Negative Vega? Understanding Options on Spreads

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The recent proliferation of hedge funds has brought to the forefront the types of trading strategies that distinguish hedge funds from the more conservative mutual funds. Two major sets of trading strategies have been in the spotlight: market-neutral trades, and trades that rely on the spread between the prices of two assets. Such strategies typically make use of exotic financial instruments. To be successfully used, the value of such an exotic financial instrument must be well understood, especially as the sensitivity of its value to underlying factors may be very different from its plain-vanilla counterpart. The purpose of this article is examine the determinants of the value of one such popular financial instrument: the option on a spread.

The spread option is an option on the difference in the price of two underlying assets. An example of such an option is the option on the crack spread, traded on the New York Mercantile Exchange (NYMEX). The spread option can be easily used to implement trading strategies—for example, a view on the volatility of the spread may be implemented by taking a position in the spread option, and making it market-neutral with appropriate positions in the underlying assets.

In comparison with plain-vanilla calls and puts, the value of an option on a spread has a complicated relation with its underlying components. In particular, the relation of the value of the spread option to the volatilities of its component assets is very different from that of calls and puts to the volatility of their underlying asset. It is well known that plain-vanilla options have positive vega, where vega is the sensitivity of the option price to changes in volatility. In other words, an increase in the volatility of the underlying asset increases the value of a call or a put. The intuition underlying why the option price is a positive monotonic function of volatility is easy to understand: as the payoff on a call is truncated, an increase in volatility increases the possible gains while the magnitude of the possible loss is strictly limited and not affected by the increase in volatility. Therefore, the net effect is that the option price increases as volatility increases. However, when we consider an option on a spread, an increase in volatility of one of the assets may not increase the option value, so that the option may exhibit *negative* vega with respect to the volatility of one of the two assets.

The specific objective of this article is to explain the relation of the value of the spread option to the volatilities of its component assets, and when and why the spread option may have a negative vega. The apparent paradox of a negative vega can be explained by realizing that the underlying asset on a spread option is the spread between the prices of the two component assets, and not the price of either one of the two assets. Although an increase in volatility of the spread will *always* increase the value of the option on the spread, an increase in the volatility of a component asset may not increase the volatility of the spread, and, therefore, may not increase the option value. Thus, the option on the spread will have a positive vega with respect to the spread volatility, but may have a negative or positive vega with respect to the volatility of one of the assets that comprise the spread.

To understand the effect of volatility changes on the option value, it is necessary to understand the relation between the volatility of the spread and the volatility of the component asset. We will show that the relation between the volatility of the underlying asset and the volatility of the spread is determined by three factors: the ratio of the individual asset volatilities, the ratio of the prices of the two assets, and the correlation between the two assets. We provide the specific conditions under which the option may have a negative or positive vega with respect to each of the component assets in the spread. We also provide an application of the analysis with respect to the option on a crack spread.

THE MODEL

In this section, we set up a simple model that allows analysis of the relation between the volatility of the spread and the individual asset volatilities.

Consider two assets, S_1 and S_2, and let the payoff of a spread option be $\mathrm{Max}(S_1 - S_2 - K, 0)$, where K is the exercise price. Let both the assets follow geometric Brownian motion,

$$dS_1 = \mu_1 S_1\, dt + \sigma_1 S_1\, dz_1 \tag{1}$$

$$dS_2 = \mu_2 S_2\, dt + \sigma_2 S_2\, dz_2 \tag{2}$$

where z_1 and z_2 are correlated Wiener processes, with correlation coefficient ρ. Define the process for the spread, $X = S_1 - S_2$. Then, by Ito's lemma, the stochastic evolution of this process is given by:

$$dX = \frac{\partial X}{\partial S_1} dS_1 + \frac{\partial X}{\partial S_2} dS_2 + \frac{1}{2}\frac{\partial^2 X}{\partial S_1 \partial S_2} dS_1\, dS_2 \tag{3}$$

This can be simplified by noting that the cross-partial derivative of X is 0 and that $\partial X/\partial S_1 = 1$ and $\partial X/\partial S_2 = -1$. Thus, we get:

$$dX = dS_1 - dS_2 \tag{4}$$

Let the variance of the spread be ω^2. Then,

$$\omega^2 = \sigma_1^2 S_1^2 + \sigma_2^2 S_2^2 - 2\rho\sigma_1\sigma_2 S_1 S_2 \tag{5}$$

where ρ is the correlation between and S_1 and S_2. By differentiating ω^2 by the variance of one of the underlying assets, we can derive the relation between the volatility of the spread, ω, and the individual volatilities of the assets, σ_1 and σ_2. Differentiating with respect to σ_1^2, we get,

$$\frac{\partial \omega^2}{\partial \sigma_1^2} = S_1^2 - \rho\frac{\sigma_2 S_2 S_1}{\sigma_1} \tag{6}$$

A similar relation holds true for the second asset. Equation (6) shows the conditions under which the spread volatility decreases with an increase in σ_1, i.e., $\partial\omega^2/\partial\sigma_1^2 < 0$. First, the two underlying assets must be positively correlated, $\rho > 0$, and second, the following relation must hold,

$$\rho\frac{\sigma_2}{\sigma_1}\frac{S_2}{S_1} > 1 \tag{7}$$

If both of the above conditions hold, then an increase in the volatility, σ_1, will decrease the volatility of the spread, and thus decrease the value of the spread option. Similarly, with respect to the other asset, if $\rho > 0$, and $\rho(\sigma_1/\sigma_2)(S_1/S_2) > 1$, then $\partial\omega^2/\partial\sigma_2^2 < 0$, and an increase in the volatility, σ_2, will decrease the volatility of the spread and the value of the option.

The analysis makes it clear that a negative vega is possible only for options on spreads where the underlying assets are positively correlated. However, for many spreads that are commonly traded, including crack spreads, the correlation between the component assets is positive. It is then necessary to examine equation (7) to note the conditions for a negative vega.

The condition in (7) indicates there are three factors that determine the sign: the ratio of the prices of the two assets, the ratio of the volatilities of the two assets, and the correlation. This condition is graphically represented in Exhibit 1. Exhibit 1 delineates the region where an increase in volatility, σ_1, increases or decreases the volatility of the spread, ω, for a fixed ratio of σ_1/σ_2 and varying correlations and ratio of the prices, S_1/S_2. The graphs may be interpreted as follows. The region below each graph shows the region over which $\partial\omega/\partial\sigma_1 < 0$, and the region above the graph corresponds to $\partial\omega/\partial\sigma > 0$. For example, consider the solid line that corresponds to the graph for $\sigma_1/\sigma_2 = 0.8$. If the correlation between the returns is 0.8, then the graph indicates that $\partial\omega/\partial\sigma_1$ is negative for $S_2/S_1 < 1$, and positive otherwise.

We can make several conclusions from Exhibit 1 and equation (7). If $S_1 > S_2$ and $\sigma_1 > \sigma_2$, then an increase in σ_1 will increase the volatility of the spread, irrespective of the magnitude of the correlation between the two assets, i.e., $\partial\omega/\partial\sigma_1 > 0$. Second, if $S_1 < S_2$ and $\sigma_1 < \sigma_2$, then $\partial\omega/\partial\sigma_1$ will be positive for low values of the correlation and negative for high values of the correlation. This also implies that there exists a value of the correlation coefficient, ρ, for which a change in the volatility, σ_1, has no effect on the volatility of the spread. Third, when the spread between the prices is positive ($S_1 > S_2$) but the spread between the volatilities is negative ($\sigma_1 < \sigma_2$), or vice versa ($S_1 < S_2$ and $\sigma_1 > \sigma_2$), then $\partial\omega/\partial\sigma_1$ may be negative only if $\sigma_2/\sigma_1 > S_1/S_2$.

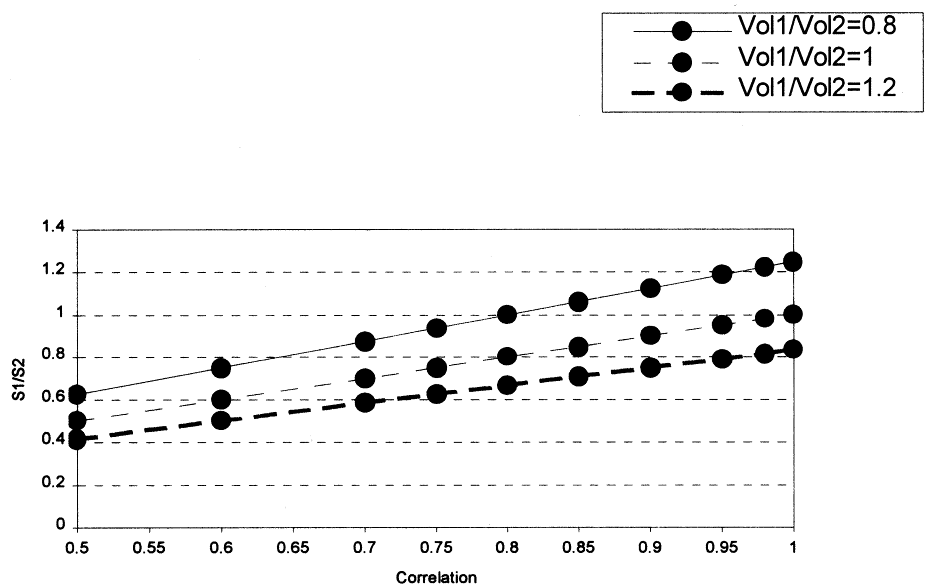
As the price of both the assets, the volatilities, and the correlation fluctuate with market conditions, it is possible that an increase in volatility of an asset in the spread may increase the price of the option at one time, and decrease it at another. This makes it critical for a spread option trader to continually monitor the volatility risk of his position. Below we provide an application of the analysis to one of the most important options on a spread, the crack spread.

AN APPLICATION: CRACK SPREADS

The crack spread is the difference between the price of a refined crude product, like heating oil or gasoline, and the price of crude petroleum itself. The option on the crack spread has traded on the New York Mercantile Exchange since October 7, 1994.

Consider, as an example, the crack spread between gasoline (GAS) and crude oil (WTI). On October 1, 1998, the December 1998 settlement on the NYMEX was $S_{GAS} = 19.31$ for gasoline and $S_{WTI} = 15.50$ for crude, so that $S_{GAS}/S_{WTI} = 1.25$. We can

EXHIBIT 1
Region for Negative Vega



use Exhibit 1 to estimate the conditions under which the option on the spread will have a negative or positive vega with respect to the volatility of gasoline (σ_{GAS}) or crude (σ_{WTI}).

First, consider the analysis with respect to the sign of $\partial\omega/\partial\sigma_{GAS}$. Noting that the spread between gasoline and crude is always positive, Exhibit 1 indicates that $\partial\omega/\partial\sigma_{GAS}$ will be negative only if the volatility of gasoline is less than the volatility of crude ($\sigma_{GAS}/\sigma_{WTI} < 1$) and if the correlation is close to 1. On average, the volatility of gasoline is greater than the volatility of crude. During the period of January, 1990 to October, 1998, the historical volatility of gasoline averaged 37% as compared with 33% for crude oil (also see Garbade [1991, 1992]). Although it is possible that over shorter periods, the volatility of crude exceeds that of gasoline, it is unlikely that such periods would correspond to high correlations. Therefore, we may conclude that, except under unusual market circumstances, an increase in the volatility of gasoline will increase the volatility of the spread and thus the value of the crack spread option.

Next, consider the effect of an increase in the volatility of crude. Noting that the ratio of the price of crude to the price of HO is 0.80, and that the ratio of the volatility of crude to the volatility is likely to be less than 1, Exhibit 1 indicates that the sign of the vega will be positive for low values of the correlation coefficient and negative for high values of the correlation coefficient. For example, if $\sigma_{WTI}/\sigma_{GAS} = 0.9$, then $\partial\omega/\partial\sigma_{WTI}$ is negative for $\rho > 0.72$, and positive otherwise. The correlation depends on the contract that is traded, and is usually higher for the far-off contracts. Garbade [1992] reports correlations between crude oil and gasoline ranging from 0.54 to 0.93, with the correlation increasing for further away contracts. The range of correlation indicates that, depending on prevalent market conditions, an increase in the volatility of crude could either increase *or* decrease the volatility of the spread and, thus, the value of the spread option.

The analysis has important implications for the trading and risk management of spread positions, for although it is relatively easy to hedge the price risk in the spread by delta-hedging the option, it is far more difficult to hedge the volatility risk. Between the two assets in the crack spread, the volatility risk of the refined product is easier to understand than the volatility risk of crude as a long position in the spread option also corresponds to a long position in the volatility of the product. However, as the above analysis indicated, the volatility risk of crude is more difficult to analyze and hedge, as an increase in the volatility of crude may either increase or decrease the value of the option. Thus, a long position in a spread, depending on market conditions, may correspond to either a long or a short position in the volatility of crude. As a result, the crack spread option is far more complicated to trade than a plain-vanilla call or put.

CONCLUSION

We have analyzed the effect of a change in the volatility of one of the underlying assets on the volatility of the spread option, and shown that, depending on market conditions, the volatility of the spread may increase or decrease. The relation between the spread volatility and the individual asset volatility depends on three factors: the ratio of the prices of the two assets, the ratio of the volatilities, and the correlation between the two assets. As all three of these factors are subject to market fluctuations, the magnitude and sign of the relation also fluctuates. An application to the crack spread between gasoline and crude oil indicates that it is likely that an increase in the volatility of the product will increase the value of the option, while an increase in the volatility of crude oil has an uncertain effect. Depending on the magnitude of the price spread, the volatilities and the correlation, an increase in the volatility of crude oil may increase or decrease the value of the option.

ENDNOTE

The author thanks Thomas Henker for comments.

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