Stellar Evolution

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With the understanding of the basic physical processes in stars, we now proceed to study their evolution. In particular, we will focus on discussing how such processes are related to key characteristics seen in the HRD.

1 Star Formation

From the virial theorem, $2E = -\Omega$, we have

$$\frac{3kTM}{\mu m_A} = \int_0^M \frac{GM_r}{r} dM_r \tag{1}$$

for the hydrostatic equilibrium of a gas sphere with a total mass M. Assuming that the density is constant, the right side of the equation is $3/5(GM^2/R)$. If the left side is smaller than the right side, the cloud would collapse. For the given chemical composition, ρ and T, this criterion gives the minimum mass (called Jeans mass) of the cloud to undergo a gravitational collapse:

$$M > M_J \equiv \left(\frac{3}{4\pi\rho}\right)^{1/2} \left(\frac{5kT}{G\mu m_A}\right)^{3/2}. \tag{2}$$

For typical temperatures and densities of large molecular clouds, $M_J \sim 10^5 M_\odot$ with a collapse time scale of $t_{ff} \approx (G\rho)^{-1/2}$. Such mass clouds may be formed in spiral density waves and other density perturbations (e.g., caused by the expansion of a supernova remnant or superbubble).

What exactly happens during the collapse depends very much on the temperature evolution of the cloud. Initially, the cooling processes (due to molecular and dust radiation) are very efficient. If the cooling time scale t_{cool} is much shorter than t_{ff}, the collapse is approximately isothermal. As $M_J \propto \rho^{-1/2}$ decreases, inhomogeneities with mass larger than the actual M_J will collapse by themselves with their local t_{ff}, different from the initial t_{ff} of the whole cloud. This fragmentation process will continue as long as the local t_{col} is shorter than the local t_{ff}, producing increasingly smaller collapsing subunits.

Eventually the density of subunits becomes so large that they become optically thick and the evolution becomes adiabatic (i.e., $T \propto \rho^{2/3}$ for an ideal gas), then $M_J \propto \rho^{1/2}$. As the density has to increase, the evolution will always reach a point when $M = M_J$, when we assume that a stellar object is born. From this moment on the cloud would start to evolve in hydrostatic equilibrium.

This way a giant molecular cloud can form a group of stars with their mass distribution being determined by the fragmentation process. The process depends on the physical and chemical properties of the cloud (ambient pressure, magnetic field, rotation, composition, dust fraction, stellar feedback, etc.). Much of the process is yet to be understood. We cannot yet theoretically determine the initial mass function (IMF) of stars. The classic expression for the IMF, determined empirically is the so-called Salpeter's law

$$dn/dM = CM^{-x} \tag{3}$$

where $x = 2.35$ for $M/M_\odot \geq 0.5$ and $x = 1.3$ for $0.1 \leq M/M_\odot \leq 0.5$ in the solar neighborhood.

However, whether or not the IMF is universal is a question yet to be answered. While the IMF in galactic disks in the MW and nearby galaxies seem to be quite consistent, there are good reasons and even lines of evidence suggesting different IMFs in more extreme environments (e.g., bottom-light in the Galactic center and top-cutoff in outer disks; Krumholz, M. R. & McKee, C. F. 2008, Nature, 451,1082). This has strong implications for understanding the star formation at high z, the mass to light ratio, etc.

2 Young Stellar Objects

Objects on the way to become stars, but extract energy primarily from gravitational contraction are called Young stellar objects (YSOs) here. They represent the entire stellar *system* throughout all pre-main sequence (MS) evolutionary phases.

Theoretically, the formation and evolution of a YSO may be divided into four stages (as illustrated in Fig. 2.1: (a) proto-star core formation; (b) the proto-star star builds up from the inside out, forming a disk around (core still contracts and is optically thick); (c) bipolar outflows; (d) the surrounding nebula swept away. The energy source is gravitational potential energy. While the total luminosity is comparable to the solar value, the proto-star stages have the KH time scale [$\sim 2 \times 10^7 (M/M_\odot)^2 (L/L_\odot)^{-1} (R/R_\odot)^{-1}$]. They are also fully convective and is thus homogeneous chemically.

Observationally, the proto-stars are classified into classes 0, 1, and 2, according to the ratio of infrared to optical, amount of molecular gas around, inflow/outflow, etc. Class 0 proto-stars are highly obscured and have short time scales (corresponding to the stage b?); a few are known. The class 1 and 2 proto-stars are already living partly on nuclear energy (c and d); but the total luminosity is still dominated by gravitational energy. The *low-mass* YSO prototype is T Tauri. We still know little about high-mass YSOs, which evolve very fast and interact strongly with their environments.

Signatures of YSOs:

- variability on hours and days due to temperature irregularities on both the stellar surface and disk
- emission lines from the disk and/or outflow
- more infrared luminosity due to dust emission
- high level of magnetic field triggered activities (flares, spots, corona ejection, etc) due to fast rotation and convection
- Strong X-ray emission from hot corona.

The structure of a YSO changes with its evolution. During the so-called protostar evolutionary stage, the optically thick stellar core that forms during the adiabatic contraction phase and grows during the accretion phase. The YSO is fully convective and evolves along the so-called Hayashi track in the HR diagram. During the collapse the density increases inwards. The optically thick phase is reached first in the central region, which leads to the formation of a more-or-less hydrostatic core with free falling gas surrounding it. The energy released by the core (now obeying the virial theorem) is absorbed by the envelope and radiated away as infrared radiation. Because of the heavy obscuration by the surrounding dusty gas, stars in this stage cannot be directly observed in optical and probably even in near-IR. The steady increase of the central temperature causes the dissociation of the H_2, then the ionization of H, and the first and second ionization of He. The sum of the energy involved in all these processes has to be at most equal to the energy available to the star through the virial theorem. The simple estimate gives the maximum initial radius R^i_{max} of a YSO has to be

$$\frac{R^i_{max}}{R_\odot} \approx 50\frac{M}{M_\odot} \tag{4}$$

Therefore, the luminosity can be very large and hence usually requires convection. If accretion of matter to the forming star may be neglected, the object follows a path on the HRD with the effective temperature similar to that given by

$$T_{eff} \propto \kappa_0^{-4/51}\mu^{13/51}(M/M_\odot)^{7/51}(L/L_\odot)^{1/102} \tag{5}$$

as was discussed for stellar envelopes in the last chapter. These paths as seen in Fig. 2.2 are known as "Hayashi tracks" and are those taken by T Tauri stars.

The effect of the chemical composition is reflected by the value of μ and κ_0. The increase of the metallicity (that causes an increase of the opacity) shifts the track to the lower T_{eff}, whereas an increase of the helium abundance at constant metallicity (that increases μ and decreases κ_0, because the opacity of He is lower than the hydrogen opacity) has the opposite (and less relevant) effect.

In the subsequent pre-main sequence (PMS) stage, the YSO has formed a radiative core, though still growing with time. When the radiative core appears, the star is no longer fully convective, and it has to depart from its Hayashi track, which forms the rightmost boundary to the evolution of stars in the HRD. As the center temperature increases due to the virial theorem, the path is almost horizontal on the HRD.

When the temperature reaches the order of 10^6 K, deuterium is transformed into ^{3}He by proton captures. The exact location when this happens depends on the stellar mass. In any case, the energy generation of this burning is comparably low and does not significantly change the evolution track.

Brown dwarfs, which are only able to burn deuterium (at $T \sim 10^6$ K with masses $\sim 0.05 - 0.1 M_\odot$) may still be called stars.

3 the Main Sequence

The CD-ROM that came with the text book (HKT) contains some nice and informative description and movies from stellar evolution modeling (e.g., CD-ROM/StellarEvolnDemo/index.html) They cover the Main Sequence (MS) and evolved stages (although some of the movies are missing).

Zero-age Main Sequence stars (ZAMS) are those stars who arrived at the MS recently (position point 1 in Fig. 2.5 of HKT; other points will be mentioned later).

3.1 Dependence on chemical composition

How does the metallicity of ZAMS stars affect their color and luminosity? The metallicity chiefly affects the opacity, or the amount of bound-free absorption, which only comes from metals. The smaller opacity allows the energy to escape more easily (so the star appears bluer). The lower opacity also reduces the pressure; hence the luminosity of the star needs to be increased to balance its gravity. (The effect is slightly less for higher mass stars, since the amount of energy generation is proportional to the metal abundance in CNO burning, but the smaller opacity is still the most important factor.)

But the nuclear burning also changes the abundances of elements and hence the molecular weight (μ). Here we take the luminosity evolution of the sun in the MS to illustrate how this change affects the luminosity, Following the discussion in Ch. 1, the virial theorem analysis gives the relation

$$T \propto \mu M^{2/3} \rho^{1/3} \tag{6}$$

for an ideal gas star. Furthermore, if we assume that radiative diffusion controls the energy flow, then

$$L \propto \frac{RT^4}{\kappa \rho}. \tag{7}$$

If Kramers' is the dominant opacity, then we have

$$L \propto \frac{M^{32/6} \rho^{1/6} \mu^{15/2}}{\kappa_0}. \tag{8}$$

Here the mass of the star is fixed, while κ does not vary strongly with the abundances. Neglecting the weak dependence on ρ, the above relation can be written in time-dependent form

$$\frac{L(t)}{L(0)} = \left[\frac{\mu(t)}{\mu(0)}\right]^{15/2}. \tag{9}$$

To see how μ varies with time, we assume that the bulk of the stellar interior is completely ionized and neglect the metal content that is small compared to hydrogen and helium. Then we have

$$\mu = \frac{4}{3+5X} \tag{10}$$

We can then get

$$\frac{d\mu}{dt} = -\frac{5}{4}\mu^2 \frac{dX}{dt} = \frac{5}{4}\mu^2 \frac{L}{MQ}, \tag{11}$$

where $Q = 6 \times 10^{18}$ ergs is the energy released from converting every gram of hydrogen to helium. This equation, together with Eq. 9, gives

$$\frac{dL(t)}{dt} = \frac{75}{8} \frac{\mu(0) L^{1+17/15}(t)}{MQL^{-1+17/15}(0)}, \tag{12}$$

with solution

$$L(t) = L(0)\left[1 - \frac{85}{8}\frac{\mu(0)L(0)}{MQ}t\right]^{-15/17}. \tag{13}$$

Expressing luminosities in units of the present value $L_\odot$ and the present age of 4.6×10^9 years, and letting $\mu \approx 0.6$, then gives

$$\frac{L(t)}{L_\odot} = \frac{L(0)}{L_\odot}\left[1 - 0.3\frac{L(0)}{L_\odot}\frac{t}{t_\odot}\right]^{-15/17}. \tag{14}$$

So the luminosity of the sun on the ZAMS must be $L(0) \approx 0.79L_{\odot}$ from this solution, which is very close to the value, 0.73, from the numerical model quoted above. The model shows that the core of the sun is indeed radiative and that the convection zone occupies only the outer 30% of the radius (but only 2% of the mass).

Another way to look at the above is to consider what happens as the mean molecular weight increases with time in the hydrogen-burning core. If $P \propto \rho T/\mu$, then the increase in μ must be compensated by an increase in ρT to maintain the hydrostatic equilibrium of the star. The result would then be a compression of the core with a corresponding increase in density. Further according to the virial theorem, Eq. 6, the increase of μ and ρ must lead to an increase in T, hence the energy generation rate and the total luminosity.

When the mass fraction of hydrogen in a stellar core declines to $X \sim 0.05$ (point 2 on the evolutionary track; e.g., see Fig. 2.5 in HKT), the MS phase has ended, and the star begins to undergo rapid changes.

3.2 Dependence on stellar mass

The mass of a star is the deciding parameter in the stellar evolution (see Fig. 2.4). The mass determines what the central temperature can reach, hence what nuclear reactions can occur and how fast they can run.

We can roughly estimate the mass dependence of the luminosity, following a similar arguments based on the virial theorem, hydrostatic equilibrium state, and the EoS of the ideal gas. The radiative heat transfer equation can be written as

$$L \propto M^{-1} R^4 T^4, \tag{15}$$

where κ is assumed to be constant in the nuclear burning region, while the hydrostatic equilibrium condition as

$$P \propto \frac{M^2}{R^4}. \tag{16}$$

One then find $L \propto M^{\eta}$ with $\eta = 3$ if the pressure is primarily due to the ideal gas (i.e., for stars with masses lower than $\sim 10M_{\odot}$; $T \propto P/\rho \propto M/R$) or $= 1$ if the radiation pressure dominates (for more massive stars; $T \propto P^{1/4} \propto M^{1/2}/R$). These exponents are close to the empirical measurements (e.g., $\eta \sim 3.5$ for stars of a few solar masses; Ch. 1). The small difference is due to the structure change caused by the convection, which makes the nuclear burning more efficient.

To see how T_{eff} depends on L, we assume that $\epsilon = \epsilon_0 \rho T^{\nu}$ (e.g., for CNO with $\nu = 18$) as an example. Then $L \propto \epsilon M = M^{2+\nu} R^{-(\nu+3)}$ (assuming $T \propto M/R$). Equating this to $L \propto M^3$ and then using $L \propto R^2 T_{eff}^4$, we have

$$R \propto M^{(\nu-1)/(\nu+3)} = M^{0.81} \tag{17}$$

we can obtain

$$\left(\frac{T_{eff}}{T_{eff,\odot}}\right) = \left(\frac{L}{L_\odot}\right)^{\frac{(\nu+11)}{12(\nu+3)}} = \left(\frac{L}{L_\odot}\right)^{0.12} \quad (18)$$

where the final exponents are evaluated, assuming $\nu = 18$. This insensitivity of T_{eff} to L is a consequence that the radius increases with mass, which follows the central temperature being roughly constant because of the strong temperature sensitivity of the CNO cycle to the temperature. Nevertheless, the exponent, 0.12, is still a factor larger than that for the Hayashi track or the RGB and AGB.

Since a star's luminosity on the MS does not change much, we can estimate its MS lifetime from simple timescale arguments and the mass-luminosity relation. If $L \propto M^\eta$, then

$$\tau_{MS} = 10^{10} \text{ yrs}(M/M_\odot)^{1-\eta} \quad (19)$$

Clearly, the MS lifetime of a star is a strong function of its mass.

The MS lifetime of a star with a mass of 0.8 $M_\odot$ is comparable to the age of the Universe. Thus we are primarily concerned with stars more massive than this. While practically most of relatively low-mass stars are close to ZAMSs, massive stars burn hydrogen much faster, especially via the CNO cycle, which occurs in a much smaller region than do the pp-chains because of the much steeper temperature dependence. The requirement for fast energy transport drives convection in the stellar core. In addition to the mass effect, metallicity and mass loss can also significantly affect the stellar evolution.

While we focus here on the evolution of isolated stars, it should be noted that if a star is in a close binary then the story can change drastically.

4 Leaving MS (low and intermediate mass stars)

From point 2 to 3 (overall contracting phase): As X becomes less than 0.05, the nuclear energy generated is not sufficient to maintain the hydrostatic equilibrium, the entire star begins to contract. The increasing gravity due to the contraction is kind of balanced by the heat or the luminosity due to the conversion of gravitational energy to thermal energy. Simultaneously, the smaller stellar radius translates into a hotter effective temperature. For higher mass stars, the mass fraction of the convective core begins to shrink rapidly. At point 3, the mass fraction of hydrogen in the core is $\sim 1\%$. The core becomes nearly isothermal.

From point 3 to 4 (think shell phase): While this is happening, the hydrogen rich material around the core is drawn inward and eventually ignites in a thick shell, containing $\sim 5\%$ of the stars mass. Much of the energy from shell burning then goes into pushing matter away in both directions. As a result, the luminosity of the star does not increase; instead the outer part of the star expands. This thick shell phase continues with the shell moving outward in mass, until the core contains $\sim 10\%$ of the stellar mass (point 4). This is the Schönberg-Chandrasekhar limit. Stars with larger (by mass fraction) cores will reach this point faster than stars with small cores.

The Schönberg-Chandrasekhar limit

At the exhaustion of central H, a star is left with an He core surrounded by an H-burning shell and then an H rich envelope. Given that there is no nuclear burning in the core, its thermal stratification is nearly isothermal. There exists an upper limit to the ratio M_c/M_t, which can be qualitatively understood as follows. We have learned that the virial theorem in case of non-vanishing surface pressure P_0 can be expressed as

$$P_0 = K_1 \frac{M_c T_c}{R_c^3} - K_2 \frac{M_c^2}{R_c^4}, \tag{20}$$

where the K_1 and K_2 are constants. For given values of M_c and T_c, P_0 attains a maximum value $P_{0,m} = K_3 \frac{T_c^4}{M_c^2}$ when the core radius $R_c = K_4 M_c/T_c$ (where K_3 and K_4 are constants). For the star to be in equilibrium, $P_{0,m}$ must be larger than, or at least equal to, the pressure P_e exerted by the envelope on the interface with the core. Assuming that the core contains only a small fraction of the total stellar mass M_t so that we can roughly approximate $P_e \propto M_t^2/R^4$ (from the hydrostatic equilibrium of the entire star) and $T_c \propto M_t/R$ (from the virial theorem), where R is the total radius of the star. Hence at the interface,, $P_e \propto T_c^4/M_t^2$. Therefore, the condition $P_e \leq P_{0,m}$ dictates the existence of an upper limit to M_c/M_t.

The exact value of the Schönberg-Chandrasekhar limit depends on the ratio between the mean molecular weight in the envelope and in the isothermal core:

$$\left(\frac{M_c}{M_t}\right)_{SC} = 0.37 \left(\frac{\mu_e}{\mu_c}\right)^2 \tag{21}$$

At the end of the MS phase of a solar chemical composition object, $\mu_e \sim 0.6$ and $\mu_c \sim 1.3$ (the core is essentially made of pure helium), the limit is thus equal to $(M_c/M_t)_{SC} \sim 0.08$. A star with the total mass larger than $\sim 2.5 - 3 M_\odot$ will evolve to have a ratio equal to the limit and will then contract on the Kelvin-Helmholtz timescale. This contraction leads to the temperature increase up to $\sim 10^8$ K, when He fusion is ignited.

From point 4 to 5: This region in the HRD corresponds to the “Hertzsprung Gap” because of the short (KH) evolution time scale, which is $\sim 10^{-3}$ of the nuclear time scale for stars of about 1.5 $M_\odot$. Point 5 corresponds to the limiting Hayashi line, when the volume of the star is dominated by the convection in the envelope, similar to YSOs; the size of the radiative cores may be negligible.

From point 5 to 6: As the envelope cools due to expansion, the opacity in the envelope increases (due to the Kramers opacity), so by the time the star reaches the base of the giant branch (point 5), convection dominates energy transport. The thermal energy trapped by this opacity causes the star to further expand. (Note: the expansion comes solely at the expense of thermal energy of the envelope.) Meanwhile, near the surface, H^- opacity dominates, and, as the star cools, the surface opacity becomes less. The energy blanketed by the atmosphere escapes, and the luminosity of the star further increases.

The first dredge-up: As the star ascends the giant branch, the decrease in envelope temperature due to expansion guarantees that energy transport will be by convection. The convective envelope continues to grow, until it almost reaches down to the hydrogen burning shell. In higher mass stars, the core decreased in size during MS evolution, leaving behind processed CNO. As a result, the surface abundance of ^{14}N grows at the expense of ^{12}C, as the processed material gets mixed onto the surface. This is called the first dredge-up. Typically, this dredge-up will change the surface CNO ratio from $1/2 : 1/6 : 1$ to $1/3 : 1/3 : 1$; this result is roughly independent of stellar mass.

RGB bump was theoretically predicted by Thomas (1967) and Iben (1968) as a region in which evolution through the RGB is stalled for a time when the H-burning shell passes the H abundance inhomogeneity envelope. This is produced by the stellar outer convection zone at the H-burning shell RGB stage after the first dredge-up of a star in a globular cluster. The position in luminosity of the RGB bump is a function of metal abundance, helium abundance, and stellar mass (and hence cluster age), as well as any additional parameters that determine the maximum inward extent of the convection envelope or the position of the H-burning shell.

Why does the star's luminosity increase sharply, starting from the base of the RGB? Based on dimensional analysis (Chapter 1), the energy generation in a thin shell of ideal gas around a degenerate core is

$$L = KM_c^z \tag{22}$$

with $z \sim 8$ for CNO burning shells ($z \sim 15$ for helium burning shells). The shell burning continually increases the mass of the core with

$$\frac{dM_c}{dt} = \frac{L}{XQ}, \tag{23}$$

where X is the mass fraction of the fuel and Q is the energy yield per mass. Replacing M_c with L (Eq. 22) and then integrating Eq. 23, we obtain

$$L = L_0 \left(1 + \frac{(1-z)K^{1/z}(t-t_0)}{XQL_0^{(1-z)/z}}\right)^{z/(1-z)}, \tag{24}$$

where L_0 is the luminosity when $t = t_0$. This luminosity increases sharply when the star ascends the RGB.

Mass loss by red giants: It is an observational fact that stars on the red giant branch undergo mass loss in the form of a slow (between 5 and 30 km s^{-1}) wind. Mass loss rates for stars of $\sim 1M_\odot$ can reach $\sim 10^{-8}M_\odot yr^{-1}$ at the tip of the giant branch. The total amount of mass lost during the giant branch phase can be $\sim 0.2M_\odot$. This rate is high enough so that a star at the tip of the giant branch will be surrounded by a circum-stellar shell, which can redden and extinct the star. In the past, the geometry of this shell has been assumed to be spherical, but a large number of stars are now known to have circum-stellar mass distributions that are bipolar. (This includes the ring around SN 1987A.)

Point 6 and beyond:

To have the He burning, we need much higher temperature and density than for the Hydrogen burning, because the large Coulomb barrier and three-bodies to come together in 10^{-16} s; ^{8}Be is not stable. The path depends on the (initial) mass again, depending on the race between the temperature (from $\sim 10^7$ K to $\sim 10^8$ K, ignition T of Helium) and density ($\sim 10^2$ to $\sim 10^6$ cm^{-3} – the degeneracy density). Remembering the scaling: the higher the mass, the lower the density.

For $M \lesssim 1.5M_\odot$, the temperature is reached after the partial degeneracy in the He core; Initially (until the degeneracy is overcome with the temperature), the He burning increases the temperature, but without reducing the density, leading to a "He flash" (when the luminosity reaches $\sim 10^{3.4}L_\odot$) and then to the HB. The mass of the core at this time is $\sim 0.5M_\odot$. The flash luminosity $\sim 10^{11}L_\odot$ is all absorbed in the expansion of the non-degenerate outer layers. As the flash proceeds, the degeneracy in the core is removed, and the core expands.

For $M \gtrsim 1.5M_\odot$: temperature wins, He-burning is triggered gently. The luminosity increase is small, because of little core contraction. The end product also depending the mass:

- $M \leq 8M_\odot$: C-O WD, PNe
- $M > 8M_\odot$: nuclear burning continues all the way to Fe.

Horizontal Branch:

Lower mass stars which do undergo the He flash quickly change their structure and land on the zero-age horizontal branch (ZAHB). These stars burn helium to carbon in their core and also have a hydrogen-burning shell. Because the core is supported by the helium fusion, it is larger, and the pressure at the hydrogen burning shell is smaller. As a result, the luminosity of the star drops from its pre-helium flash luminosity to $\sim 100L_\odot$. The restructuring of the star to this stable configuration occurs on a Kelvin-Helmholtz timescale.

Because of the large luminosity associated with helium burning, the central regions of horizontal branch stars are convective. A star with a helium core of $\sim 0.5M_\odot$ will have a convective helium burning core of $\sim 0.1M_\odot$.

The effective temperature of a ZAHB star depends principally on its envelope mass (especially when the mass of the envelope is small). Stars with low mass envelopes will be extremely blue, with log $T_{eff} \geq 4.3$. Stars with large envelope masses ($\sim 0.4M_\odot$) appear near the base of the red giant branch.

On average, a star spends about 20% of the life time on the HB and has a luminosity of $\sim 10^2$ times the MS counterpart. While $Q_{HB} \sim 0.1Q_{MS}$, where does the energy come from? It turns out that much of the luminosity of a HB star arises from the H-shell burning (up to $\sim 80\%$).

In the observable HR diagram (i.e., V vs. $B - V$), the extreme blue end of the horizontal branch turns downward, and becomes almost vertical. This is mostly a bolometric correct effect; for stars with extremely small envelope masses, most of the luminosity comes out in the ultraviolet.

HB color and second parameter problem

What determines the color of the HB of a SSP? Presumably, clusters with many blue horizontal branch stars are dominated by stars with small envelopes, while clusters whose horizontal branch stars are in a "red clump" have large envelope mass stars, evolved from stars with relatively large masses ($\gtrsim 3M_\odot$) and metallicity.

Red clump stars are the (relatively metal-rich and/or young population I counterparts to HB stars (which belong to population II). Red clump stars are in the helium core burning stage and have metallicity greater than about 10% solar. Above this value, red clump location in a CMD is fairly insensitive to the metallicity. They look redder because opacity rises with Z. They nestle up against the RGB in HR diagrams for old, open cluster, making a clump of dots of similar luminosity.

The metallicity is the main ('the first') parameter controlling the color distribution of HB stars, due mainly to the envelope opacity and secondarily to the expected correlation of the evolving stellar mass (hence more massive stellar envelope) and the metallicity (e.g., higher metallicity stars evolve more slowly than lower ones). Thus the color distribution of HB stars could in principle be used a measurement of the age. However, observations show that at a given metallicity clusters of apparently the same age have different HB colors, which is the origin of the so-called second parameter problem. The nature of the problem is not clear. But the color difference could result from different mass-loss laws, which may depend on stellar rotation or dynamic interaction within the clusters, or from initial He abundances?

Helium burning phase of higher mass stars:

Stars with the helium burning ignited under non-degenerate conditions will also causes the core to expand, which decreases the pressure at the hydrogen-burning shell, which causes the shell luminosity, which contributes most to the star's luminosity, to drop, which causes the outer layers to contract. The star also evolves blue-ward off the red giant branch.

During their post red-giant branch evolution, stars between 3 and $10M_\odot$ perform "loops" in the HR diagram. The size and timescale of the loop depends on the mass; higher mass stars are more luminous, evolve more quickly, and have larger loops. Because this stage of evolution occurs on a nuclear timescale, there is a much higher probability of finding a star in these loops than finding a star in the Hertzsprung Gap. The precise nature of the loops, however, depends critically on the opacity, energy generation, metallicity, and theory of convection. Consequently, the modeling of these loops is extremely uncertain.

The Asymptotic Giant Branch:

When core helium burning ceases, the central core of the star will contract, and helium will begin burning in a thick shell. The luminosity from this shell will cause the region outside of it to expand. When this happens, the temperature and density of the hydrogen shell will decrease, and hydrogen burning will be extinguished.

The C-O core's evolution course and its consequences are similar to the Helium core's: core contraction, heating of the envelope, which then expands and cools, leading to the convection. The star then becomes redder and brighter.

Three outstanding characteristics for AGB stars:

- Nuclear burning occurs in two shells – a thermally unstable configuration – leading to a long series of thermal pulses.
- The luminosity is uniquely determined by the core mass. For $M_c > 0.5M_\odot$,

  $$\frac{L}{L_\odot} = 6 \times 10^4 \left(\frac{M_c}{M_\odot} - 0.5 \right), \tag{25}$$

  which is of the order of the Eddington luminosity.
- A strong stellar wind as a result of the high radiation pressure in the envelope (and the thermal pulses).

  Indeed, such superwind is confirmed in observations of stars which eject mass at rates of the order of 10^{-4} $M_\odot$ yr^{-1}. The high mass loss rate of some AGB stars, at least those from relatively low mass progenitors, is associated with a pulsation instability in the envelope (stars known as *Miras*), similar to that of RR Lyrae (low-mass He shell burning) and Cepheids (high-mass He shell burning).

As a consequence of the superwind, stars of initial mass in the range 1 $M_\odot \lesssim M \lesssim 10$ $M_\odot$ are left with C-O cores of mass between 0.6 $M_\odot$ and 1.1 $M_\odot$.

The core of a star at the end of its AGB phase is surrounded by an extended shell, which is then illuminated by intense UV radiation from the contracting central star. Such a shining nebula is called *planetary nebula*.

The central star, or *planetary nebula nucleus*, initially moves toward higher temperatures, powered by nuclear burning in the thin shell still left on top of the C-O core. When the mass of the shell decreases below a critical mass of the order of 10^{-3} $M_\odot$ to 10^{-4} $M_\odot$, the shell can no longer maintain the high temperature for the nuclear burning and the luminosity of the star drops.

At the same time, the nebula, expanding at a rate of ~ 10 km s^{-1}, gradually disperses, after some $10^4 - 10^5$ yrs.

5 The evolution of high mass stars ($\geq 10 M_\odot$)

- Helium ignition occurs on the giant branch for intermediate mass stars; higher mass stars ($M \geq 15 M_\odot$) can ignite helium in the Hertzsprung Gap, close to the main sequence.
- important role played by mass loss. Mass loss time scale shorter than the MS timescale. The MS evolutionary paths of stars of initial masses exceeding $30 M_\odot$ converge toward that of a $30 M_\odot$ star; similar He cores; Wolf-Rayet stars with CNO products (helium and nitrogen) exposed.
- nearly horizontal evolutionary track in the H-R diagram, since the luminosity is already close to the Eddington limit.
- undergo all the major burning stages, ending with a growing iron core surrounded by layers of different compositions; electrons do not become degenerate until the core consists of iron.

When the degenerate core's mass surpasses the Chandrasekhar limit (or close to it), the core contracts rapidly. No further source of nuclear energy in the iron core, the temperature rises from the contraction, but not fast enough. It collapses on a time scale of seconds!

Mass loss

The mass loss is particularly important in the study of massive stars. WR stars are examples.

correlation: $\log[\dot{M} v_\infty R^{1/2}] \propto \log[L]$

How $\dot{M}$ and v_w may be measured? GC survey as an example.

The number of AGB stars is proportional to the lifetime of stars in this stage. Using the conversion factor for hydrogen to carbon, $\dot{M}_c = L/Q = 1.2 \times 10^{-11}$ M$_\odot$ yr$^{-1}\frac{L}{L_\odot}$, the lifetime can be estimated as

$$\tau_{AGB} = (1.4 \times 10^6 \text{ yr})\ln\left(\frac{M_c - 0.5M_\odot}{M_{c,0} - 0.5M_\odot}\right) \tag{26}$$

Evolution calculations show that a relation exists between the initial core mass and the initial mass of the star M_0, of the form

$$M_{c,0} = a + bM_0, \tag{27}$$

where a and b are constants. Hence τ_{AGB} is essentially a function of the initial stellar mass. So if the IMF is assumed, we know the the relative population of AGB stars. Compared with the observed, one can conclude that the core mass can grow by only about 0.1 M$_\odot$, indicating the mass loss must be very intense.

6 End-products of stars

It is the core mass of a star that decides the outcome at the end of the stellar evolution.

6.1 White dwarfs

Observations show two peaks in the mass distribution of WDs, corresponding to the two origins:

- from stars with initial masses in $\sim 0.8 - 1M_\odot$; He-burning never gets started because of too low central temperature: hydrogen burning only, hence hydrogen-rich envelopes; WD masses of 0.2-0.4$M_\odot$.
- from stars with initial masses in $\sim 1 - 10M_\odot$, which undergo the AGB phase, accounting for most of the WDs observed. More hydrogen-rich envelopes than helium-rich envelopes (25%), because the relatively long duty fraction of hydrogen-shell burning than He-burning during the AGB pulse cycles. WD mass range peaks at 0.6$M_\odot$.

The internal energy source: thermal energy stored by the ions (as the heat capacity of the electrons is negligible).

WD structure and cooling:

- an isothermal degenerate electron core. Why is this a reasonable assumption?
- a thermal radiative envelope with negligible mass and energy source.

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}, \tag{28}$$

Replacing dr with the hydrostatic equation, using the Kramers' opacity, and integrate the equation from the surface, where $P = T = 0$, inward, we have

$$P \propto \left(\frac{M}{L}\right)^{1/2} T^{17/4}. \tag{29}$$

Reversing back to the density,

$$\rho \propto \left(\frac{M}{L}\right)^{1/2} T^{13/4}, \tag{30}$$

which holds down to r_b, where the ideal electron pressure and the degenerate electron pressure are the same:

$$\frac{\rho}{\mu_e m_A} kT = K(\rho/\mu_e)^{5/3} \tag{31}$$

where K is just a constant. Since $T_b = T_c$ to prevent a jump in temperature, eliminating ρ between the above two equations, obtain

$$\frac{L/L_\odot}{M/M_\odot} \approx 9 \times 10^{-3} (T_c/10^7 K)^{7/2} \tag{32}$$

Neglecting the mass and energy in the atmosphere, the total thermal energy is

$$U_I = \frac{3MkT_c}{2\mu_I m_A}. \tag{33}$$

$$L = -\frac{dU_I}{dt} \tag{34}$$

Therefore,

$$\tau_{cool} \propto (1/T_c^{5/2} - 1/T_{c,0}^{5/2}) \tag{35}$$

For $T_c \ll T_{c,0}$, we have

$$\tau_{cool} = 2.5 \times 10^6 \text{ yr} \left(\frac{M/M_\odot}{L/L_\odot} \right)^{5/7} \tag{36}$$

For example, about 2×10^9 yrs would be required for the luminosity of a $1M_\odot$ WD to drop to $10^{-4}L_\odot$.

Afterward, the cooling can be accelerated by crystallization. The WD quickly becomes invisible.

6.2 Supernovae (SNe)

Two basic types:

- Type Ia: only metal lines; no hydrogen lines in its spectrum; observed in all kinds of galaxies and regions inside a galaxy; rather uniform light curves.
- Type Ib/Ic supernovae are distinguished from Type Ia by the lack of an absorption line of singly-ionized silicon at a wavelength of 635.5 nm. As Type Ib/Ic supernovae age, they also display lines from elements such as oxygen, calcium and magnesium. Type Ic supernovae are distinguished from Type Ib in that the former also lack lines of helium at 587.6 nm.
- Type II and Type Ib,c: strong hydrogen emission and absorption lines and nearly always found in recent massive star formation regions.
  - So related to Pop I — core collapse. Evidence: pulsars and neutrinos (from SN1987A)
  - Eject more mass, but at slower speed.
  - Slightly fainter. Light-curves are much less uniform.
  - Relatively easy to be picked up in radio and X-ray, usually at later times than the visible light peak.

More physically, Type II and Type Ib,c together are called "core-collapsed" SNe.

6.2.1 Core-collapsed SNe

As the core collapses, instabilities occur

- Because of the high degeneracy of the gas, the temperature rises unrestrained. In time, it becomes sufficiently high for the photo-disintegration of iron nuclei

$$100\text{MeV} +^{56}\text{Fe} \rightarrow 13^4\text{He} + 4\text{n} \tag{37}$$

- electron capture by heavy nuclei (inverse beta-decay) deprives the core of its main pressure source. The increase of the density forces the degenerate electrons to ever-higher momentum state - hence higher energy states, exceeding the neutron proton mass difference.

- Eventually, the density becomes so high that free protons to capture free electrons and turn into neutrons. Not only does this process absorb energy, but it also reduces the number of particles.

- the rapid energy loss from neutrinos further deprives the thermal pressure support.

The star contracting from a density of $\sim 10^9$ g cm^{-3} and ending up with a neutron star with a size of $\sim$ 10 km and a density $\sim 3 \times 10^{14}$ g cm^{-3}, comparable to the nuclear matter density. The neutron degeneracy pressure balances the gravity!

The total gravitational energy release from the collapse is $\sim 3 \times 10^{53}$ ergs, more than enough to dissolve all the synthesized nuclear materials $\sim 2 \times 10^{52}$. But how a fraction of this energy may be used to drive the explosion is not clear. A few possibilities: 1) bouncing shock wave, 2) trapped neutrinos, and 3) jets.

Neutron stars

Neutron stars are believed to be the stellar remnants of massive stars, with initial mass in the range of $\sim 10-25M_\odot$. Neutron stars, determined by the stellar evolution, are generally in the mass range of $\sim 1.2 - 2.5M_\odot$; the average mass of neutron stars in binary systems is of about $1.4M_\odot$. Such a neutron star has a radius of $\sim$ 10 km, depending on the assumed exact equation of state, an issue of still much interest. A newly born neutron star is expected to have fast rotation and strong magnetic field. Such magnetized and fast rotating neutron stars explain the presence of pulsars. The life time of a pulsar is typically on the order of 10^7 years, depending on the magnetic field, which determines the spin-down rate. The exact evolution of the magnetic field in a young neutron star is still very uncertain. But the magnetic field eventually decays.

A "dead" neutron star may become "alive" again in a binary system. The star may accrete matter from its companion and can be observed as an X-ray binary. The accretion leads to the angular momentum transfer and the spin-up of the neutron star. As a result, the neutron star may become a pulsar again, typically with a period of a few to a few tens of ms. Because of the weakness of such an old neutron star, the spin rate is extremely stable and decreases very slowly.

SN1987A:

First observed visually on Feb. 24, 1987 in the LMC. Kind of unique light-curve and intrinsically dimmer, compared with the "normal" Type II SNe. Progenitor: B3 I blue supergiant (16-20 $M_\odot$).

The explosion leads to the synthesis of heavy elements in the ejecta, chiefly ^{56}Ni, which decays into ^{56}Co (half-life of 77 days) and then to ^{56}Fe. These decays give the major energy source that keeps the expanding ejecta bright.

Another key evidence for the core collapse and the formation of a neutron star is the detection of the neutrinos about *a quarter of a day* before optical discovery. But the neutron star is so far not detected.

6.2.2 Pair-Instability Supernovae (PISNe)

The hotter a star's core becomes, the higher energy the gamma rays it produces. When the mass of a star exceeds about $100M_\odot$, the produced gamma rays become so energetic, their interaction with atomic nucleus can lead to the production of electron-position pairs. Even though these pairs are released into the star's core and usually recombine (releasing back into gamma rays) in very short time periods, the speed at which energy (radiation) transfers through a gas is highly dependent on the average distance between interactions. Then the distance that gamma rays travel in the gas starts to decrease instead of increasing. This decrease in the distance that gamma rays travel is an instability, and causes a feedback loop: as gamma ray travel distance decreases, the temperature at the core increases, and this increases the generation of the nuclear energy and hence the gamma ray energy and further decreases the distance that gammas can travel.

For a star in the mass range of $\sim 100 - 130M_\odot$, the instability most likely leads to partial collapse and pressure pulses. This process tends to eject parts of the outer layers of the star until it becomes light enough to collapse in a normal SN.

For a star in the mass range of $\sim 130 - 250M_\odot$, the collapse caused by the pair instability proceeds to allow runaway nuclear fusion to burn the star's core in a few seconds, creating a thermonuclear explosion[1]. With more thermal energy released than the stars' gravitational binding energy, it is completely disrupted; no black hole or other remnant is left behind. A PISN is sometimes called a "hypernova", a term that used to refer an exceptionally energetic explosion with an inferred energy over 100 SNe. But the inference of such an energy is not necessarily physical, depending on the assumption of the isotropy of the radiation.

In addition to the immediate energy release, a large fraction of the star's core is transformed to ^{56}Ni, a radioactive isotope which decays with a half-life of 6.1 days into ^{56}Co, which has a half-life of 77 days, and then further decays to the stable isotope ^{56}Fe. Thus a PISN may be distinguished from other SNe by its very long duration to peak brightness, together with its brightness due to the production of much more radioactive Ni. For the hypernova SN 2006gy, studies indicate that perhaps 40 solar masses of the original star were released as ^{56}Ni, almost the entire mass of the star's core regions. Collision between the exploding star core and gas it ejected earlier, and radioactive decay, release most of the visible light.

For a star in the mass range of $\gtrsim 250M_\odot$, a different reaction mechanism, photo-disintegration, results after collapse. This endothermic reaction (energy-absorbing) causes the star to continue collapse into a black hole rather than exploding due to thermonuclear reactions.

The pair instability happens in low metallicity stars, with low to moderate rotation rates. Simulations show that there is vigorous mixing of heavy elements from deep in the interior of the star to its outer layers in the CC case but not in the PISN. If these heavy elements appear in the emission spectra of the light curve just after shock breakout they would be a clear signature of a low-mass Pop III progenitor rather than a very massive one. Thus the detection of PISNe could directly constrain the primordial IMF for the first time, which is key to the formation of the first galaxies, early cosmological re-ionization, and the chemical enrichment of the primeval IGM.

Stars formed by collision mergers having a metallicity Z between 0.02 and 0.001 may also end their lives as PISNe if their mass is in the appropriate range. High metallicity stars are probably unstable due to the Eddington limit, and would tend to shed mass during the formation process.

6.2.3 Type I SNe

The lack of hydrogens in the spectra of such SNe strong indicate that they result from the collapse of "undressed" cores due to strong stellar winds and/or by transferring to companions.

Energy source: explosive fusion of close to 1 $M_\odot$ carbon and oxygen to iron-peak elements, especially ^{56}Ni. Indeed, the formation of each ^{56}Ni from Carbon will generate $\sim 8 \times 10^{-5}$ erg; 1 $M_\odot$ would generate about 10^{52} erg, with a pretty to spare for a SN.

What causes this *explosive* burning? The fuel must be degenerate at ignition, as in a "He-flash".

Where do we expect to find this amount of carbon and oxygen? a WD. But a WD with mass $< M_\infty$, or the Chandrasekhar limiting mass, will just sit and cool off for the age of the Universe.

How to make a WD add mass? 1) merging two WDs: accounting for the absence of hydrogen. But there may not be enough of them with enough masses and tight enough to merge over the age of the Universe. 2) accretion: most of the accreted materials is fused to carbon and oxygen during nova and possibly ejected. So all these need to lead to the increase of the WD mass.

Is a neutron star expected? No. But a leftover WD is a possibility, if the explosion is only partial and off-center.

While WDs, whether due to merger or accretion, are likely to be the origins of the Type Ia SNe, Type Ib and Ic probably arise from WRs and are indeed found in recent massive star forming regions.

References

[1] C.L. Fryer, S.E. Woosley, A. Heger 2001, "Pair-Instability Supernovae, Gravity Waves, and Gamma-Ray Transients", ApJ, 550, 372

7 Review

Key Concepts: Jeans mass, initial mass function, Salpeter's law, Hayashi track, Brown dwarfs, ZAMS, the Schönberg-Chandrasekhar limit, the Eddington limit, Hertzsprung Gap, Wolf-Rayet stars, (p-, e-, r-, and s-) processes

What are the basic signatures of low-mass YSOs?

What is the basic reason for the mass fragmentation of a collapsing cloud? When is the fragmentation expected to stop?

Qualitatively what is the basic structure of a proto-star?

What may be the structure change of a YSO that could end its evolution along the Hayashi track?

How does the lifetime of a star depend on its mass?

Qualitatively describe the track of a star from its proto-star stage to its death in the HR diagram. How does the track depend on the initial mass of the star? What are the relatively time durations that the star spend on different evolutionary stages?

The location of the Hayashi track, the MS, or the red-giant branch is sensitive to the chemical composition of the stars. Does T_{eff} increase or decrease with the increase of the metallicity or the decrease of the He abundance? Why?

How does the metallicity of ZAMS stars affect their color and luminosity?

How does the luminosity of a MS star depend on its mass? Can you characterize this dependence from a simple dimensional analysis, assuming that the radiative heat transfer dominates in the star?

What is the first dredge-up? How does it affect the observed surface abundances of the elements?

Why do some stars undergo He-flash, while others don't? Has He-flash actually been observed?

What is the horizontal branch? Why does it tend to have a narrow luminosity range?

What are the differences between the red-giant and asymptotic giant branches? What is the main heating transfer mechanism in these branches? Why?

At which evolutionary stage of a star is a planetary nebula expected to form? What keeps such a nebula bright visually?

There are two major types of white dwarfs, one hydrogen-rich and the other helium-rich? What are their relative proportions? What physical reasons are believed to be responsible for the proportions?

What the internal energy source of a white dwarf that keeps it bright? Why is the interior close to be isothermal?

What are the main differences of the post-MS evolution of massive stars ($\geq 10M_\odot$) from that of lower mass ones?

In an HR diagram, name the nuclear burning states along the evolutionary tracks for low and high mass stars, separately.

What are the key observational signatures that distinguish Type I and Type II supernovae? Why are Type Ib,c supernovae also believed to arise from the collapse of massive stars?

What is a pair-instability supernova? Why is it proposed to be related to Pop III stars?

What is the energy source that keeps a supernova bright for $\sim 10^2$ days or longer?

How would the radius of a star change if its opacity were increased by a small amount?

How would themain sequence lifetime of star change if the mass loss at the surface was enhanced?