

standard probabilistic practice of saying that E provides evidence for H just in case $P(H|E) > P(H)$.) Whether one's resulting credence in the theory of quantum mechanics would be so low that one would reject quantum mechanics would depend on details like one's prior probability for Theory X. Even with a probability shift in favour of Theory X, one's credences could still be such that quantum mechanics is judged vastly more probable than Theory X.

Just as the observation of the decay can provide evidence for Theory X and against quantum mechanics, the observation that we are in a small civilization can provide evidence for or against various physical theories. For example, consider Theory Y, which predicts that all civilizations are small. The fact that our civilization is small could constitute evidence in favour of Theory Y and against inflationary theory. But depending on such factors as one's prior probability for Theory Y, one's credences could still be such that inflationary theory is judged vastly more probable than Theory Y. Thus, that our civilization is small could lead one to decrease one's credence in inflationary theory, but nevertheless that credence could still be high.

The issue we are discussing in the previous paragraph is a general issue about theory comparison in science; if that is the core of Olum's paper then there is nothing surprising here. It would be surprising if there were a conflict between anthropic reasoning and observation, but Olum's argument fails to show that this is the case.

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Reference

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Does Frege have too many thoughts? A Cantorian problem revisited

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In two recent papers in this journal, Adam Rieger (2002) and Nicholas Denyer (2003) discuss a problem with Frege's ontology. According to

Rieger, Frege regards thoughts as objects. But this is impossible, because one can generate a distinct thought for every concept, in violation of Cantor's theorem whereby there must be more concepts than objects. Denyer points out that Rieger's argument involves a questionable hidden assumption. I will suggest that Denyer won that battle, but that the war to save Frege was lost a long time ago.

Rieger's argument is as follows. For each concept, consider the thought that Ben Lomond falls under that concept. He assumes (problematically, as we shall see) that the thought so generated is distinct for each concept. Call these the Ben Lomond thoughts. Now consider the concept of being an *ordinary Ben Lomond thought*. Something is an ordinary Ben Lomond thought iff it is a Ben Lomond thought but does not fall under the concept it attributes to Ben Lomond. From this concept we can generate a Ben Lomond thought, viz. the thought that Ben Lomond is an ordinary Ben Lomond thought. The problem is of course whether or not this resulting Ben Lomond thought is ordinary. It's ordinary iff it does not fall under the concept it attributes to Ben. But it attributes ordinariness to Ben, so it's ordinary iff it is not.

Denyer retorts that the problem doesn't have to be traced back to treating thoughts as objects, and can instead be traced back to the naive assumption that each Ben Lomond thought is associated with exactly one concept. Denyer gives a more complicated example, but consider the thought that Ben Lomond loves Ben Lomond. This thought can be decomposed as attributing any of three concepts to Ben Lomond: the concept something falls under iff it loves Ben Lomond, the concept something falls under iff Ben Lomond loves it, or the concept something falls under iff it loves itself. Without the assumption that the Ben Lomond thought generated for each concept is distinct, the antinomy does not go through. It is also fairly easy to see that there is no violation of Cantor's theorem, because without this assumption the proof that there are at least as many objects as concepts evaporates.

Moreover, as he admits in a footnote, Rieger also speaks loosely by suggesting that for each concept, there is such a thing as *the thought* that Ben Lomond falls under that concept. As Frege (1979: 118–25) makes clear, the sense/reference distinction applies also to concept expressions. Fregean concepts are functions and have extensional identity conditions. The expressions '... is non-self-identical' and '... is a unicorn' refer to the same concept, despite having different senses. So the thoughts expressed by 'Ben Lomond is non-self-identical', and 'Ben Lomond is a unicorn', attribute the same concept to Ben, but are surely distinct. This point, however, is not as crucial. If Frege is committed to *more than one* thought for each concept, that would, if anything, make the Cantorian problems worse, not better.

While Rieger's specific attempt at a Cantorian construction does not work, there are surely better ones. Instead of considering Ben Lomond thoughts, we can consider thoughts that *universally generalize upon* concepts. For each pair consisting of a concept C , and a sense that presents it C^* , we can generate a distinct thought by considering the thought that *everything is C* , with C^* being the operant sense presenting C . Call the thoughts so generated *universal thoughts*. Now define a concept O' that something falls under if and only if it is a universal thought that does not fall under the concept it generalizes upon. We must assume that there is at least one sense O'^* , presenting O' . This may seem questionable, but consider that the sense of the expression '*... is a universal thought that does not fall under the concept it generalizes upon*' would appear to be one such sense. Hence, there is a universal thought *everything is O'* (as presented through O'^*). Does it fall under O' ? Apparently, it does just in case it does not.

Does this construction suffer the same defect as Rieger's? I fail to see how. Frege understands quantifiers as second-level concepts. The sense expressed by '*everything is ...*', a sense whose referent is a second-level concept, has the sort of incompleteness that is filled by a sense presenting a first-level concept. The thought expressed would seem to differ for each such sense placed in that spot. *Everything is such that it loves Lomond* is a distinct thought from *everything is such that Lomond loves it*, and so forth. Here, the worries about multiple possible 'decompositions' do not seem to arise. Frege is in trouble.

The charge that Frege's ontology falls prey to this sort of Cantorian problem is not new. A few months after reporting Russell's paradox, Russell despaired in a letter to Frege that 'from Cantor's proposition that any class contains more subclasses than objects we can elicit constantly new contradictions' (see Frege 1980: 147), and reports on his discovery of a Cantorian paradox regarding *propositions*, suggesting that, *mutatis mutandis*, this should be a worry for Frege's theory of thoughts. In the ensuing correspondence, Russell formulates the paradox in terms of both generating a distinct proposition for every class, as well as generating a distinct proposition for every propositional function. Frege never quite fully appreciated Russell's point, but nevertheless Russell had put his finger on a genuine issue Frege *should have* been concerned with, as I have argued elsewhere (Klement 2001, 2002: chs. 5–6).

Years later, the same sort of Cantorian problem regarding thoughts or propositions plagued Alonzo Church's initial attempts to devise a 'Logic of Sense and Denotation'. Church (1951) had (plausibly enough) included as an axiom in one of his formulations the principle that distinct universal thoughts are generated by each sense presenting a function onto truth-values. Accordingly, John Myhill (1958) used, essentially, the Cantorian

construction I describe above to show that Church's system was inconsistent. Hence, C. A. Anderson (1980) has dubbed this paradox 'the Russell-Myhill antinomy'.

In some of my recent work, I have attempted to highlight all the possible ways someone with broadly Fregean commitments could escape from such paradoxes (Klement 2002, 2003). With regard to the particular Cantorian construction sketched above, the most initially promising responses would be (1) denying there is any such concept as O' , (2) denying that there is any sense O'^* presenting the concept O' (since, if there is no such sense, there can be no thoughts about O'), (3) denying that the concept O' is the only concept that the thought that *everything is O'* generalizes over, and (4) denying that the thought that *everything is O'* is of the appropriate logical type for the question as to whether it falls under O' to be meaningful. We can form complex function expressions that, at least apparently, are satisfied by all and only things that are O' . Hence, solution (1) would entail giving up the standard comprehension principle used in higher-order function logic; this could be done, but I doubt the historical Frege would have gone along happily. Solution (2) is similarly difficult to swallow, since the complex function expressions satisfied by all and only things that are O' would seem to have O' as referent, and therefore must express senses that present O' . Denyer's solution to Rieger's version would point us towards attempting solution (3), but I have already cast suspicion on such an attempt.

That only leaves solution (4). Perhaps it is the solution Rieger would advocate, since he suggests that the problem is that thoughts are objects. If O' is a concept that takes objects as argument, but the thought that *everything is O'* is not an object, then the question doesn't arise as to whether it falls under O' . But the issue is really more complicated than that. Presumably if thoughts are of a different logical type, then we must merely distinguish *concepts that apply (or not) to objects* from *concepts that apply (or not) to thoughts*. Which one is O' ? Recall that O' is the function mapping all universal thoughts not falling under the concept they generalize upon to the True, and everything else (in the appropriate logical category) to the False. So O' can be considered in the latter category, and so denying that thoughts are objects provides no help (modulo an interpretation of 'everything is ...' to limit itself to the appropriate logical category). Indeed, Church's type-theory made such distinctions as between type o_0 (truth-values), type o_1 (senses whose referents are truth-values, i.e., thoughts), type o_2 (senses whose referents are thoughts), and so on, as well as between types t_0 (individuals), t_1 (senses whose referents are individuals), t_2 (senses whose referents are senses of type t_1), etc. These type distinctions provided him with no help in replying to Myhill. Solution (4) will only work either if thoughts are not placed in any logical category

at all (e.g. if we simply eschew them altogether), or if thoughts are split into different *ramified orders* so that no thought can involve quantification over a range of thoughts in which it is included. Russell accepted the ‘harsh and highly artificial’ (Russell 1903: 528) solution given by ramification only after years of searching for others; there’s no indication Frege would have been happy with it at all.

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Knowledge in borderline cases

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1. Vagueness gives rise to borderline cases. If *F* is vague, then there will typically be cases to which *F* clearly applies, cases to which it clearly doesn’t apply and intermediate cases which we don’t know how to classify.