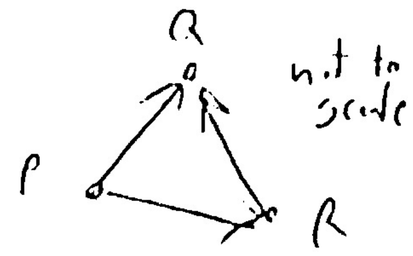
Math 233 Worksheet
Sections: 12.1-12.2



1. Classify the triangle having vertices $P(1,1,1)$, $Q(2,3,1)$, and $R(0,1,4)$ as scalene, isosceles, or equilateral. Is PQR a right triangle?

$\vec{PQ} = \langle 2-1, 3-1, 1-1 \rangle = \langle 1,2,0 \rangle$

$\vec{PR} = \langle 0-1, 1-1, 4-1 \rangle = \langle -1,0,3 \rangle$

$\vec{RQ} = \langle 2-0, 3-1, 1-4 \rangle = \langle 2,2,-3 \rangle$

$|\vec{PQ}| = \sqrt{1^2+2^2} = \sqrt{5}$

$|\vec{PR}| = \sqrt{(-1)^2+3^2} = \sqrt{10}$

$|\vec{RQ}| = \sqrt{2^2+2^2+(-3)^2} = \sqrt{17}$

Scalene since sides all different length

not right since $(\sqrt{5})^2 + (\sqrt{10})^2 \neq (\sqrt{17})^2$

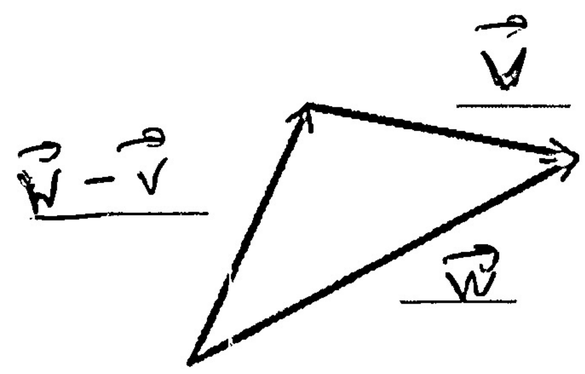
2. Does the equation $x^2 - 2x + y^2 + 2y + z^2 - 4z = 43$ represent a sphere? If so, then find the center and radius of the sphere. If not, then explain why the equation does not represent a sphere.

$(x^2 - 2x \underline{+1}) + (y^2 + 2y \underline{+1}) + (z^2 - 4z \underline{+4}) = 43 + 1 + 1 + 4$

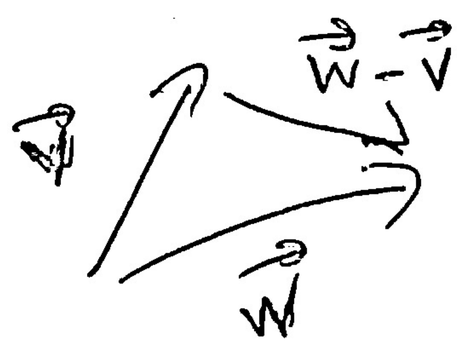
$(x-1)^2 + (y+1)^2 + (z-2)^2 = 49$

sphere w/ center $(1,-1,2)$ and radius $\sqrt{49} = 7$

3. Depicted below are the vectors $\vec{v}$, $\vec{w}$, and $\vec{w} - \vec{v}$. Label each vector appropriately. Is your answer the only possible case? If not, can you come up with another correct combination?



could also have



these are the only two possibilities

$\vec{a} = \langle 0-1, -1-0, 2-(-3)\rangle = \langle -1,-1,5\rangle$.

4. Suppose vector $\vec{a}$ represents $\vec{AB}$ where $A = (1, 0, -3)$ and $B = (0, -1, 2)$. Let $\vec{b} = 3\vec{j} - 4\vec{k}$.

   (a) What is the unit vector for $\vec{b}$?

$$|\vec{b}| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5 \quad \text{so} \quad \frac{\vec{b}}{|\vec{b}|} = \left\langle \frac{3}{5}, -\frac{4}{5}, 0 \right\rangle$$

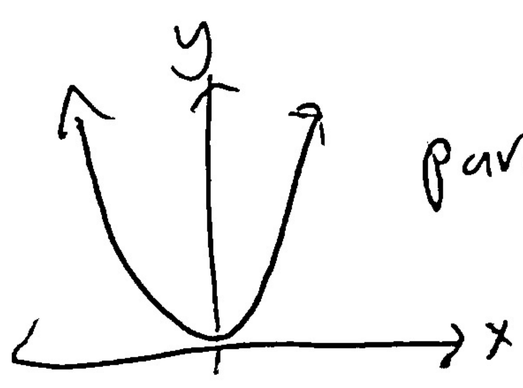
   (b) Calculate $3\vec{a} - 2\vec{b}$

$$3\langle -1,-1,5\rangle - 2\langle 3,-4,0\rangle = \langle -3-6, -3+8, 15-0\rangle$$
$$= \langle -9, 5, 15\rangle$$

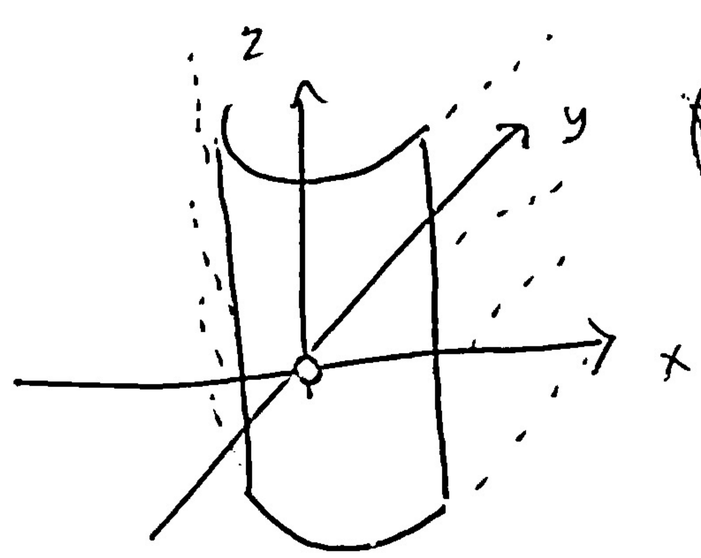
   (c) Find $|3\vec{a} - 2\vec{b}|$.

$$\| \quad \sqrt{(-9)^2 + (5)^2 + (15)^2} = \sqrt{81 + 25 + 225} = \sqrt{331}$$

5. What does the equation $y = x^2$ represent in 2D? What about in 3D?



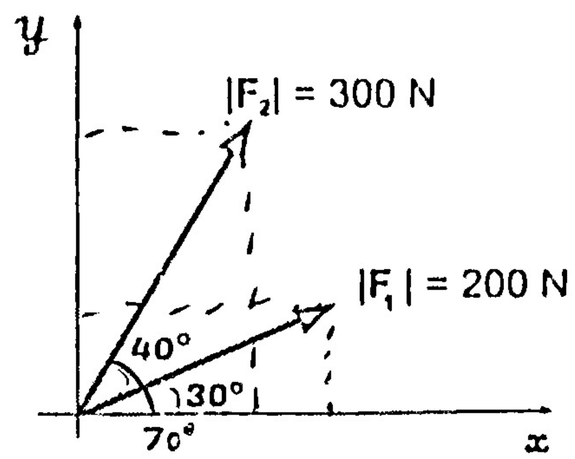
parabola in 2D



parabolic cylinder in 3D

(any trace $z = c$ is a parabola)

6. Suppose that two forces are applied to a point, as shown in the figure. Find the magnitude of the resultant force and the angle θ that it makes with the positive $x-$ axis.



$$|F_{1_x}| = 200\cos(30^\circ)\ N \qquad |F_{2_x}| = 300\cos(\overset{30^\circ+40^\circ}{\cancel{70^\circ}})\ N$$

$$|F_{1_y}| = 200\sin(30^\circ)\ N \qquad |F_{2_y}| = 300\sin(\cancel{70^\circ})\ N$$

call the resultant force G. Then using a calculator, we have

$$|G_x| = |F_{1x}| + |F_{2x}|$$
$$\approx 275.8\ N$$

$$|G_y| = |F_{1y}| + |F_{2y}|$$
$$\approx 381.9\ N$$

then

$$|G| = \sqrt{|G_x|^2 + |G_y|^2}$$
$$\approx 471.1\ N$$

and

$$\theta = \arctan\left(\frac{|G_y|}{|G_x|}\right)$$
$$\approx 0.945 \text{ radians}$$
$$\text{or } 54.16^\circ$$