Conditionals as Modal Modifiers

1. Conditional Constructions: Basic Terminology, Questions, and Assumptions

(1) Conditional
A conditional or conditional construction or conditional sentence is sentence of the form ‘if $S_1$, (then) $S_2$.’

Examples:

a. If Dave takes his medicine, (then) he will get better.

b. If Dave were a doctor, (then) his parents would be proud of him.

c. If Dave had simply listened to me, (then) he wouldn’t be in this mess right now.

d. If you don’t hand in the exam, (then) you won’t get an A for the class.

(2) Overarching Question What are the semantics of conditionals?

Sub-Questions:

a. What are the truth-conditions of conditional sentences?

b. How are these truth-conditions compositionally derived from the meanings of the component parts of the conditional?

(3) The Anatomy of a Conditional Construction

a. Antecedent (or Protasis): ‘(if) $S_1$’ The ‘If-Clause’

b. Consequent (or Apodosis): ‘(then) $S_2$’ The Main Clause

(4) Special Case We Will Ignore: Relevance Conditionals (a.k.a. ‘Biscuit Conditionals’)

a. Examples:

   (i) If you’re hungry, there is pizza in the fridge.
   (ii) If you scare easily, don’t watch *It Follows*.
   (iii) If you hate him, then why did you marry him?

b. Special Meaning: If antecedent is true, then consequent is relevant.

   • We’ll follow most of the semantic literature in bracketing these kinds of conditionals as having a different analysis from the ones in (1).

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1 These notes are based upon material in von Fintel & Heim (2021), pages 25-28 and 47-53.
(5) **The Contribution of Tense and Aspect**

Certain subtle aspects of the conditional’s meaning can be affected the choice of tense and/or aspect in either the antecedent or the consequent.

- Broadly (and informally) speaking, the choice of tense/aspect can communicate ‘how realistic’ the antecedent and/or consequent propositions are…

(6) **Some Selected Categories of Conditional**

a. **Future Neutral Vivid**

(i) **Example:** If Dave *takes* his medicine, then he *will* get better.

(ii) **Structure:** Antecedent and consequent are present tense.

(iii) **Semantic Property (Roughly / Controversially Stated):**

The antecedent and consequent might be true now, or might be false. Both are presented as equally ‘likely’.

b. **Future Less Vivid**

(i) **Example:** If Dave *took* his medicine, then he *would* get better.

(ii) **Structure:** Antecedent and consequent are past tense (subjunctive)

(iii) **Semantic Property (Roughly / Controversially Stated):**

The antecedent and consequent might be true now, or they might be false. But, their actual truth is held out as ‘remote’ in some sense.

c. **(Past) Counterfactuals**

(i) **Example:**
If Dave *had taken* his medicine, then he *would have* gotten better.

(ii) **Structure:** Antecedent and consequent are past perfect (pluperfect).

(iii) **Semantic Property (Roughly / Controversially Stated):**

The antecedent is false, and the consequent is false.

For our purposes in this discussion, we will ignore the differences between these subtypes, and so we will ignore the contribution that tense/aspect make to the conditional…

- However, it remains a *major* topic in the formal semantics of conditionals… (Iatridou 2000, Ippolito 2003, Arregui 2009, Schulz 2014, Prince 2019…)
2. An Extensional Semantics for Conditionals, and Its Famous Problems

If you’ve ever taken an introductory logic class, you’ve been exposed to the following extensional (truth-value operator) semantics for conditionals.

(7) **Extensional Truth-Conditions for Conditionals**

\[
[[ \text{If } S_1 \text{ then } S_2 ]] = T \text{ iff } \text{ either } [[ S_1 ]] = F \text{ or } [[ S_2 ]] = T
\]

(8) **Truth-Table Representation of the Truth-Conditions in (7)**

<table>
<thead>
<tr>
<th>$\text{If } S_1 \text{ then } S_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

(9) **Conceptual Motivation**

- Intuitively, “If $S_1$ then $S_2$” says that when $S_1$ is true, $S_2$ also has to be true.
- Consequently, “If $S_1$ then $S_2$” will be false if $S_1$ is true while $S_2$ is false.

(10) **Empirical Motivation**

In many cases, ‘If $S_1$ then $S_2$’ does feel equivalent to ‘Either NOT $S_1$ or $S_2$’

a. (i) If the Red Sox don’t keep Ortiz, then there’s no chance for the pennant.
   (ii) Either the Red Sox keep Ortiz, or there’s no chance for the pennant.

b. (i) If you don’t hand in the exam, then you won’t get an A.
   (ii) Either you hand in the exam, or you won’t get an A.

However, as is well known, the simple extensional semantics in (7) has many insuperable problems.

2.1 First Problem for the Extensional Semantics: Falsehood Condition is Too Strong

(11) **The ‘Falsehood’-Conditions Predicted by (7)**

\[
[[ \text{If } S_1 \text{ then } S_2 ]] = F \text{ iff } \text{ Both } [[ S_1 ]] = T \text{ and } [[ S_2 ]] = F
\]

However, contrary to the prediction in (11), it seems that conditionals can be false event when:

- The antecedent is false, or
- The consequent is true.
(12) **False Conditionals Whose Antecedents are (Possibly) False**  
a. If the sun explodes tomorrow, then Alpha Centauri will explode tomorrow too.  
b. If Seth were a phonologist, then Alpha Centauri would explode.  
c. If McCain had won the 2008 election, then Alpha Centauri would have exploded.

(13) **False Conditionals Whose Consequents are (Possibly) True**  
a. If I wear the same sweater every day, then the sun will not explode next week.  
b. If UMass burns down, then we’ll have a nice spring.  
c. If Biden brushes his teeth, then the Republicans won’t confirm his nominee.

(14) **Key Observation**  
• In each of (12a,b,c), we can plausibly assume that their antecedents are false  
• In each of (13a,b,c), we can plausibly assume that their consequents are true  
• Nevertheless, each allows an interpretation which is false.

(15) **Key Intuition**  
The conditionals in (12)-(13) are false because the consequent doesn’t have any kind of ‘causal’ or ‘logical’ dependance on the antecedent proposition!

(16) **Some Discussion**  
• The natural intuition in (15) first led philosophers / logicians to explore a *modal* semantics for conditionals…  
• Under such an analysis, the conditional antecedent (‘if-clause’) semantically introduces universal quantification over all the worlds where the antecedent is true (16a)  
  a. **Rough Modal Semantics for Conditionals:**  
  \[
  [[ \text{If } S_1 \text{ then } S_2 ]]^w = T \quad \text{iff} \quad \forall w' \in \{ w'' : w'' \text{ is ‘relevantly similar’ to } w \} : [[ S_1 ]]^{w'} = T, \text{ then } [[ S_2 ]]^{w'} = T
  \]  
  • As the reader can confirm, this semantics in (16a) correctly predicts the falsity of the conditionals in (12)-(13)  
  • As discussed by von Fintel & Heim (2021: 25-28) this semantics also correctly captures a variety of facts regarding counterfactual conditionals like (6c)  
  • **However, although a modal semantics for conditionals is on the right track, there is evidence that the modality doesn’t arise from the ‘if-clause/marker’ itself…**
2.2 Second Problem for the Extensional Semantics: Consequents Containing Modals

Let us begin by considering the scenario in (17)...

(17) The Scenario: Lost on the Highway

Sandy and Kim are driving to the town of Lockhart for a party. They’ve gotten lost, though, and no longer know (exactly) what road they are on. **However, they do know the following:** They are either on Route 87 or Route 91. And, they’ve just passed a park. Sandy checks the map and sees the following picture:

![Map of Route 87 and Route 91]

- Route 87 passes by two parks, one in Lockhart and one in Petersburg
- Route 91 passes by one park, in Evanston

(18) The Crucial Data

If, after looking at the map, Sandy uttered (16a), she would be saying something true. If, after looking at the map, Sandy uttered (16b), she would be saying something false.

a. If we are on Route 87, then we might be in Lockhart. (True in (17))

b. If we are on Route 91, then we might be in Lockhart (False in (17))

So, we need our semantics for conditionals to predict that (18a) is true in scenario (17), while sentence (18b) is false...

- Let’s focus on (18b), and assume a structure where the modal *might* is interpreted within the consequent (*i.e.*, a surface scope LF)
(19) **Truth-Conditions Predicted by Our Extensional Account, Part 1**

a. **Structure at LF:**

   
   \[
   \begin{array}{ll}
   [ [ & \text{If we are on Route 91} ] [ \text{then we might be in Lockhart} ] ] \\
   \end{array}
   \]

b. **Predicted Truth Conditions (Rough, Informal Statement)**

   Either we are not on Route 91, or
   
   there are worlds compatible with our knowledge where we are in Lockhart.

(20) **Problem For Predicted Truth-Conditions in (19b):**

   The truth-conditions in (19b) are *extremely weak*. They hold as long as it is consistent with our knowledge that we are in Lockhart.

   - Moreover, note that in scenario (17), *it is consistent with Sandy’s knowledge that she is in Lockhart*
   - Therefore, the truth-conditions in (19b) wrongly predict that (18b) is true in (17)!

(21) **Truth-Conditions Predicted by Our Extensional Account, Part 2**

a. **Structure at LF:**

   
   \[
   \begin{array}{ll}
   \text{might} & [ [ \text{If we are on Route 91} ] [ \text{then we be in Lockhart} ] ] \\
   \end{array}
   \]

b. **Predicted Truth Conditions (Rough, Informal Statement)**

   There are worlds \(w’\) compatible with our knowledge in \(w\) such that either:
   
   (i) We are not on Route 91 in \(w’\), or
   (ii) We are in Lockhart in \(w’\)

(22) **Problem For Predicted Truth-Conditions in (21b):**

   Again, the truth-conditions in (20b) are *extremely weak*. They hold as long as it is consistent with our knowledge that we are not on Route 91.

   - And in scenario (17), *it is consistent with Sandy’s knowledge that she is on Route 87*
   - Therefore, the truth-conditions in (21b) wrongly predict that (18b) is true in (17)!
So, clearly, sentences (18a,b) do not have the meanings predicted by our extensional semantics...

But what exactly are the truth-conditions of (18a,b)?

(23) Crucial, Guiding Intuition:

- Sentence (18a) is true in context (17) because:

  If Sandy and Kim learned that they were on Route 87,
  they would conclude (given the map) that they might be in Lockhart.

- Sentence (18b) is false in context (17) because:

  If Sandy and Kim learned that they were on Route 91,
  they wouldn’t conclude (given the map) that they might be in Lockhart.

In the remainder of this handout, we will develop a compositional semantics for conditionals that captures the core intuition in (23)...

3. Conditional Antecedents as Modal Modifiers

Following our guiding intuition in (23), we might paraphrase the (intuitive) meaning of (18a,b) as follows:

(24) Paraphrases of the Truth-Conditions of (18a,b)

a. (i) Sentence: If we are on Route 87, then we might be in Lockhart.

   (ii) Paraphrase of Truth Conditions:

   If we (provisionally) add to our knowledge that we are on Route 87,
   then our knowledge would be consistent with our being in Lockhart.

b. (i) Sentence: If we are on Route 91, then we might be in Lockhart.

   (ii) Paraphrase of Truth Conditions:

   If we (provisionally) add to our knowledge that we are on Route 91,
   then our knowledge would be consistent with our being in Lockhart.

Given our background theory of epistemic modality, we could formalize these truth-conditional statements as in (25a,b)...
(25) **Formalized Restatement of the Paraphrases**

a. \[ [[ (18a) / (24ai) ]]^w = T \text{ iff } \exists w' \in \{ w'' : \text{what we know in } w \text{ is true in } w'' \text{ and we are on Route 87 in } w'' \} : \text{we are in Lockhart in } w'. \]

There is some world \( w' \) that is consistent with what we know and where we are on Route 87, and we are in Lockhart in \( w' \).

b. \[ [[ (18b) / (24bi) ]]^w = T \text{ iff } \exists w' \in \{ w'' : \text{what we know in } w \text{ is true in } w'' \text{ and we are on Route 91 in } w'' \} : \text{we are in Lockhart in } w'. \]

There is some world \( w' \) that is consistent with what we know and where we are on Route 91, and we are in Lockhart in \( w' \).

(26) **Key Observations:**

a. The truth-conditions in (25a) do indeed hold in scenario (17). Therefore, (25a) correctly predicts that (18a) is true in that scenario.

b. The truth-conditions in (25b) do not hold in scenario (17). Therefore, (25b) correctly predicts that (18b) is false in that scenario.

(27) **New Goal**

- Let us accept the truth-conditions in (25) as accurate.
- Let us, then, develop a system that will derive these truth-conditions…

3.1 **The Conditional Antecedent and the Modal Base**

(28) **Restatement of Goal**

- Clearly, to derive (25a,b), we want to somehow add the propositional content of the ‘if-clause’ to the modal base of the epistemic modal in (18a,b)

- There are many ways we could imagine doing that compositionally. A number of possibilities are discussed by von Fintel & Heim (2021).

- For our purposes, however, let’s just consider the following method.
The Semantics of ‘If’

\[
[[ \text{if} ]]^{w,t,g} = [\lambda p_{<s,t>} : \lambda B_{<s,<st,t>)} : \lambda w : B(w) \cup \{ p \} ]
\]

- Under this semantics, ‘if’ takes as argument a proposition \( p \) and a modal base (‘conversational background’) \( B \ldots \)
- It then returns a function from worlds to sets of propositions (type \( <s, \langle<s,t>,t\rangle> \)), which for any world \( w \), returns the set consisting of:
  1. All the propositions in \( B(w) \), combined with
  2. The proposition \( p \)

Illustration:

a. Syntax of a Conditional Antecedent:

```
BaseP
    BASE1
     CP
      C
       if
         S
           We are on Route 91
```

b. Semantics of Conditional Antecedent:

- Assume that \( g(1) = [\lambda w' : \{ p : \text{we know p in } w' \} ] \)

\[
[[ \text{ BaseP } ]]^{w,t,g} =
\]
\[
[[ \text{if} ]]^{w,t,g} ([[ \lambda w' : [[S]]^{w,t,g} ]]) ([[ BASE1 ]]^{w,t,g}) =
\]
\[
[[ \text{if} ]]^{w,t,g} (\lambda w' : \text{we are on route 91 in } w') (g(1)) =
\]
\[
[\lambda p_{<s,t>} : \lambda B_{<s,<st,t>)} : \lambda w : B(w) \cup \{ p \} ] ([\lambda w' : \text{we are on route 91 in } w'])(g(1)) =
\]
\[
[\lambda w : g(1)(w) \cup \{ \lambda w' : \text{we are on route 91 in } w' \} ] =
\]
\[
[\lambda w : \{ p : \text{we know p in } w \} \cup \{ \lambda w' : \text{we are on route 91 in } w' \} ]
\]

The function that maps a world \( w \) onto the following set of propositions:
- The propositions known in \( w \) combined with the proposition ‘we are on route 91’
3.2 The Compositional Semantics of Conditional Constructions

For simplicity’s sake, we will ignore the contribution of the ordering source to the semantics of an epistemic modal...

(31) a. **Sentence:** If we are on Route 87, we might be in Lockhart

b. **Syntax:**

```
S
   /\         \
ModalP VP
   /\         \
  Modal BaseP
     /\         \
   BASE1 CP
         /\         \
     C  if      S
```

```
we are on Route 87
```

```
if
```

```
we are in Lockhart
```

we are on Route 87

c. **Semantics:**

\[
[[ S ]]^{w,t,g} = T \quad \text{iff}
\]

\[
[[ \text{ModalP} ]^{w,t,g} ( [ \lambda w' : [[S]]^{w',t,g} ] ) = T \quad \text{iff}
\]

\[
[[ \text{ModalP} ]^{w,t,g} ( [ \lambda w' : \text{we are in Lockhart in } w' ] ) = T \quad \text{iff}
\]

\[
[[ \text{might} ]^{w,t,g} ( [ [ \text{BaseP} ]^{w,t,g} ( [ \lambda w' : \text{we are in Lockhart in } w' ] ) = T \quad \text{iff}
\]

\[
[ \lambda B_{<s,<t,>}: \lambda p : \exists w' \in \bigcap B(w) : p(w') = T ]
\]

\[
( [ [ \text{BaseP} ]^{w,t,g} ( [ \lambda w' : \text{we are in Lockhart in } w' ] ) = T \quad \text{iff}
\]

\[
\exists w' \in \bigcap [ [ \text{BaseP} ]^{w,t,g}(w) : \text{we are in Lockhart in } w' \quad \text{iff}
\]

\[
\exists w' \in \bigcap [ \lambda w : \{ p : \text{we know } p \text{ in } w \}
\]

\[
\cup \{ [ \lambda w' : \text{we are on route 87 in } w' ] \} ](w) : \text{we are in Lockhart in } w' \quad \text{iff}
\]

\[
\exists w' \in \bigcap [ \{ p : \text{we know } p \text{ in } w \} \cup \{ [ \lambda w' : \text{we are on route 87 in } w' ] \} ] : \text{we are in Lockhart in } w' \quad \text{iff}
\]

\[
\exists w' \in \{ w'' : \text{what we know in } w \text{ is true in } w'' \text{ and we are on Route 87 in } w'' \} : \text{we are in Lockhart in } w'.
\]
Interim Summary:

- As we see in (31), the semantics for ‘if’ in (29) is able to derive the desired truth-conditional statement in (25a) for sentence (18a)!

- As the reader can confirm, via a parallel calculation, we can derive the desired truth-conditional statement in (25b) for sentence (18b)!

- In this way, our semantics is able to correctly predict the reported truth-value judgments for (18a,b) in scenario (17)!

4. Further Discussion of the Analysis

In addition to the key results in (32), our proposed semantics for conditioinals also makes accurate predictions regarding the sentences in (33) below.

Conditionals that Contain Strong (Epistemic) Modals in the Consequents

a. Sentences:
   (i) If we are on Route 87, then we must be in Lockhart.
   (ii) If we are on Route 91, then we must not be in Lockhart.

b. Judgments:
   (i) Sentence (33ai) is false in scenario (17)
   (ii) Sentence (33aii) is true in scenario (17)

As the reader can confirm, our semantics predicts that the sentences in (33a) will have the truth-conditions below...

Truth-Conditions Predicted for the Sentences in (33a)

a. Truth Conditions for (33ai)
   \[ \forall w' \in \{ w : \text{we know in } w \text{ is true in } w' \text{ and we are on Route 87 in } w' \} : \text{we are in Lockhart in } w' \]

   In all worlds w’ that are consistent with what we know and where we are on Route 87, we are in Lockhart in w’.

b. Truth Conditions for (33aii)
   \[ \forall w' \in \{ w : \text{we know in } w \text{ is true in } w' \text{ and we are on Route 91 in } w' \} : \text{we are not in Lockhart in } w' \]

   In all worlds w’ that are consistent with what we know and where we are on Route 91, we are not in Lockhart in w’.
(35) **Truth-Conditions in (34) Predict Judgments in (33b)**

a. **For Sentence (33ai)**
   In context (17), there is a possible world which is consistent with the information on the map, and in which Sandy & Kim are on Route 87, but where they are in Petersburg – not Lockhart.

b. **For Sentence (33aii)**
   Consider any possible world which is consistent with the information on the map, and in which Sandy & Kim are on Route 91. Clearly, since 91 doesn’t go through Lockhart, in any such possible world, they are not in Lockhart.

(36) **Major Question**

- The system developed above works very well for sentences where ‘if-clauses’ combine with sentences containing epistemic modals…

- But what about sentences where the consequent of the conditional doesn’t appear to contain a modal?

(37) **A Common Response: The Ubiquity of Modals**

Actually, whenever you have a conditional construction, there is some (possibly covert) modal operator whose ‘BASE’ the antecedent is modifying / restricting.

- Thus, while the conditional sentence as a whole does indeed have a modal semantics, the modality is contributed by an independent modal operator in the consequent (16)

(38) **Basic Supporting Evidence for (37)**

- In the most canonical examples of English conditionals (1), the consequent contains some form of the auxiliary ‘WOLL’ (i.e., ‘will’, ‘won’t’, ‘would’, ‘wouldn’t’, etc.)

- As we discussed in our unit on tense & aspect, this auxiliary in English is a kind of modal auxiliary.

(39) a. **Natural Follow-Up Question:**
   Now that we have a more detailed theory of modal semantics on the table, what exactly is the modal semantics of English ‘WOLL’?

b. **The Commonly-Held Answer:**
   English ‘WOLL’ primarily allows for circumstantial readings. Thus, it has a circumstantial base and a stereotypical ordering source.
(40) **Potential Challenge to (37)**
It does seem possible for conditional constructions in English not to contain any modal auxiliary in their consequents.

(41) **Conditionals with No (Overt) Modals Elements in their Consequents**

a. If it is three o’clock, then Dave is in NYC.
b. If the red blotches don’t go away, then you have eczema.
c. If the Red Sox trade Ortiz, there’s no chance for the pennant.
d. If Dave is worried, then I’m definitely worried.

(42) **Observation:**
Nevertheless, each of the conditionals in (41) does seem to be equivalent to a conditional that contains an overt modal in its consequent.

(43) **Semantically Equivalent Conditionals that Do Contains Modals**

a. If it is three o’clock, then Dave must be in NYC.  
   (‘epistemic’ must)
b. If the red blotches don’t go away, then you must have eczema.  (‘epistemic’ must)
c. If the Red Sox trade Ortiz, there will be no chance for the pennant.
d. If Dave is worried, then I will definitely be worried.

(44) **Key Idea: Covert Modals in English Conditionals**

- Perhaps the underlying structure of the conditionals in (41) is actually identical to the intuitively equivalent ones in (43)…

- That is, each of the conditionals in (41) contain some kind of ‘phonologically null’ modal operator:

a. If it is three o’clock, then Dave ØMUST [ is in NYC ].
b. If the red blotches don’t go away, then you ØMUST [ have eczema ].
c. If the Red Sox trade Ortiz, there ØWILL [ is no chance for the pennant ].
d. If Dave is worried, then I ØWILL [ am definitely worried ].

“The history of the conditional is the story of a syntactic mistake. There is no two-place “if...then” connective in the logical forms of natural languages. “If”-clauses are devices for restricting the domains of various operators. **Whenever there is no explicit operator, we have to posit one.** As shown above, epistemic modals are candidates for such hidden operators…”