k}$ and $\vec{v} = 2\vec{i} + 3\vec{j} + 6\vec{k}$.

(a) Use the dot product to find $\cos\theta$.

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta \Rightarrow 2\cdot 2 + 3\cdot 3 - 6\cdot 6 = (2^2 + 3^2 + 6^2)\cos\theta$$

$$\frac{-23}{49} = \cos\theta$$

(b) Use the cross product to find $\sin\theta$.

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta \Rightarrow |\langle 36, -24, 0\rangle| = 49 \sin\theta$$

$$\frac{12\sqrt{13}}{49} = \sin\theta$$

(c) Confirm that $\sin^2\theta + \cos^2\theta = 1$.

$$\left[-\frac{23}{49}\right]^2 + \left[\frac{12\sqrt{13}}{49}\right]^2 = \frac{529 + 144\cdot 13}{49^2}$$

$$\parallel$$

$$\frac{2401}{2401} = 1 \quad \checkmark$$