Simple Stellar Populations

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1 Theoretical isochrones

Simple Stellar Population consists of stars born at the same time and having the same *initial* element composition. Stars of different masses follow different evolutionary tracks.

In a theoretical HRD, stars of a SSP are located along an **isochrone**, from the Greek word meaning "same age". This line connects the points belong to the various theoretical tracks at the same age. By applying to each point a set of appropriate bolometric corrections, the isochrone can then be converted to an observational CMD.

The **TO** point is the the bluest point along the isochrone MS, where the central hydrogen is exhausted.

For a particular isochrone, the stellar mass range is very large along the MS, whereas the mass evolving along RGB and successive phases is approximately constant; i.e., the stars there essentially all evolved from the same ZAMS mass.

The evolutionary phase of a star along an isochrone may be characterized with the curvilinear coordinate s, starting from at the bottom of the ZAMS and increasing when moving towards more advanced phases (the above value is not important, though). Its value is at a point is uniquely determined by the age of this isochrone t and the ZAMS mass M of the star, $s = s(t, M)$. This function can then be inverted to write $t = t(M, s)$. Then we have

$$dt(M,s) = \left(\frac{dt}{dM}\right)_s dM + \left(\frac{dt}{ds}\right)_M ds = 0 \tag{1}$$

from which we obtain

$$\left(\frac{dM}{ds}\right)_t = -\left(\frac{dM}{dt}\right)_s \left(\frac{dt}{ds}\right)_M \tag{2}$$

where $\left(\frac{dM}{dt}\right)_s$ is the ZAMS mass change rate for stars entering the phase, and $\left(\frac{dt}{ds}\right)_M$ is the time that a star with the M stay in the phase.

Indeed, theoretical isochrones also predict the relative number of stars at different evolutionary phases, in addition to the location in the HRD and CMD of an SSP. The number of stars in a particular phase interval is

$$\frac{dN}{ds} = -\frac{dN}{dM}\left(\frac{dM}{dt}\right)_s \left(\frac{dt}{ds}\right)_M \tag{3}$$

where $\frac{dN}{dM}$ is the IMF. For a generic post-MS phase, the first two terms are constant. Therefore, the ratio between the number of stars in two different post-MS phase intervals along the isochrone is simply equal to t_{PMS1}/t_{PMS2}.

Note that the mass used here needs to be ZAMS mass, whereas the value of the evolving mass (which is related to the T_{eff}, L, and the surface gravity) is always used to determine the bolometric corrections to a generic photometric system.

2 Old SSPs

'old' here denotes ages greater than ~ 4 Gyr (corresponding to masses lower than $\sim 1.3 M_{\odot}$. This encompasses the look back time of the universe from redshift $z = 0$ up to $z \sim 2$.

The age is also far from the so-called RGB phase transition. Theoretical models suggest that two special events significantly mark the spectral evolution of a SSP during its lifetime. The first, after $\sim 10^8$ yrs, due to the sudden appearance of red and bright Asymptotic Giant Branch (AGB) stars (AGB transition), and the second, after $\sim 6 \times 10^8$ yrs (with the corresponding stellar mass $M \lesssim 2.5 M_{\odot}$ and with He-burning starting from degenerate cores), due to the development of the full Red Giant Branch (RGB transition).

Important properties:

- the lower MS – starting from ~ 2 mag below the TO – and RGB are unaffected by age, but are sensitive to Z.
- the brightness of the ZAHB is also unaffected by age, but sensitive to Z. The brightness is determined mostly by the He core mass at the He flash, which decreases with increasing Z.
- the brightness and color of the TO are affected by both age and Z. A star of certain Z and mass behaves somewhat like a star of lower Z and lower mass, in terms of the MS lifetime, brightness, and color.

These properties can be used as tools to determine such important parameters as ages, Z, and distances to SSPs, providing the foundation for understanding the cosmic evolution.

2.1 Age estimates

We need to compare an observed CMD of an SSP with theoretical isochrones. A natural approach would be to fit the whole theoretical isochrone to the observed CMD, even for a generic SSP. But this approach is not practical because of the systematic uncertainties in both the model and in measurements. An empirical CMD typically shows non-negligible scatters, caused by photometric errors, blending effects, and the presence of unresolved binaries, hence causing significant errors in the parameter determination of a fit. Sometimes, however, one cannot avoid to use the approach to gain information about the CMD in complicated situations than the SSP (e.g., to estimate the star formation history of a galaxy), in which such errors may be tolerated.

To facilitate a comparison for a SSP, it is customary to use a 'fiducial line' (connecting peaks in color or magnitude ranges in the empirical CMD) as the observed CMD, in a way that is resistant to the errors mentioned above.

Alternatively, one can focus on some age-sensitive features of the models that are less affected by the uncertainties. The direct comparison of the model and observed TO positions, either in color or in brightness, needs accurate correction for the effects and extinction distance. Thus it is preferred to use differential quantities like the magnitude (usually V) difference between ZAHB and TO or the color [usually either $(B-V)$ or $(V-I)$] difference between the TO and the base of the RGB. The difference changes with the age of the SSP.

Use the vertical method as an example. The ZAHB in a cluster CMD is the lower envelope of the HB star distribution.

$$V_{\rm ZAHB} =< V_{\rm HB} > +0.05[{\rm Fe/H}] + 0.20 \tag{4}$$

where $< V_{\rm HB} >$ is the mean level of objects in the RR Lyrae instability strip or at its red side. The blue side is not horizontal in a typical CMD (because of the choice of the color) and is thus cannot be used.

2.2 Abundances of Light Elements

Light Elements, such as He and Li, and their isotopes originate primarily in the primordial nucleosynthesis during the Big Bang. The abundance measurements of important abundances can therefore provide the observational constraints on the theory and on the cosmological baryon density in particular.

The initial He abundance Y affects the evolution of stars, and hence the properties of SSPs. We have learned that the effect is chiefly due to the dependence of the molecular weight on Y; a star with a higher Y appears brighter and bluer (if other parameters remain the same). This is manifested in the change of the isochrone of an SSP. An increase of Y decreases the mass of a star evolving at the TO and in post-MS phases and shifts the MS to lower left and the ZAHB to upward in a CMD (Fig. 9.17 in SC). These shifts can thus be used as measurements of Y:

The parameter **R**, defined as the number ratio of HB stars to RGB stars brighter than the ZAHB level, increases with Y. This parameter has a very weak dependence on [Fe/H], except for the effect due to the RGB bump that moves across the ZAHB level from higher to lower luminosities at [Fe/H] ~ -1, depending on the age of the SSP.

The parameter Δ is the magnitude difference between ZAHB and the MS at a given color; the color should be selected to minimize the effect of age. However, this parameter, sensitive to metallicity, is not suited for absolute Y determination.

The parameter **A** is defined to be the luminosity-to-mass ratio of a RR Lyrae star, $A = \log(L/L_\odot) - 0.81\log(M/M_\odot)$ [1]. Increasing Y increases the luminosity of the HB (hence the luminosity of RR Lyrae stars), which is only partly offset by the dependence of M on Y. The stellar evolution model predicts $A \propto 1.20Y$ and $\log P = 11.497 + 0.84A - 3.481 \log T_{eff}$), from which we can determine A and then Y if both P and T_{eff} can measured observationally. The error in Y arises chiefly from the difficulty in getting an accurate T_{eff}.

While old stars are too cold to show He lines in their spectra, spectroscopy can be used for Li abundance measurements. Lithium is a very fragile element, easily destroyed in stellar interiors when $T \gtrsim 2.5 \times 10^6$ K. Such temperature can be achieved in stellar cores during their pre-MS contraction and even in outer regions during the MS phase. Therefore, if the surface convection zone extends to such a region at some points of the stellar evolution, depending on the opacity and hence the temperature of a star, the Li abundance at its stellar surface can then be substantially altered. Indeed, the measurements of the Li abundance as a function of T_{eff} shows a relative flat part and then a sharp down-turn at $T_{eff} < 5700$ K.

There exist discrepancies between the Big Bang nucleosynthesis predictions and stellar measurements: 1) ^{7}Li is a factor of four below the prediction, though observed abundances in old halo nuclei are stable to about 5%; 2) observed ^{4}He abundance is typically below the prediction. Such discrepancies could arise from the interaction of the species with dark matter (e.g., sterile neutrino species), though very weakly, and/or their decay products, or from more mundane issues, including uncertainties in the nuclear physics.

2.3 Reddening and metallicity estimates

The extinction affects both the brightness and the color. But both are also affected by Z; e.g., metal-poor stars appear bluer and brighter. The brightness further depends on a reliable determination of the distance. We thus need to break the degeneracy. It helps to have extra information about the stellar intrinsic spectral properties. Of course, spectroscopy may allow us to estimate the metallicity of individual stars. But this is not always practical, especially for a distant stellar cluster. In such a case, we may still be able to get the information, photometrically.

Reddening measurements based on Strömgrem photometry: This photometry system is designed to isolate parts of the stellar spectra from which to build color indices sensitive to specific properties of the stars. The system consists of four filers *uvby* (≈ 200 Å wide each), plus a pair of narrow filters β_n (30 Å) and β_w (100 Å), centered at 4860Å, that measure the strength of the H_β line and its adjacent continuum. The age-dependent $\beta(= \beta_w - \beta_n)$ index is sensitive to T_{eff} (hence defining the intrinsic spectral shape), but not to reddening. In contrast, the measurements of the broad-band color indices, such as $E(b-y)$, can be used to determine the extinction $E(B-V)$, in an iteration fashion.

The dereddened CMD can then be used to measure Z. For example, the slope of the RGB of an old SSP is affected by Z and are insensitive to age. It is then customary to use a reference that is empirically (and mostly locally) constructed, based on clusters with good photometric and spectroscopic determinations of [Fe/H].

2.4 Distance Estimates

Except for geometrical methods such as the parallax technique (do you know any other techniques?), which can only used for relatively nearby stars, any distance measurement needs a standard candle. Here we discuss properties of isochrones that can be used to make them as a stellar standard candle. The key difficulties are the variation of stellar properties (e.g., age and Z) from one parent stellar system to another. Therefore, one needs to single out various classes of stellar objects whose brightness is predicted to depend only on the initial chemical composition – a quantity that can be estimated more easily than age. Such developed methods can be used to determine distances within the distance to the Virgo Cluster. What standard candles may you find in a CMD?

MS-fitting method

Once the Y and Z are determined, the low MS can be used as a template and compared to the observed MS in an SSP with the initial chemical composition. The difference between the absolute magnitudes of the template MS and the apparent magnitudes of the observed one gives the distance modulus. This is the so-called MS-fitting method.

Because of the steep slope of the lower MS, a small uncertainty in the color can affect the measurement a lot. An uncertainty in colors (not due to the uncertainty in the reddening) translate into an uncertainty of five times larger in the derived distance modulus.

The uncertainty related to the error on the reddening estimate is smaller, of the order of $\Delta(m - M)_0 \sim 2.0\Delta E(B - V)$, because of the cancellation due to magnitude correction $\Delta V \sim 3.1\Delta E(B - V)$.

The template can be theoretical and empirical. To accurately correct for the composition effect, an empirical MS is built by considering local field stars of known [Fe/H] (e.g., determined from spectroscopy) with distances determined geometrically. However, one has to shift the position of these template stars of various [Fe/H] values to the location they would have at the metallicity of a SSP, for which the distance is to be measured. For not too large metallicity ranges (order of 1 dex), an appropriate color shift is sufficient, based various empirical or theoretical relationships between the color shift and the [Fe/H] value.

Typical errors on the best MS-fitting distance are of the order of $\sim 0.07 - 0.08$ mag.

Tip of the RGB method

The bolometric luminosity of the tip of the RGB (TRGB) is determined by the mass of the He core at the He flash, with little dependence on the age if greater than $\sim$ 4Gyr (i.e., initial mass lower than $\sim 1.8M_\odot$). Such stars all develop very similar levels of electron degeneracy within the He core, and the mass of the He core has to reach almost the same value before He-burning ignites.

For stars with larger masses, the electron degeneracy is at a lower level as a consequence of higher T and lower ρ, resulting a strong decrease of the luminosity. After a minimum has been attained (corresponding to the the RGB transition with the stellar mass $\sim 2.5M_\odot$), the mass of the core (which is no longer degenerate) and the value of the luminosity starts to increase again with increasing total mass as a consequence of the increasing mass of the convective core during the core H-burning phase.

With increasing metallicity, for a fixed He content and stellar mass, the H-burning rate in the shell becomes more efficient. As results, the thermal condition for igniting He is reached at a lower-mass and hotter He core, but the surface luminosity ends up higher at the TRGB: $M_{bol}^{TRGB} \propto -0.19$[Fe/H] for metallicity well below solar.

In practice, the TRGB method is found to be best done in the I-band, with the weakest dependence on the metallicity, $M_I^{TRGB} \propto -0.15$[Fe/H]. Once the I-band apparent magnitude of the TRGB (I_{TRGB}) is determined from the discontinuity in the observed luminosity function (LF), and an extinction correction (to be independently determined) is applied to get $I_{0,TRGB}$, the distance modulus can be estimated from

$$(m - M)_0 = I_{0,TRGB} - M_I^{TRGB} \tag{5}$$

The method has been applied to the field halo population of external galaxies and is used for galaxies up to the Virgo distance with the HST.

Horizontal branch fitting method As in case of TRGB stars, the ZAHB brightness of low mass stars is determined by the value of the He core mass at the He flash, depending only on the initial metallicity:

$$M_V(ZAHB) = 0.17[\mathrm{Fe/H}] + 0.78. \tag{6}$$

The estimated distances for a sample of Galactic GCs are consistent with present MS-fitting distances.

In addition, both red clump and RGB bump have also been used for distance measurements. Examples are the determination of the orientation of the stellar bar in the central region of our Galaxy, using red clump stars. Furthermore, they have been used to map out extinction toward the Galactic nuclear region.

2.5 Luminosity functions and estimates of the IMF

LF of SSPs are a traditional tool to assess the level of agreement between theoretical stellar evolution models and real stars. In particular, the shape of the LF of post-MS evolutionary phases of an SSP is fully determined by the evolutionary times of the single mass evolving along those phases, independent of the choice of the IMF.

There are a few distinct diagnostics. The shape of the TO and its brightness are affected by the age; the SGB population along a young isochrone (hence higher initial mass) causes a more peaked shape before the dropping into the RGB than along an old one.

The local maximum along the RGB is due to the RGB bump. Comparison of the predicted bump brightness with SSP observations provides a powerful test for the extension of the surface convection; the higher metallicity corresponds to higher opacity, hence deeper the convection reaches.

The slope of the post-MS LFs is essentially independent of age and of the initial Z, because the luminosity evolution is pre-dominantly due to the growth of the electron degenerate He core mass. In contrast, the star counts along the MS is sensitive to the IMF due to the large mass range covered. After the observed and theoretical LFs are normalized to the same number of stars along the RGB (where the shape of the LF is unaffected by age and choice of the IMF), the IMF of an SSP (old or young) can be determined by comparing the shape of the observed LF along the MS with the theoretical counterparts.

In practice, one needs to consider the dynamic (mass-segregation) effects on the LF of a particular region of a GC or even in the central regions of galactic bulges (such as in M31). More massive objects tend to sink toward the cluster center, whereas lower mass ones move outward. Therefore, one actually determine the so-called Present Day Mass function (PDMF). The relationship between the PDMF and IMF has to be carefully deduced from the dynamical modeling of the observed population. The dynamic effect is also important for central regions of young clusters, even on 10^6 year time scales.

3 Young SSPs

3.1 Age estimates

The isochrones of 'young' SSPs (younger than $\sim$ 4 Gyr; e.g., typically open clusters) are distinctly different from those for old SSPs. Precise age determination for young SSPs are difficult. The young isochrones show the hook-like feature at the TO due to the overall contraction of young stars when the hydrogen abundance in the core becomes $X < 0.05$ but before the complete exhaustion. For very young ages the vertical TO region covers a large luminosity range. The SGB and RGB phase is almost completely depopulated because of the much faster evolutionary timescale (the He core mass at the end of the central H-burning reaches the Schönberg-Chandrasekhar limit; He-burning starts before the core could become degenerate). The horizontal method is of no use.

The vertical method may still be used if a sizable sample of red clump stars is present, although the dependence on the age and metallicity is different from that of old SSPs. For ages between $\sim$ 0.5 and 4 Gyr the He-burning phase is usually a red clump of stars (because of the presence of relatively massive envelopes). For younger stars, the phase moves progressively to the blue side of the CMD, describing increasingly larger loops to the blue.

If the distance and extinction to the observed SSP are known (e.g., clusters in the Galactic bulge and in external galaxies), a direct fit of theoretical isochrones to the TO can be used to estimate the age.

3.2 Metallicity and reddening estimates

Similar methods as used for the old SSPs may be used for faint MS stars of young SSPs.

3.3 Distance estimates

The TRGB and He-burning phases are of less use because they are both affected by the SSP age (the TRGB does not exist for ages below $\sim 0.1 - 1$ Gyr).

The most important technique is to make use of the Cepheid period-luminosity relationships, whenever a number of such variables is detected in the SSP.

In fact, only a few Cepheids in the Galaxy have parallax error below 30%. They also have large uncertainties in the extinction correction. The template P-L relationship traditionally used has been determined on the Cepheids i the LMC.

References

[1] Caputo, F.; Cayrel, R.; Cayrel de Strobel, G. 1983, "The galactic globular cluster system - Helium content versus metallicity", A&A, 123, 135

4 Review

Key concepts: simple stellar population, isochrone, TO of an isochrone.

What determines the stellar number density along the post-MS isochrone?

How do various features in a CMD depend on the parameters of an isochrone?

What are some basic ways to estimate the age, metallicity, and distance?

Using the high-resolution HST imaging capability, what may be the way to determine the distance to a dwarf spheroid galaxy that is probably in our Local Group and consists of primarily stars more than 4 Gyr old?

How may a present day mass function of a stellar cluster different from its IMF?

What are the so-called red-clump stars?