Tractatus Logico-Philosophicus
Logisch-philosophische Abhandlung
By Ludwig Wittgenstein

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Introduction

By Bertrand Russell, F.R.S.

Mr. Wittgenstein’s *Tractatus Logico-Philosophicus*, whether or not it prove to give the ultimate truth on the matters with which it deals, certainly deserves, by its breadth and scope and profundity, to be considered an important event in the philosophical world. Starting from the principles of Symbolism and the relations which are necessary between words and things in any language, it applies the result of this inquiry to various departments of traditional philosophy, showing in each case how traditional philosophy and traditional solutions arise out of ignorance of the principles of Symbolism and out of misuse of language.

The logical structure of propositions and the nature of logical inference are first dealt with. Thence we pass successively to Theory of Knowledge, Principles of Physics, Ethics, and finally the Mystical (*das Mystische*).

In order to understand Mr. Wittgenstein’s book, it is necessary to realize what is the problem with which he is concerned. In the part of his theory which deals with Symbolism he is concerned with the conditions which would have to be fulfilled by a logically perfect language. There are various problems as regards language. First, there is the problem what actually occurs in our minds when we use language with the intention of meaning something by it; this problem belongs to psychology. Secondly, there is the problem as to what is the relation subsisting between thoughts, words, or sentences, and that which they refer to or mean; this problem belongs to epistemology. Thirdly, there is the problem of using sentences so as to convey truth rather than falsehood; this belongs to the special sciences dealing with the subject-matter of the sentences in question. Fourthly, there is the question: what relation must one fact (such as a sentence) have to another in order to be capable of being a symbol for that other? This last is a logical question, and is the one with which Mr. Wittgenstein is concerned. He is concerned with the conditions for accurate Symbolism, i.e. for Symbolism in which a sentence “means” something quite definite. In practice, language is always more or less vague, so that what we assert is never quite precise. Thus, logic has two problems to deal with in regard to Symbolism: (1) the conditions for sense rather than nonsense in combinations of symbols; (2) the conditions for uniqueness of meaning or reference in symbols or combinations of symbols. A logically perfect language has rules of syntax which prevent nonsense, and has single symbols which always have a definite and unique meaning. Mr. Wittgenstein is concerned with the conditions for a logically perfect language—not that any language is logically perfect, or that we believe ourselves capable, here and now, of constructing a logically perfect language, but that the whole function of language is to have meaning, and it only fulfills this function in proportion as it approaches to the ideal language which we postulate.

The essential business of language is to assert or deny facts. Given the syntax of language, the meaning of a sentence is determined as soon as the meaning of the component words is known. In order that a certain sentence should assert a certain fact there must, however the language may be constructed, be something in common between the structure of the sentence and the structure of the fact. This is perhaps the most fundamental thesis of Mr. Wittgenstein’s theory. That which has to be in common between the sentence and the fact cannot, he contends, be itself in turn *said* in language. It can, in his phraseology, only be *shown*, not said, for whatever we may say will still need to have the same structure.

The first requisite of an ideal language would be that there should be one name for every simple, and never the same name for two different simples. A name is a simple symbol in the sense that it has no parts which are themselves symbols. In a logically perfect language nothing that is not simple will have a simple symbol. The symbol for the whole will be a “complex”, containing the symbols for the parts. (In speaking of a “complex” we are, as will appear later, sinning against the rules of philosophical grammar, but this is unavoidable at the outset. “Most propositions and questions that have been written about philosophical matters are not false but senseless. We cannot, therefore,
answer questions of this kind at all, but only state their senselessness. Most questions and propositions of the philosophers result from the fact that we do not understand the logic of our language. They are of the same kind as the question whether the Good is more or less identical than the Beautiful” (4.003). What is complex in the world is a fact. Facts which are not compounded of other facts are what Mr. Wittgenstein calls Sachverhalte, whereas a fact which may consist of two or more facts is a Tatsache: thus, for example “Socrates is wise” is a Sachverhalt, as well as a Tatsache, whereas “Socrates is wise and Plato is his pupil” is a Tatsache but not a Sachverhalt.

He compares linguistic expression to projection in geometry. A geometrical figure may be projected in many ways: each of these ways corresponds to a different language, but the projective properties of the original figure remain unchanged whichever of these ways may be adopted. These projective properties correspond to that which in his theory the proposition and the fact must have in common, if the proposition is to assert the fact.

In certain elementary ways this is, of course, obvious. It is impossible, for example, to make a statement about two men (assuming for the moment that the men may be treated as simples), without employing two names, and if you are going to assert a relation between the two men it will be necessary that the sentence in which you make the assertion shall establish a relation between the two names. If we say “Plato loves Socrates”, the word “loves” which occurs between the word “Plato” and the word “Socrates” establishes a certain relation between these two words, and it is owing to this fact that our sentence is able to assert a relation between the persons named by the words “Plato” and “Socrates”. “We must not say, the complex sign ‘aRb’ says that ‘a stands in a certain relation R to b’, but we must say, that ‘a’ stands in a certain relation to ‘b’ says that aRb” (3.1432).

Mr. Wittgenstein begins his theory of Symbolism with the statement (2.1): “We make to ourselves pictures of facts.” A picture, he says, is a model of the reality, and to the objects in the reality correspond the elements of the picture: the picture itself is a fact. The fact that things have a certain relation to each other is represented by the fact that in the picture its elements have a certain relation to one another. “In the picture and the pictured there must be something identical in order that the one can be a picture of the other at all. What the picture must have in common with reality in order to be able to represent it after its manner—rightly or falsely—is its form of representation” (2.161, 2.17).

We speak of a logical picture of a reality when we wish to imply only so much resemblance as is essential to its being a picture in any sense, that is to say, when we wish to imply no more than identity of logical form. The logical picture of a fact, he says, is a Gedanke. A picture can correspond or not correspond with the fact and be accordingly true or false, but in both cases it shares the logical form with the fact. The sense in which he speaks of pictures is illustrated by his statement: “The gramophone record, the musical thought, the score, the waves of sound, all stand to one another in that pictorial internal relation which holds between language and the world. To all of them the logical structure is common. (Like the two youths, their two horses and their lilies in the story. They are all in a certain sense one)” (4.014). The possibility of a proposition representing a fact rests upon the fact that in it objects are represented by signs. The so-called logical “constants” are not represented by signs, but are themselves present in the proposition as in the fact. The proposition and the fact must exhibit the same logical “manifold”, and this cannot be itself represented since it has to be in common between the fact and the picture. Mr. Wittgenstein maintains that everything properly philosophical belongs to what can only be shown, or to what is in common between a fact and its logical picture. It results from this view that nothing correct can be said in philosophy. Every philosophical proposition is bad grammar, and the best that we can hope to achieve by philosophical discussion is to lead people to see that philosophical discussion is a mistake. “Philosophy is not one of the natural sciences. (The word ‘philosophy’ must mean something which stands above or below, but not beside the natural sciences.) The object of philosophy is the logical clarification of thoughts. Philosophy is not a theory but an activity. A philosophical work consists essentially of elucidations. The result of philosophy is not a number of ‘philosophical propositions’, but to make propositions clear. Philosophy should make clear and delimit sharply the thoughts which otherwise are, as it were, opaque and blurred” (4.111 and 4.112). In accordance with this principle the
things that have to be said in leading the reader to understand Mr. Wittgenstein's theory are all of them things which that theory itself condemns as meaningless. With this proviso we will endeavour to convey the picture of the world which seems to underlie his system.

The world consists of facts: facts cannot strictly speaking be defined, but we can explain what we mean by saying that facts are what makes propositions true, or false. Facts may contain parts which are facts or may contain no such parts; for example: “Socrates was a wise Athenian”, consists of the two facts, “Socrates was wise”, and “Socrates was an Athenian.” A fact which has no parts that are facts is called by Mr. Wittgenstein a Sachverhalt. This is the same thing that he calls an atomic fact. An atomic fact, although it contains no parts that are facts, nevertheless does contain parts. If we may regard “Socrates is wise” as an atomic fact we perceive that it contains the constituents “Socrates” and “wise”. If an atomic fact is analyzed as fully as possible (theoretical, not practical possibility is meant) the constituents finally reached may be called “simples” or “objects”. It is a logical necessity demanded by theory, like an electron. His ground for maintaining that there must be simples is that every complex presupposes a fact. It is not necessarily assumed that the complexity of facts is finite; even if every fact consisted of an infinite number of atomic facts and if every atomic fact consisted of an infinite number of objects there would still be objects and atomic facts (4.2211). The assertion that there is a certain complex reduces to the assertion that its constituents are related in a certain way, which is the assertion of a fact: thus if we give a name to the complex the name only has meaning in virtue of the truth of a certain proposition, namely the proposition asserting the relatedness of the constituents of the complex. Thus the naming of complexes presupposes propositions, while propositions presuppose the naming of simples. In this way the naming of simples is shown to be what is logically first in logic.

The world is fully described if all atomic facts are known, together with the fact that these are all of them. The world is not described by merely naming all the objects in it; it is necessary also to know the atomic facts of which these objects are constituents. Given this totality of atomic facts, every true proposition, however complex, can theoretically be inferred. A proposition (true or false) asserting an atomic fact is called an atomic proposition. All atomic propositions are logically independent of each other. No atomic proposition implies any other or is inconsistent with any other. Thus the whole business of logical inference is concerned with propositions which are not atomic. Such propositions may be called molecular.

Wittgenstein’s theory of molecular propositions turns upon his theory of the construction of truth-functions.

A truth-function of a proposition $p$ is a proposition containing $p$ and such that its truth or falsehood depends only upon the truth or falsehood of $p$, and similarly a truth-function of several propositions $p, q, r, \ldots$ is one containing $p, q, r, \ldots$ and such that its truth or falsehood depends only upon the truth or falsehood of $p, q, r, \ldots$. It might seem at first sight as though there were other functions of propositions besides truth-functions; such, for example, would be “A believes $p$”, for in general A will believe some true propositions and some false ones: unless he is an exceptionally gifted individual, we cannot infer that $p$ is true from the fact that he believes it or that $p$ is false from the fact that he does not believe it. Other apparent exceptions would be such as “$p$ is a very complex proposition” or “$p$ is a proposition about Socrates”. Mr. Wittgenstein maintains, however, for reasons which will appear presently, that such exceptions are only apparent, and that every function of a proposition is really a truth-function. It follows that if we can define truth-functions generally, we can obtain a general definition of all propositions in terms of the original set of atomic propositions. This Wittgenstein proceeds to do.

It has been shown by Dr. Sheffer (Trans. Am. Math. Soc., Vol. XIV, pp. 481–488) that all truth-functions of a given set of propositions can be constructed out of either of the two functions “not-$p$ or not-$q$” or “not-$p$ and not-$q$”. Wittgenstein makes use of the latter, assuming a knowledge of Dr. Sheffer’s work. The manner in which other truth-functions are constructed out of “not-$p$ and not-$q$” is easy to see. “Not-$p$ and not-$q$” is equivalent to “not-$p$, or not-$q$”, hence we obtain a definition of negation in terms of our primitive function: hence we can define “$p$ or $q$”, since this is the negation of “not-$p$ and not-$q$”, i.e. of our primitive function. The development of other truth-functions out of “not-$p$ and “$p$ or $q$” is given in detail at the beginning of Principia Mathematica. This gives all that is wanted when the propositions which are argu-
ments to our truth-function are given by enumeration. Wittgenstein, however, by a very interesting analysis succeeds in extending the process to general propositions, i.e. to cases where the propositions which are arguments to our truth-function are not given by enumeration but are given as all those satisfying some condition. For example, let \( fx \) be a propositional function (i.e. a function whose values are propositions), such as “\( x \) is human”—then the various values of \( fx \) form a set of propositions. We may extend the idea “not-\( p \) and not-\( q \)” so as to apply to the simultaneous denial of all the propositions which are values of \( fx \). In this way we arrive at the proposition which is ordinarily represented in mathematical logic by the words “\( fx \) is false for all values of \( x \)”. The negation of this would be the proposition “there is at least one \( x \) for which \( fx \) is true” which is represented by “(\( \exists x \). \( fx \))”. If we had started with not-\( fx \) instead of \( fx \) we should have arrived at the proposition “\( fx \) is true for all values of \( x \)” which is represented by “(\( x \). \( fx \))”. Wittgenstein’s method of dealing with general propositions [i.e. “(\( x \). \( fx \))” and “(\( \exists x \). \( fx \))”] differs from previous methods by the fact that the generality comes only in specifying the set of propositions concerned, and when this has been done the building up of truth-functions proceeds exactly as it would in the ease of a finite number of enumerated arguments \( p, q, r, \ldots \).

Mr. Wittgenstein’s explanation of his symbolism at this point is not quite fully given in the text. The symbol he uses is \([ \bar{p}, \tilde{\xi}, N(\tilde{\xi})] \). The following is the explanation of this symbol:

- \( \bar{p} \) stands for all atomic propositions.
- \( \tilde{\xi} \) stands for any set of propositions.
- \( N(\tilde{\xi}) \) stands for the negation of all the propositions making up \( \tilde{\xi} \).

The whole symbol \([ \bar{p}, \tilde{\xi}, N(\tilde{\xi})] \) means whatever can be obtained by taking any selection of atomic propositions, negating them all, then taking any selection of the set of propositions now obtained, together with any of the originals—and so on indefinitely. This is, he says, the general truth-function and also the general form of proposition. What is meant is somewhat less complicated than it sounds. The symbol is intended to describe a process by the help of which, given the atomic propositions, all others can be manufactured. The process depends upon:

(a). Sheffer’s proof that all truth-functions can be obtained out of simultaneous negation, i.e. out of “not-\( p \) and not-\( q \)”;

(b). Mr. Wittgenstein’s theory of the derivation of general propositions from conjunctions and disjunctions;

(c). The assertion that a proposition can only occur in another proposition as argument to a truth-function. Given these three foundations, it follows that all propositions which are not atomic can be derived from such as are, by a uniform process, and it is this process which is indicated by Mr. Wittgenstein’s symbol.

From this uniform method of construction we arrive at an amazing simplification of the theory of inference, as well as a definition of the sort of propositions that belong to logic. The method of generation which has just been described, enables Wittgenstein to say that all propositions can be constructed in the above manner from atomic propositions, and in this way the totality of propositions is defined. (The apparent exceptions which we mentioned above are dealt with in a manner which we shall consider later.) Wittgenstein is enabled to assert that propositions are all that follows from the totality of atomic propositions (together with the fact that it is the totality of them); that a proposition is always a truth-function of atomic propositions; and that if \( p \) follows from \( q \) the meaning of \( p \) is contained in the meaning of \( q \), from which of course it results that nothing can be deduced from an atomic proposition. All the propositions of logic, he maintains, are tautologies, such, for example, as “\( p \) or not \( p \)”.

The fact that nothing can be deduced from an atomic proposition has interesting applications, for example, to causality. There cannot, in Wittgenstein’s logic, be any such thing as a causal nexus. “The events of the future”, he says, “cannot be inferred from those of the present. Superstition is the belief in the causal nexus.” That the sun will rise to-morrow is a hypothesis. We do not in fact know whether it will rise, since there is no compulsion according to which one thing must happen because another happens.

Let us now take up another subject—that of names. In Wittgenstein’s theoretical logical language, names are only given to simples. We do not give two names to one thing, or one name to two things. There is no way whatever, according to him, by which we can describe the totality of things that can be named, in other words, the totality of
that whatever can be said has to be about bounded portions of the world. This view may have been originally suggested by notation, and if so, that is much in its favor, for a good notation has a subtlety and suggestiveness which at times make it seem almost like a live teacher. Notational irregularities are often the first sign of philosophical errors, and a perfect notation would be a substitute for thought. But although notation may have first suggested to Mr. Wittgenstein the limitation of logic to things within the world as opposed to the world as a whole, yet the view, once suggested, is seen to have much else to recommend it. Whether it is ultimately true I do not, for my part, profess to know. In this Introduction I am concerned to expound it, not to pronounce upon it. According to this view we could only say things about the world as a whole if we could get outside the world, if, that is to say, it ceased to be for us the whole world. Our world may be bounded for some superior being who can survey it from above, but for us, however finite it may be, it cannot have a boundary, since it has nothing outside it. Wittgenstein uses, as an analogy, the field of vision. Our field of vision does not, for us, have a visual boundary, just because there is nothing outside it, and in like manner our logical world has no logical boundary because our logic knows of nothing outside it. These considerations lead him to a somewhat curious discussion of Solipsism. Logic, he says, fills the world. The boundaries of the world are also its boundaries. In logic, therefore, we cannot say, there is this and this in the world, but not that, for to say so would apparently presuppose that we exclude certain possibilities, and this cannot be the case, since it would require that logic should go beyond the boundaries of the world as if it could contemplate these boundaries from the other side also. What we cannot think we cannot think, therefore we also cannot say what we cannot think.

This, he says, gives the key to solipsism. What Solipsism intends is quite correct, but this cannot be said, it can only be shown. That the world is my world appears in the fact that the boundaries of language (the only language I understand) indicate the boundaries of my world. The metaphysical subject does not belong to the world but is a boundary of the world.

We must take up next the question of molecular propositions which are at first sight not truth-functions, of the propositions that they
Wittgenstein introduces this subject in the statement of his position, namely, that all molecular functions are truth-functions. He says (5.54): “In the general propositional form, propositions occur in a proposition only as bases of truth-operations.” At first sight, he goes on to explain, it seems as if a proposition could also occur in other ways, e.g. “A believes p.” Here it seems superficially as if the proposition p stood in a sort of relation to the object A. “But it is clear that ‘A believes that p,’ ‘A thinks p,’ ‘A says p’ are of the form “‘p’ says p”; and here we have no co-ordination of a fact and an object, but a co-ordination of facts by means of a co-ordination of their objects” (5.542).

What Mr. Wittgenstein says here is said so shortly that its point is not likely to be clear to those who have not in mind the controversies with which he is concerned. The theory which which he is disagreeing will be found in my articles on the nature of truth and falsehood in Philosophical Essays and Proceedings of the Aristotelian Society, 1906–7. The problem at issue is the problem of the logical form of belief, i.e. what is the schema representing what occurs when a man believes. Of course, the problem applies not only to belief, but also to a host of other mental phenomena which may be called propositional attitudes: doubting, considering, desiring, etc. In all these cases it seems natural to express the phenomenon in the form “A doubts p”, “A considers p”, “A desires p”, etc., which makes it appear as though we were dealing with a relation between a person and a proposition. This cannot, of course, be the ultimate analysis, since persons are fictions and so are propositions, except in the sense in which they are facts on their own account. A proposition, considered as a fact on its own account, may be a set of words which a man says over to himself, or a complex image, or train of images passing through his mind, or a set of incipient bodily movements. It may be any one of innumerable different things. The proposition as a fact on its own account, for example, the actual set of words the man pronounces to himself, is not relevant to logic. What is relevant to logic is that common element among all these facts, which enables him, as we say, to mean the fact which the proposition asserts. To psychology, of course, more is relevant; for a symbol does not mean what it symbolizes in virtue of a logical relation alone, but in virtue also of a psychological relation of intention, or association, or what-not. The psychological part of meaning, however, does not concern the logician. What does concern him in this problem of belief is the logical schema. It is clear that, when a person believes a proposition, the person, considered as a metaphysical subject, does not have to be assumed in order to explain what is happening. What has to be explained is the relation between the set of words which is the proposition considered as a fact on its own account, and the “objective” fact which makes the proposition true or false. This reduces ultimately to the question of the meaning of propositions, that is to say, the meaning of propositions is the only non-psychological portion of the problem involved in the analysis of belief. This problem is simply one of a relation of two facts, namely, the relation between the series of words used by the believer and the fact which makes these words true or false. The series of words is a fact just as much as what makes it true or false is a fact. The relation between these two facts is not unanalyzable, since the meaning of a proposition results from the meaning of its constituent words. The meaning of the series of words which is a proposition is a function of the meaning of the separate words. Accordingly, the proposition as a whole does not really enter into what has to be explained in explaining the meaning of a propositions. It would perhaps help to suggest the point of view which I am trying to indicate, to say that in the cases which have been considering the proposition occurs as a fact, not as a proposition. Such a statement, however, must not be taken too literally. The real point is that in believing, desiring, etc., what is logically fundamental is the relation of a proposition considered as a fact, to the fact which makes it true or false, and that this relation of two facts is reducible to a relation of their constituents. Thus the proposition does not occur at all in the same sense in which it occurs in a truth-function.

There are some respects, in which, as it seems to me, Mr. Wittgenstein’s theory stands in need of greater technical development. This applies in particular to his theory of number (6.02ff.) which, as it stands, is only capable of dealing with finite numbers. No logic can be considered adequate until it has been shown to be capable of dealing with transfinite numbers. I do not think there is anything in Mr. Wittgenstein’s system to make it impossible for him to fill this lacuna.

More interesting than such questions of comparative detail is Mr.
Wittgenstein’s attitude towards the mystical. His attitude upon this grows naturally out of his doctrine in pure logic, according to which the logical proposition is a picture (true or false) of the fact, and has in common with the fact a certain structure. It is this common structure which makes it capable of being a picture of the fact, but the structure cannot itself be put into words, since it is a structure of words, as well as of the fact to which they refer. Everything, therefore, which is involved in the very idea of the expressiveness of language must remain incapable of being expressed in language, and is, therefore, inexpressible in a perfectly precise sense. This inexpressible contains, according to Mr. Wittgenstein, the whole of logic and philosophy. The right method of teaching philosophy, he says, would be to confine oneself to propositions of the sciences, stated with all possible clearness and exactness, leaving philosophical assertions to the learner, and proving to him, whenever he made them, that they are meaningless. It is true that the fate of Socrates might befall a man who attempted this method of teaching, but we are not to be deterred by that fear, if it is the only right method. It is not this that causes some hesitation in accepting Mr. Wittgenstein’s position, in spite of the very powerful arguments which he brings to its support. What causes hesitation is the fact that, after all, Mr. Wittgenstein manages to say a good deal about what cannot be said, thus suggesting to the sceptical reader that possibly there may be some loophole through a hierarchy of languages, or by some other exit. The whole subject of ethics, for example, is placed by Mr. Wittgenstein in the mystical, inexpressible region. Nevertheless he is capable of conveying his ethical opinions. His defence would be that what he calls the mystical can be shown, although it cannot be said. It may be that this defence is adequate, but, for my part, I confess that it leaves me with a certain sense of intellectual discomfort.

There is one purely logical problem in regard to which these difficulties are peculiarly acute. I mean the problem of generality. In the theory of generality it is necessary to consider all propositions of the form \( fx \) where \( fx \) is a given propositional function. This belongs to the part of logic which can be expressed, according to Mr. Wittgenstein’s system. But the totality of possible values of \( x \) which might seem to be involved in the totality of propositions of the form \( fx \) is not admitted by Mr. Wittgenstein among the things that can be spoken of, for this is no other than the totality of things in the world, and thus involves the attempt to conceive the world as a whole; “the feeling of the world as a bounded whole is the mystical”; hence the totality of the values of \( x \) is mystical (6.45). This is expressly argued when Mr. Wittgenstein denies that we can make propositions as to how many things there are in the world, as for example, that there are more than three.

These difficulties suggest to my mind some such possibility as this: that every language has, as Mr. Wittgenstein says, a structure concerning which in the language, nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit. Mr. Wittgenstein would of course reply that his whole theory is applicable unchanged to the totality of such languages. The only retort would be to deny that there is any such totality. The totalities concerning which Mr. Wittgenstein holds that it is impossible to speak logically are nevertheless thought by him to exist, and are the subject-matter of his mysticism. The totality resulting from our hierarchy would be not merely logically inexpressible, but a fiction, a mere delusion, and in this way the supposed sphere of the mystical would be abolished. Such a hypothesis is very difficult, and I can see objections to it which at the moment I do not know how to answer. Yet I do not see how any easier hypothesis can escape from Mr. Wittgenstein’s conclusions. Even if this very difficult hypothesis should prove tenable, it would leave untouched a very large part of Mr. Wittgenstein’s theory, though possibly not the part upon which he himself would wish to lay most stress. As one with a long experience of the difficulties of logic and of the deceptiveness of theories which seem irrefutable, I find myself unable to be sure of the rightness of a theory, merely on the ground that I cannot see any point on which it is wrong. But to have constructed a theory of logic which is not at any point obviously wrong is to have achieved a work of extraordinary difficulty and importance. This merit, in my opinion, belongs to Mr. Wittgenstein’s book, and makes it one which no serious philosopher can afford to neglect.

Bertrand Russell.

May 1922.
Tractatus Logico-Philosophicus

DEDICATED
TO THE MEMORY OF MY FRIEND

DAVID H. PINSENT

Motto: ...und alles, was man weiss, nicht bloss rauschen und brausen gehört hat, lässt sich in drei Worten sagen. —KÜRNBERGER.
This book will perhaps only be understood by those who have themselves already thought the thoughts which are expressed in it—or similar thoughts. It is therefore not a textbook. Its object would be attained if there were one person who read it with understanding and to whom it afforded pleasure.

The book deals with the problems of philosophy and shows, as I believe, that the method of formulating these problems rests on the misunderstanding of the logic of our language. Its whole meaning could be summed up somewhat as follows: What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent.

The book will, therefore, draw a limit to thinking, or rather—not to thinking, but to the expression of thoughts; for, in order to draw a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought).

The limit can, therefore, only be drawn in language and what lies on the other side of the limit will be simply nonsense.

How far my efforts agree with those of other philosophers I will not decide. Indeed what I have here written makes no claim to novelty in points of detail; and therefore I give no sources, because it is indifferent to me whether what I have thought has already been thought before me by another.

Thus the aim of the book is to draw a limit to thought, or rather—not to thought, but to the expression of thoughts: for in order to be able to draw a limit to thought, we should have to find both sides of the limit thinkable (i.e. we should have to be able to think what cannot be thought).

It will therefore only be in language that the limit can be drawn, and what lies on the other side of the limit will simply be nonsense.

I do not wish to judge how far my efforts coincide with those of other philosophers. Indeed, what I have written here makes no claim to novelty in detail, and the reason why I give no sources is that it is a matter of indifference to me whether the thoughts that I have had have been anticipated by someone else.
ein anderer gedacht hat.

Nur das will ich erwähnen, dass ich den
großartigen Werken Freges und den Arbei-
ten meines Freundes Herrn Bertrand Russell
einen großen Teil der Anregung zu meinen
Gedanken schulde.

Wenn diese Arbeit einen Wert hat, so be-
steht er in Zweierlei. Erstens darin, dass in ihr
Gedanken ausgedrückt sind, und dieser Wert
wird umso größer sein, je besser die Gedanken
ausgedrückt sind. Je mehr der Nagel auf den
Kopf getroffen ist. – Hier bin ich mir bewusst,
weit hinter dem Möglichen zurückgeblieben zu
sein. Einfach darum, weil meine Kraft zur Be-
wältigung der Aufgabe zu gering ist. – Mögen
andere kommen und es besser machen.

Dagegen scheint mir die Wahrheit der
hier mitgeteilten Gedanken unantastbar und
definitiv. Ich bin also der Meinung, die Proble-
me im Wesentlichen endgültig gelöst zu ha-
ben. Und wenn ich mich hierin nicht irre, so
besteht nun der Wert dieser Arbeit zweitens
darin, dass sie zeigt, wie wenig damit getan
ist, dass diese Probleme gelöst sind.

L. W.
Vienna, 1918

I will only mention that to the great
works of Frege and the writings of my friend
Bertrand Russell I owe in large measure the
stimulation of my thoughts.

If this work has a value it consists in two
things. First that in it thoughts are expressed,
and this value will be the greater the better
the thoughts are expressed. The more the
nail has been hit on the head.—Here I am
conscious that I have fallen far short of the
possible. Simply because my powers are in-
sufficient to cope with the task.—May others
come and do it better.

On the other hand the truth of the thoughts
communicated here seems to me unassailable
and definitive. I am, therefore, of the opinion
that the problems have in essentials been fi-
nally solved. And if I am not mistaken in this,
then the value of this work Secondly consists
in the fact that it shows how little has been
done when these problems have been solved.

L. W.
Vienna, 1918

I will only mention that I am indebted to
Frege’s great works and of the writings of my
friend Mr. Bertrand Russell for much of the
stimulation of my thoughts.

If this work has any value, it consists in
two things: the first is that thoughts are ex-
pressed in it, and on this score the better the
thoughts are expressed—the more the nail has
been hit on the head—the greater will be its
value.—Here I am conscious of having fallen
a long way short of what is possible. Simply
because my powers are too slight for the ac-
 complishment of the task.—May others come
and do it better.

On the other hand the truth of the thoughts
that are here communicated seems to me unas-
sailable and definitive. I therefore believe my-
self to have found, on all essential points, the
final solution of the problems. And if I am
not mistaken in this belief, then the second
thing in which the value of this work consists
is that it shows how little is achieved when
these problems are solved.

L. W.
<table>
<thead>
<tr>
<th>German</th>
<th>Ogden</th>
<th>Pears/McGuinness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>Die Welt ist alles, was der Fall ist.</td>
<td>The world is all that is the case.</td>
</tr>
<tr>
<td>1.1</td>
<td>Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.</td>
<td>The world is the totality of facts, not of things.</td>
</tr>
<tr>
<td>1.11</td>
<td>Die Welt ist durch die Tatsachen bestimmt und dadurch, dass es alle Tatsachen sind.</td>
<td>The world is determined by the facts, and by these being all the facts.</td>
</tr>
<tr>
<td>1.12</td>
<td>Denn, die Gesamtheit der Tatsachen bestimmt, was der Fall ist und auch, was alles nicht der Fall ist.</td>
<td>For the totality of facts determines both what is the case, and also all that is not the case.</td>
</tr>
<tr>
<td>1.13</td>
<td>Die Tatsachen im logischen Raum sind die Welt.</td>
<td>The facts in logical space are the world.</td>
</tr>
<tr>
<td>1.2</td>
<td>Die Welt zerfällt in Tatsachen.</td>
<td>The world divides into facts.</td>
</tr>
<tr>
<td>1.21</td>
<td>Eines kann der Fall sein oder nicht der Fall sein und alles übrige gleich bleiben.</td>
<td>Each item can be the case or not the case while everything else remains the same.</td>
</tr>
<tr>
<td>2</td>
<td>Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten.</td>
<td>What is the case—a fact—is the existence of states of affairs.</td>
</tr>
<tr>
<td>2.01</td>
<td>Der Sachverhalt ist eine Verbindung von Gegenständen. (Sachen, Dingen.)</td>
<td>A state of affairs (a state of things) is a combination of objects (things).</td>
</tr>
<tr>
<td>2.011</td>
<td>Es ist dem Ding wesentlich, der Bestandteil eines Sachverhaltes sein zu können.</td>
<td>It is essential to things that they should be possible constituents of states of affairs.</td>
</tr>
<tr>
<td>2.012</td>
<td>In der Logik ist nichts zufällig: Wenn das Ding im Sachverhalt vorkommen kann, so muss die Möglichkeit des Sachverhaltes im Ding bereits präjudiziert sein.</td>
<td>In logic nothing is accidental: if a thing can occur in an atomic fact the possibility of that atomic fact must already be prejudged in the thing.</td>
</tr>
<tr>
<td>2.0121</td>
<td>Es erschiene gleichsam als Zufall, wenn dem Ding, das allein für sich bestehen</td>
<td>It would seem to be a sort of accident, if it turned out that a situation would fit a</td>
</tr>
</tbody>
</table>

* [German] Die Decimalzahlen als Nummern der einzelnen Sätze deuten das logische Gewicht der Sätze an, den Nachdruck, der auf ihnen in meiner Darstellung liegt. Die Sätze n.1, n.2, n.3, etc., sind Bemerkungen zum Satze No. n; die Sätze n.m1, n.m2, etc. Bemerkungen zum Satze No. n.m; und so weiter. / [Ogden] The decimal figures as numbers of the separate propositions indicate the logical importance of the propositions, the emphasis laid upon them in my exposition. The propositions n.1, n.2, n.3, etc., are comments on proposition No. n; the propositions n.m1, n.m2, etc., are comments on the proposition No. n.m; and so on. / [Pears & McGuinness] The decimal numbers assigned to the individual propositions indicate the logical importance of the propositions, the stress laid on them in my exposition. The propositions n.1, n.2, n.3, etc. are comments on proposition no. n; the propositions n.m1, n.m2, etc. are comments on proposition no. n.m; and so on.
könnte, nachträglich eine Sachlage passen würde.

Wenn die Dinge in Sachverhalten vorkommen können, so muss dies schon in ihnen liegen.

(Etwas Logisches kann nicht nur möglich sein. Die Logik handelt von jeder Möglichkeit und alle Möglichkeiten sind ihre Tatsachen.)

Wie wir uns räumliche Gegenstände überhaupt nicht außerhalb des Raumes, zeitliche nicht außerhalb der Zeit denken können, so können wir uns keinen Gegenstand außerhalb der Möglichkeit seiner Verbindung mit anderen denken.

Wenn ich mir den Gegenstand im Verbande des Sachverhalts denken kann, so kann ich ihn nicht außerhalb der Möglichkeit dieses Verbandes denken.

Das Ding ist selbständig, insofern es in allen möglichen Sachlagen vorkommen kann, aber diese Form der Selbständigkeit ist eine Form des Zusammenhangs mit dem Sachverhalt, eine Form der Unselbständigkeit. (Es ist unmöglich, dass Worte in zwei verschiedenen Weisen auftreten, allein und im Satz.)

2.0122

Wenn ich den Gegenstand kenne, so kenne ich auch sämtliche Möglichkeiten seines Vorkommens in Sachverhalten.

(Jede solche Möglichkeit muss in der Natur des Gegenstandes liegen.)

Es kann nicht nachträglich eine neue Möglichkeit gefunden werden.

2.0123

Um einen Gegenstand zu kennen, muss ich zwar nicht seine externen – aber ich muss alle seine internen Eigenschaften

2.01231

| können, nachträglich eine Sachlage passen würde. | on its own account, subsequently a state of affairs could be made to fit. |
| Wenn die Dinge in Sachverhalten vorkommen können, so muss dies schon in ihnen liegen. | If things can occur in atomic facts, this possibility must already lie in them. |
| Wenn ich mir den Gegenstand im Verbande des Sachverhalts denken kann, so kann ich ihn nicht außerhalb der Möglichkeit dieses Verbandes denken. | (A logical entity cannot be merely possible. Logic treats of every possibility, and all possibilities are its facts.) |
| Das Ding ist selbständig, insofern es in allen möglichen Sachlagen vorkommen kann, aber diese Form der Selbständigkeit ist eine Form des Zusammenhangs mit dem Sachverhalt, eine Form der Unselbständigkeit. (Es ist unmöglich, dass Worte in zwei verschiedenen Weisen auftreten, allein und im Satz.) | Just as we cannot think of spatial objects at all apart from space, or temporal objects apart from time, so we cannot think of any object apart from the possibility of its connexion with other things. |
| Wenn ich den Gegenstand kenne, so kenne ich auch sämtliche Möglichkeiten seines Vorkommens in Sachverhalten. | If I can think of an object in the context of an atomic fact, I cannot think of it apart from the possibility of this context. |
| Um einen Gegenstand zu kennen, muss ich zwar nicht seine externen – aber ich muss alle seine internen Eigenschaften. | The thing is independent, in so far as it can occur in all possible circumstances, but this form of independence is a form of connexion with the atomic fact, a form of dependence. (It is impossible for words to occur in two different ways, alone and in the proposition.) |
| Wenn ich mir den Gegenstand im Verbande des Sachverhalts denken kann, so kann ich ihn nicht außerhalb der Möglichkeit dieses Verbandes denken. | If I can think of an object, then I also know all the possibilities of its occurrence in atomic facts. |
| Das Ding ist selbständig, insofern es in allen möglichen Sachlagen vorkommen kann, aber diese Form der Selbständigkeit ist eine Form des Zusammenhangs mit dem Sachverhalt, eine Form der Unselbständigkeit. (Es ist unmöglich, dass Worte in zwei verschiedenen Weisen auftreten, allein und im Satz.) | (Every such possibility must lie in the nature of the object.) |
| Wenn ich den Gegenstand kenne, so kenne ich auch sämtliche Möglichkeiten seines Vorkommens in Sachverhalten. | A new possibility cannot subsequently be found. |
| Um einen Gegenstand zu kennen, muss ich zwar nicht seine externen – aber ich muss alle seine internen Eigenschaften. | In order to know an object, I must know not its external but all its internal qualities. |
| Wenn ich mir den Gegenstand im Verbande des Sachverhalts denken kann, so kann ich ihn nicht außerhalb der Möglichkeit dieses Verbandes denken. | If I know an object, then I also know all its possible occurrences in states of affairs. |
| Das Ding ist selbständig, insofern es in allen möglichen Sachlagen vorkommen kann, aber diese Form der Selbständigkeit ist eine Form des Zusammenhangs mit dem Sachverhalt, eine Form der Unselbständigkeit. (Es ist unmöglich, dass Worte in zwei verschiedenen Weisen auftreten, allein und im Satz.) | (Every one of these possibilities must be part of the nature of the object.) |
| Um einen Gegenstand zu kennen, muss ich zwar nicht seine externen – aber ich muss alle seine internen Eigenschaften. | If I am to know an object, though I need not know its external properties, I must know all its internal properties. |
2.0124 Sind alle Gegenstände gegeben, so sind damit auch alle möglich Sachverhalte gegeben.

2.013 Jedes Ding ist, gleichsam, in einem Raume möglicher Sachverhalte. Diesen Raum kann ich mir leer denken, nicht aber das Ding ohne den Raum.

2.0131 Der räumliche Gegenstand muss im unendlichen Raume liegen. (Der Raumpunkt ist eine Argumentstelle.)

2.014 Die Gegenstände enthalten die Möglichkeit aller Sachlagen.

2.0141 Die Möglichkeit eines Vorkommens in Sachverhalten, ist die Form des Gegenstandes.

2.02 Der Gegenstand ist einfach.

2.0201 Jede Aussage über Komplexe lässt sich in eine Aussage über deren Bestandteile und in diejenigen Sätze zerlegen, welche die Komplexe vollständig beschreiben.

2.021 Die Gegenstände bilden die Substanz der Welt. Darum können sie nicht zusammengesetzt sein.

2.0211 Hätte die Welt keine Substanz, so würde, ob ein Satz Sinn hat, davon abhängen, ob ein anderer Satz wahr ist.

2.0212 Es wäre dann unmöglich, ein Bild der Welt (wahr oder falsch) zu entwerfen.

---

If all objects are given, then thereby are all possible atomic facts also given.

Every thing is, as it were, in a space of possible atomic facts. I can think of this space as empty, but not of the thing without the space.

A spatial object must lie in infinite space. (A point in space is an argument place.)

A speck in a visual field need not be red, but it must have a colour; it has, so to speak, a colour space round it. A tone must have a pitch, the object of the sense of touch a hardness, etc.

Objects contain the possibility of all states of affairs.

The possibility of its occurrence in atomic facts is the form of the object.

The object is simple.

Every statement about complexes can be analysed into a statement about their constituent parts, and into those propositions which completely describe the complexes.

Objects form the substance of the world. Therefore they cannot be compound.

If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.

It would then be impossible to form a picture of the world (true or false).

---

If all objects are given, then at the same time all possible states of affairs are also given.

Each thing is, as it were, in a space of possible states of affairs. This space I can imagine empty, but I cannot imagine the thing without the space.

A spatial object must be situated in infinite space. (A spatial point is an argument place.)

A speck in the visual field, thought it need not be red, must have some colour: it is, so to speak, surrounded by colour-space. Notes must have some pitch, objects of the sense of touch some degree of hardness, and so on.

Objects contain the possibility of all situations.

The possibility of its occurring in states of affairs is the form of an object.

Objects are simple.

Every statement about complexes can be resolved into a statement about their constituents and into the propositions that describe the complexes completely.

Objects make up the substance of the world. That is why they cannot be composite.

If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.

In that case we could not sketch any picture of the world (true or false).
It is clear that however different from the real one an imagined world may be, it must have something—a form—in common with the real world.

This fixed form consists of the objects.

The substance of the world can only determine a form and not any material properties. For these are first presented by the propositions—first formed by the configuration of the Gegenstände.

Roughly speaking: objects are colourless.

Two objects of the same logical form are—apart from their external properties—only differentiated from one another in that they are different.

Either a thing has properties which no other has, and then one can distinguish it straight away from the others by a description and refer to it; or, on the other hand, there are several things which have the totality of their properties in common, and then it is quite impossible to point to any one of them.

For if a thing is not distinguished by anything, I cannot distinguish it—for otherwise it would be distinguished.

Substance is what exists independently of what is the case.

It is form and content.

Space, time and colour (being coloured) are forms of objects.

Only if there are objects can there be a fixed form of the world.
Das Feste, das Bestehende und der Gegenstand sind Eins.

Der Gegenstand ist das Feste, Bestehende; die Konfiguration ist das Wechselnde, Unbeständige.

Die Konfiguration der Gegenstände bildet den Sachverhalt.

Im Sachverhalt hängen die Gegenstände ineinander, wie die Glieder einer Kette.

Im Sachverhalt verhalten sich die Gegenstände in bestimmter Art und Weise zueinander.

Die Art und Weise, wie die Gegenstände im Sachverhalt zusammenhängen, ist die Struktur des Sachverhaltes.

Die Form ist die Möglichkeit der Struktur.

Die Struktur der Tatsache besteht aus den Strukturen der Sachverhalte.

Die Gesamtheit der bestehenden Sachverhalte ist die Welt.

Die Gesamtheit der bestehenden Sachverhalte bestimmt auch, welche Sachverhalte nicht bestehen.

Das Bestehen und Nichtbestehen von Sachverhalten ist die Wirklichkeit.

(Das Bestehen von Sachverhalten nennt man auch eine positive, das Nichtbestehen eine negative Tatsache.)

Die Sachverhalte sind von einander unabhängig.

Aus dem Bestehen oder Nichtbestehen eines Sachverhaltes kann nicht auf das Bestehen oder Nichtbestehen eines anderen geschlossen werden.

Die gesamte Wirklichkeit ist die Welt.

Wir machen uns Bilder der Tatsachen.

The fixed, the existent and the object are one.

The object is the fixed, the existent; the configuration is the changing, the variable.

The configuration of the objects forms the atomic fact.

In the atomic fact objects hang one in another, like the links of a chain.

In the atomic fact the objects are combined in a definite way.

The way in which objects hang together in the atomic fact is the structure of the atomic fact.

The form is the possibility of the structure.

The structure of the fact consists of the structures of the atomic facts.

The totality of existent atomic facts is the world.

The totality of existent atomic facts also determines which atomic facts do not exist.

The existence and non-existence of atomic facts is the reality.

(The existence of atomic facts we also call a positive fact, their non-existence a negative fact.)

Atomic facts are independent of one another.

From the existence or non-existence of an atomic fact we cannot infer the existence or non-existence of another.

The total reality is the world.

We make to ourselves pictures of facts.

Objects, the unalterable, and the subsistent are one and the same.

Objects are what is unalterable and subsistent; their configuration is what is changing and unstable.

The configuration of objects produces states of affairs.

In a state of affairs objects fit into one another like the links of a chain.

In a state of affairs objects stand in a determinate relation to one another.

The determinate way in which objects are connected in a state of affairs is the structure of the state of affairs.

Form is the possibility of structure.

The structure of a fact consists of the structures of states of affairs.

The totality of existing states of affairs is the world.

The totality of existing states of affairs also determines which states of affairs do not exist.

The existence and non-existence of states of affairs is reality.

(We call the existence of states of affairs a positive fact, and their non-existence a negative fact.)

States of affairs are independent of one another.

From the existence or non-existence of one state of affairs it is impossible to infer the existence or non-existence of another.

The sum-total of reality is the world.

We picture facts to ourselves.
Das Bild stellt die Sachlage im logischen Raum, das Bestehen und Nichtbestehen von Sachverhalten vor.

Das Bild ist ein Modell der Wirklichkeit.

Den Gegenständen entsprechen im Bild die Elemente des Bildes.

Die Elemente des Bildes vertreten im Bild die Gegenstände.

Das Bild besteht darin, dass sich seine Elemente in bestimmter Art und Weise zu einander verhalten.

Das Bild ist eine Tatsache.

Dass sich die Elemente des Bildes in bestimmter Art und Weise zu einander verhalten, stellt vor, dass sich die Sachen so zu einander verhalten.

Dieser Zusammenhang der Elemente des Bildes heiße seine Struktur und ihre Möglichkeit seine Form der Abbildung.

Die Form der Abbildung ist die Möglichkeit, dass sich die Dinge so zu einander verhalten, wie die Elemente des Bildes.

Das Bild ist so mit der Wirklichkeit verknüpft – es reicht bis zu ihr.

Es ist wie ein Maßstab an die Wirklichkeit angelegt.

Nur die äußersten Punkte der Teilstriche berühren den zu messenden Gegenstand.

Nach dieser Auffassung gehört also zum Bilde auch noch die abbildende Beziehung, die es zum Bild macht.

Die abbildende Beziehung besteht aus den Zuordnungen der Elemente des Bildes.

The picture presents the facts in logical space, the existence and non-existence of atomic facts.

The picture is a model of reality.

To the objects correspond in the picture the elements of the picture.

The elements of the picture stand, in the picture, for the objects.

The picture consists in the fact that its elements are combined with one another in a definite way.

The picture is a fact.

That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another.

This connexion of the elements of the picture is called its structure, and the possibility of this structure is called the form of representation of the picture.

The form of representation is the possibility that the things are combined with one another as are the elements of the picture.

Thus the picture is linked with reality; it reaches up to it.

It is like a scale applied to reality.

Only the outermost points of the dividing lines touch the object to be measured.

According to this view the representing relation which makes it a picture, also belongs to the picture.

The representing relation consists of the co-ordinations of the elements of the picture.

That is how a picture is attached to reality; it reaches right out to it.

It is laid against reality like a measure.

Only the end-points of the graduating lines actually touch the object that is to be measured.

So a picture, conceived in this way, also includes the pictorial relationship, which makes it into a picture.

The pictorial relationship consists of the correlations of the picture’s elements.
und der Sachen.

Diese Zuordnungen sind gleichsam die Fühler der Bildelemente, mit denen das Bild die Wirklichkeit berührt.

Die Tatsache muss, um Bild zu sein, etwas mit dem Abgebildeten gemeinsam haben.

In Bild und Abgebildetem muss etwas identisch sein, damit das eine überhaupt ein Bild des anderen sein kann.

Was das Bild mit der Wirklichkeit gemein haben muss, um sie auf seine Art und Weise – richtig oder falsch – abzubilden zu können, ist seine Form der Abbildung.

Das Bild kann jede Wirklichkeit abilden, deren Form es hat.

Das räumliche Bild alles Räumliche, das farbige alles Farbige, etc.

Seine Form der Abbildung aber, kann das Bild nicht abilden; es weist sie auf.

Das Bild stellt sein Objekt von außerhalb dar (sein Standpunkt ist seine Form der Darstellung), darum stellt das Bild sein Objekt richtig oder falsch dar.

Das Bild kann sich aber nicht außerhalb seiner Form der Darstellung stellen.

Was jedes Bild, welcher Form immer, mit der Wirklichkeit gemein haben muss, um sie überhaupt – richtig oder falsch – abzubilden zu können, ist die logische Form, das ist, die Form der Wirklichkeit.

Ist die Form der Abbildung die logische Form, so heißt das Bild das logische Bild.

Jedes Bild ist auch ein logisches. (Dag-

picture and the things.

These co-ordinations are as it were the feelers of its elements with which the picture touches reality.

In order to be a picture a fact must have something in common with what it pictures.

In the picture and the pictured there must be something identical in order that the one can be a picture of the other at all.

What the picture must have in common with reality in order to be able to represent it after its manner—rightly or falsely—is its form of representation.

The picture can represent every reality whose form it has.

The spatial picture, everything spatial, the coloured, everything coloured, etc.

The picture, however, cannot represent its form of representation; it shows it forth.

The picture represents its object from without (its standpoint is its form of representation), therefore the picture represents its object rightly or falsely.

But the picture cannot place itself outside of its form of representation.

What every picture, of whatever form, must have in common with reality in order to be able to represent it at all—rightly or falsely—is the logical form, that is, the form of reality.

If the form of representation is the logical form, then the picture is called a logical picture.

Every picture is also a logical picture.

with things.

These correlations are, as it were, the feelers of the picture's elements, with which the picture touches reality.

If a fact is to be a picture, it must have something in common with what it depicts.

There must be something identical in a picture and what it depicts, to enable the one to be a picture of the other at all.

What a picture must have in common with reality, in order to be able to depict it—correctly or incorrectly—in the way that it does, is its pictorial form.

A picture can depict any reality whose form it has.

A spatial picture can depict anything spatial, a coloured one anything coloured, etc.

A picture cannot, however, depict its pictorial form: it displays it.

A picture represents its subject from a position outside it. (Its standpoint is its representational form.) That is why a picture represents its subject correctly or incorrectly.

A picture cannot, however, place itself outside its representational form.

What any picture, of whatever form, must have in common with reality, in order to be able to depict it—correctly or incorrectly—in any way at all, is logical form, i.e. the form of reality.

A picture whose pictorial form is logical form is called a logical picture.

Every picture is at the same time a log-
gegen ist z. B. nicht jedes Bild ein räumli-

ches.)

Das logische Bild kann die Welt abbilden.

Das Bild hat mit dem Abgebildeten die logische Form der Abbildung gemein.

Das Bild bildet die Wirklichkeit ab, indem es eine Möglichkeit des Bestehens und Nichtbestehens von Sachverhalten darstellt.

Das Bild stellt eine mögliche Sachlage im logischen Raume dar.

Das Bild enthält die Möglichkeit der Sachlage, die es darstellt.

Das Bild stimmt mit der Wirklichkeit überein oder nicht; es ist richtig oder unrichtig, wahr oder falsch.

Das Bild stellt dar, was es darstellt, unabhängig von seiner Wahr- oder Falschheit, durch die Form der Abbildung.

Was das Bild darstellt, ist sein Sinn.

In der Übereinstimmung oder Nichtübereinstimmung seines Sinnes mit der Wirklichkeit, besteht seine Wahrheit oder Falschheit.

Um zu erkennen, ob das Bild wahr oder falsch ist, müssen wir es mit der Wirklichkeit vergleichen.

Aus dem Bild allein ist nicht zu erkennen, ob es wahr oder falsch ist.

Ein a priori wahres Bild gibt es nicht.

Das logische Bild der Tatsachen ist der Gedanke.

„Ein Sachverhalt ist denkbar“ heißt:

(On the other hand, for example, not every picture is spatial.)

The logical picture can depict the world.

The picture has the logical form of representation in common with what it pictures.

The picture depicts reality by representing a possibility of the existence and non-existence of atomic facts.

The picture represents a possible state of affairs in logical space.

The picture contains the possibility of the state of affairs which it represents.

The picture agrees with reality or not; it is right or wrong, true or false.

The picture represents what it represents, independently of its truth or falsehood, through the form of representation.

What the picture represents is its sense.

In the agreement or disagreement of its sense with reality, its truth or falsity consists.

In order to discover whether the picture is true or false we must compare it with reality.

It cannot be discovered from the picture alone whether it is true or false.

There is no picture which is a priori true.

The logical picture of the facts is the thought.

"An atomic fact is thinkable"—means:

ical one. (On the other hand, not every picture is, for example, a spatial one.)

Logical pictures can depict the world.

A picture has logico-pictorial form in common with what it depicts.

A picture depicts reality by representing a possibility of existence and non-existence of states of affairs.

A picture represents a possible situation in logical space.

A picture contains the possibility of the situation that it represents.

A picture agrees with reality or fails to agree; it is correct or incorrect, true or false.

What a picture represents it represents independently of its truth or falsity, by means of its pictorial form.

What a picture represents is its sense.

The agreement or disagreement of its sense with reality constitutes its truth or falsity.

In order to tell whether a picture is true or false we must compare it with reality.

It is impossible to tell from the picture alone whether it is true or false.

There are no pictures that are true a priori.

A logical picture of facts is a thought.

‘A state of affairs is thinkable’: what
Wir können uns ein Bild von ihm machen. this means is that we can picture it to ourselves.

Die Gesamtheit der wahren Gedanken sind ein Bild der Welt.
The totality of true thoughts is a picture of the world.

Der Gedanke enthält die Möglichkeit der Sachlage, die er denkt. Was denkbar ist, ist auch möglich.
The thought contains the possibility of the state of affairs which it thinks. What is thinkable is also possible.

Wir können nichts Unlogisches denken, weil wir sonst unlogisch denken müssten.
We cannot think anything unlogical, for otherwise we should have to think illogically.

Man sagte einmal, dass Gott alles schaffen könne, nur nichts, was den logischen Gesetzen zuwider wäre. – Wir können nämlich von einer „unlogischen“ Welt nicht sagen, wie sie aussähe.
It used to be said that God could create everything, except what was contrary to the laws of logic. The truth is, we could not say of an “unlogical” world how it would look.

To present in language anything which “contradicts logic” is as impossible as in geometry to present by its co-ordinates a figure which contradicts the laws of space; or to give the co-ordinates of a point which does not exist.

It is as impossible to represent in language anything that ‘contradicts logic’ as it is in geometry to represent by its co-ordinates a figure that contradicts the laws of space, or to give the co-ordinates of a point that does not exist.

Wohl können wir einen Sachverhalt räumlich darstellen, welcher den Gesetzen der Physik, aber keinen, der den Gesetzen der Geometrie zuwiderliefe.
Though a state of affairs that would contravene the laws of physics can be represented by us spatially, one that would contravene the laws of geometry cannot.

Ein a priori richtiger Gedanke wäre ein solcher, dessen Möglichkeit seine Wahrheit bedingte.
An a priori true thought would be one whose possibility guaranteed its truth.

Nur so könnten wir a priori wissen, dass ein Gedanke wahr ist, wenn aus dem Gedanken selbst (ohne Vergleichsobjekt) seine Wahrheit zu erkennen wäre.
Only if we could know a priori that a thought is true if its truth was to be recognized from the thought itself (without an object of comparison).

Im Satz drückt sich der Gedanke sinnlich wahrnehmbar aus.
In the proposition the thought is expressed perceptibly through the senses.

Wir benützen das sinnlich wahrnehm-
We use the perceptible sign of a proposi-
bare Zeichen (Laut- oder Schriftzeichen etc.) des Satzes als Projektion der möglichen Sachlage.

Die Projektionsmethode ist das Denken des Satz-Sinnes.

3.12

Das Zeichen, durch welches wir den Gedanken ausdrücken, nenne ich das Satzzeichen. Und der Satz ist das Satzzeichen in seiner projekiven Beziehung zur Welt.

Zum Satz gehört alles, was zur Projektion gehört; aber nicht das Projizierte.

Also die Möglichkeit des Projizierten, aber nicht dieses selbst.

Im Satz ist also sein Sinn noch nicht enthalten, wohl aber die Möglichkeit, ihn auszudrücken.

(„Der Inhalt des Satzes“ heißt der Inhalt des sinnvollen Satzes.)

Im Satz ist die Form seines Sinnes enthalten, aber nicht dessen Inhalt.

3.13

Das Satzzeichen besteht darin, dass sich seine Elemente, die Wörter, in ihm auf bestimmte Art und Weise zu einander verhalten.

Das Satzzeichen ist eine Tatsache.

3.14

Der Satz ist kein Wörtergemisch. – (Wie das musikalische Thema kein Gemisch von Tönen.)

Der Satz ist artikuliert.

3.141

Nur Tatsachen können einen Sinn ausdrücken, eine Klasse von Namen kann es nicht.

3.142

Dass das Satzzeichen eine Tatsache ist, wird durch die gewöhnliche Ausdrucksform der Schrift oder des Druckes verschleiert.

3.143

(sound or written sign, etc.) of the proposition as a projection of the possible state of affairs.

The method of projection is the thinking of the sense of the proposition.

The sign through which we express the thought I call the propositional sign. And the proposition is the propositional sign in its projective relation to the world.

To the proposition belongs everything which belongs to the projection; but not what is projected.

Therefore the possibility of what is projected but not this itself.

In the proposition, therefore, its sense is not yet contained, but the possibility of expressing it.

(“The content of the proposition” means the content of the significant proposition.)

In the proposition the form of its sense is contained, but not its content.

The propositional sign consists in the fact that its elements, the words, are combined in it in a definite way.

The propositional sign is a fact.

The proposition is not a mixture of words (just as the musical theme is not a mixture of tones).

The proposition is articulate.

Only facts can express a sense, a class of names cannot.

That the propositional sign is a fact is concealed by the ordinary form of expression, written or printed.

The method of projection is to think of the sense of the proposition.

I call the sign with which we express a thought a propositional sign.—And a proposition is a propositional sign in its projective relation to the world.

A proposition includes all that the projection includes, but not what is projected.

Therefore, though what is projected is not itself included, its possibility is.

A proposition, therefore, does not actually contain its sense, but does contain the possibility of expressing it.

(‘The content of a proposition’ means the content of a proposition that has sense.)

A proposition contains the form, but not the content, of its sense.

What constitutes a propositional sign is that in it its elements (the words) stand in a determinate relation to one another.

A propositional sign is a fact.

A proposition is not a blend of words.—(Just as a theme in music is not a blend of notes.)

A proposition is articulate.

Only facts can express a sense, a set of names cannot.

Although a propositional sign is a fact, this is obscured by the usual form of expression in writing or print.
Denn im gedruckten Satz z. B. sieht das Satzzeichen nicht wesentlich verschieden aus vom Wort.

(So war es möglich, dass Frege den Satz einen zusammengesetzten Namen nannte.)

3.1431
Sehr klar wird das Wesen des Satzzeichens, wenn wir es uns, statt aus Schriftzeichen, aus räumlichen Gegenständen (etwa Tischen, Stühlen, Büchern) zusammengesetzt denken.

Die gegenseitige räumliche Lage dieser Dinge drückt dann den Sinn des Satzes aus.

3.1432

3.144
Sachlagen kann man beschreiben, nicht benennen.

(Namen gleichen Punkten, Sätze Pfeilen, sie haben Sinn.)

3.2
Im Satze kann der Gedanke so ausge- drückt sein, dass den Gegenständen des Gedankens Elemente des Satzzeichens entsprechen.

3.201
Diese Elemente nenne ich „einfache Zeichen“ und den Satz „vollständig analysiert“.

3.202
Die im Satze angewandten einfachen Zeichen heißen Namen.

3.203
Der Name bedeutet den Gegenstand. Der Gegenstand ist seine Bedeutung. („A“ ist dasselbe Zeichen wie „A“.)

3.21
Der Konfiguration der einfachen Zeichen im Satzzeichen entspricht die Konfiguration der Gegenstände in der Sachlage.

For in the printed proposition, for example, the sign of a proposition does not appear essentially different from a word.

(Thus it was possible for Frege to call the proposition a compounded name.)

The essential nature of the propositional sign becomes very clear when we imagine it made up of spatial objects (such as tables, chairs, books) instead of written signs.

The mutual spatial position of these things then expresses the sense of the proposition.

We must not say, “The complex sign ‘aRb’ says ‘a stands in relation R to b’”; but we must say, “That ‘a’ stands in a certain relation to ‘b’ says that aRb”.

States of affairs can be described but not named.

(Names resemble points; propositions resemble arrows, they have sense.)

In propositions thoughts can be so expressed that to the objects of the thoughts correspond the elements of the propositional sign.

These elements I call “simple signs” and the proposition “completely analysed”.

The simple signs employed in propositions are called names.

The name means the object. The object is its meaning. (‘A’ is the same sign as ‘A’.)

To the configuration of the simple signs in the propositional sign corresponds the configuration of the objects in the state of

For in a printed proposition, for example, no essential difference is apparent between a propositional sign and a word.

(That is what made it possible for Frege to call a proposition a composite name.)

The essence of a propositional sign is very clearly seen if we imagine one composed of spatial objects (such as tables, chairs, and books) instead of written signs.

Then the spatial arrangement of these things will express the sense of the proposition.

Instead of, ‘The complex sign “aRb” says that a stands to b in the relation R’, we ought to put, ‘That “a” stands to “b” in a certain relation says that aRb.’

Situations can be described but not given names.

(Names are like points; propositions like arrows—they have sense.)

In a proposition a thought can be expressed in such a way that elements of the propositional sign correspond to the objects of the thought.

I call such elements ‘simple signs’, and such a proposition ‘complete analysed’.

The simple signs employed in propositions are called names.

A name means an object. The object is its meaning. (‘A’ is the same sign as ‘A’.)

The configuration of objects in a situation corresponds to the configuration of simple signs in the propositional sign.
Der Name vertritt im Satz den Gegenstand.


Die Forderung der Möglichkeit der einfachen Zeichen ist die Forderung der Bestimmtheit des Sinnes.

Der Satz, welcher vom Komplex handelt, steht in interner Beziehung zum Satze, der von dessen Bestandteil handelt.

Der Komplex kann nur durch seine Beschreibung gegeben sein, und diese wird stimmen oder nicht stimmen. Der Satz, in welchem von einem Komplex die Rede ist, wird, wenn dieser nicht existiert, nicht unsinnig, sondern einfach falsch sein.

Dass ein Satzelement einen Komplex bezeichnet, kann man aus einer Unbestimmtheit in den Sätzen sehen, worin es vorkommt. Wir wissen, durch diesen Satz ist noch nicht alles bestimmt. (Die Allgemeinheitsbezeichnung enthält ja ein Urbild.)

Die Zusammenfassung des Symbols eines Komplexes in ein einfaches Symbol kann durch eine Definition ausgedrückt werden.

Es gibt eine und nur eine vollständige Analyse des Satzes.

Der Satz drückt auf bestimmte, klar angebbare Weise aus, was er ausdrückt: Der Satz ist artikuliert.

3.22

In the proposition the name represents the object.

Objects I can only name. Signs represent them. I can only speak of them. I cannot assert them. A proposition can only say how a thing is, not what it is.

The postulate of the possibility of the simple signs is the postulate of the determinateness of the sense.

A proposition about a complex stands in an internal relation to the proposition about its constituent part.

A complex can only be given by its description, and this will either be right or wrong. The proposition in which there is mention of a complex, if this does not exist, becomes not nonsense but simply false.

A proposition can be given only by its description, which will be right or wrong. A proposition that mentions a complex will not be nonsensical, if the complex does not exist, but simply false.

That a propositional element signifies a complex can be seen from an indeterminateness in the propositions in which it occurs. We know that everything is not yet determined by this proposition. (The notation for generality contains a prototype.)

The combination of the symbols of a complex in a simple symbol can be expressed by a definition.

There is one and only one complete analysis of the proposition.

The proposition expresses what it expresses in a definite and clearly specifiable way: the proposition is articulate.

3.221

In a proposition a name is the representative of an object.

Objects can only be named. Signs are their representatives. I can only speak about them: I cannot put them into words. Propositions can only say how things are, not what they are.

The requirement that simple signs be possible is the requirement that sense be determinate.

A proposition about a complex stands in an internal relation to a proposition about a constituent of the complex.

A complex can be given only by its description, which will be right or wrong. A proposition that mentions a complex will not be nonsensical, if the complex does not exist, but simply false.

When a propositional element signifies a complex, this can be seen from an indeterminateness in the propositions in which it occurs. In such cases we know that the proposition leaves something undetermined. (In fact the notation for generality contains a prototype.)

The contraction of a symbol for a complex into a simple symbol can be expressed in a definition.

A proposition has one and only one complete analysis.

What a proposition expresses it expresses in a determinate manner, which can be set out clearly: a proposition is ar-
3.26 Der Name ist durch keine Definition weiter zu zergliedern: er ist ein Urzeichen.

3.261 Jedes definierte Zeichen bezeichnet über jene Zeichen, durch welche es definiert wurde; und die Definitionen weisen den Weg.
Zwei Zeichen, ein Urzeichen, und ein durch Urzeichen definiertes, können nicht auf dieselbe Art und Weise bezeichnen. Namen kann man nicht durch Definitionen auseinanderlegen. (Kein Zeichen,welches allein, selbständig eine Bedeutung hat.)

3.262 Was in den Zeichen nicht zum Ausdruck kommt, das zeigt ihre Anwendung. Was die Zeichen verschlucken, das spricht ihre Anwendung aus.

3.263 Die Bedeutungen von Urzeichen können durch Erläuterungen erklärt werden. Erläuterungen sind Sätze, welche die Urzeichen enthalten. Sie können also nur verstanden werden, wenn die Bedeutungen dieser Zeichen bereits bekannt sind.

Nur der Satz hat Sinn; nur im Zusammenhang des Satzes ist ein Name Bedeutung.

3.3 Jeden Teil des Satzes, der seinen Sinn charakterisiert, nenne ich einen Ausdruck (ein Symbol).
(Der Satz selbst ist ein Ausdruck.)

Ausdruck ist alles, für den Sinn des Satzes wesentliche, was Sätze miteinander gemein haben können.

Der Ausdruck kennzeichnet eine Form und ist er die Form der Zeichen, welche es bilden.

3.31 Nur der Satz hat Sinn; nur im Zusammenhang des Satzes ist ein Name Bedeutung.

Jeden Teil des Satzes, der seinen Sinn charakterisiert, nenne ich einen Ausdruck (ein Symbol).

(Werter Satz selbst ist ein Ausdruck.)

Ausdruck ist alles, für den Sinn des Satzes wesentliche, was Sätze miteinander gemein haben können.

Der Ausdruck kennzeichnet eine Form und ist er die Form der Zeichen, welche es bilden.

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Nur der Satz hat Sinn; nur im Zusammenhang des Satzes ist ein Name Bedeutung.

Jeden Teil des Satzes, der seinen Sinn charakterisiert, nenne ich einen Ausdruck (ein Symbol).

(Der Satz selbst ist ein Ausdruck.)

Ausdruck ist alles, für den Sinn des Satzes wesentliche, was Sätze miteinander gemein haben können.

3.33 Nur der Satz hat Sinn; nur im Zusammenhang des Satzes ist ein Name Bedeutung.

Jeden Teil des Satzes, der seinen Sinn charakterisiert, nenne ich einen Ausdruck (ein Symbol).

(Der Satz selbst ist ein Ausdruck.)

Ausdruck ist alles, für den Sinn des Satzes wesentliche, was Sätze miteinander gemein haben können.

3.34 Die Bedeutungen von Urzeichen können durch Erläuterungen erklärt werden. Erläuterungen sind Sätze, welche die Urzeichen enthalten. Sie können also nur verstanden werden, wenn die Bedeutungen dieser Zeichen bereits bekannt sind.

Nur der Satz hat Sinn; nur im Zusammenhang des Satzes ist ein Name Bedeutung.

Jeden Teil des Satzes, der seinen Sinn charakterisiert, nenne ich einen Ausdruck (ein Symbol).

(Der Satz selbst ist ein Ausdruck.)

Ausdruck ist alles, für den Sinn des Satzes wesentliche, was Sätze miteinander gemein haben können.

3.35 Nur der Satz hat Sinn; nur im Zusammenhang des Satzes ist ein Name Bedeutung.

Jeden Teil des Satzes, der seinen Sinn charakterisiert, nenne ich einen Ausdruck (ein Symbol).

(Der Satz selbst ist ein Ausdruck.)

Ausdruck ist alles, für den Sinn des Satzes wesentliche, was Sätze miteinander gemein haben können.
und einen Inhalt.

3.311 Der Ausdruck setzt die Formen aller Sätze voraus, in welchen er vorkommen kann. Er ist das gemeinsame charakteristische Merkmal einer Klasse von Sätzen.

3.312 Er wird also dargestellt durch die allgemeine Form der Sätze, die er charakterisiert.

Und zwar wird in dieser Form der Ausdruck k o n s t a n t und alles übrige v a r i a b l e sein.

3.313 Der Ausdruck wird also durch eine Variable dargestellt, deren Werte die Sätze sind, die den Ausdruck enthalten.

(Im Grenzfall wird die Variable zur Konstanten, der Ausdruck zum Satz.)

Ich nenne eine solche Variable „Satzvariable“.


(Auch der variable Name.)

3.315 Verwandeln wir einen Bestandteil eines Satzes in eine Variable, so gibt es eine Klasse von Sätzen, welche sämtlich Werte des so entstandenen variablen Satzes sind. Diese Klasse hängt im allgemeinen noch davon ab, was wir, nach willkürlicher Übereinkunft, mit Teilen jenes Satzes meinen. Verwandeln wir aber alle jene Zeichen, deren Bedeutung willkürlich bestimmt wurde, in Variable, so gibt es nun noch immer eine solche Klasse. Diese aber ist nun von keiner Übereinkunft abhängig, sondern nur noch von der Natur des Satzes. Sie entspricht einer logischen Form –

a content.

3.312 An expression presupposes the forms of all propositions in which it can occur. It is the common characteristic mark of a class of propositions.

It is therefore represented by the general form of the propositions which it characterizes.

And in this form the expression is constant and everything else variable.

3.313 An expression is thus presented by a variable, whose values are the propositions which contain the expression.

(In the limiting case the variable becomes constant, the expression a proposition.)

I call such a variable a “propositional variable”.

3.314 An expression has meaning only in a proposition. Every variable can be conceived as a propositional variable.

(Including the variable name.)

3.315 If we change a constituent part of a proposition into a variable, there is a class of propositions which are all the values of the resulting variable proposition. This class in general still depends on what, by arbitrary agreement, we mean by parts of that proposition. But if we change all those signs, whose meaning was arbitrarily determined, into variables, there always remains such a class. But this is now no longer dependent on any agreement; it depends only on the nature of the proposition. It corresponds to a logical form, to a logical prototype.

3.315 Thus an expression is presented by means of a variable whose values are the propositions that contain the expression.

(In the limiting case the variable becomes a constant, the expression becomes a proposition.)

I call such a variable a ‘propositional variable’.

3.316 An expression has meaning only in a proposition. All variables can be construed as propositional variables.

(Even variable names.)

3.316 If we turn a constituent of a proposition into a variable, there is a class of propositions all of which are values of the resulting variable proposition. In general, this class too will be dependent on the meaning that our arbitrary conventions have given to parts of the original proposition. But if all the signs in it that have arbitrarily determined meanings are turned into variables, we shall still get a class of this kind. This one, however, is not dependent on any convention, but solely on the nature of the proposition. It corresponds to a logical form—a logical prototype.
3.316

einem logischen Urbild.

Welche Werte die Satzvariable annehmen darf, wird festgesetzt.

Die Festsetzung der Werte ist die Variable.

Die Festsetzung der Werte der Satzvariablen ist die Angabe der Sätze, deren gemeinsames Merkmal die Variable ist.

Die Festsetzung ist eine Beschreibung dieser Sätze.

Die Festsetzung wird also nur von Symbolen, nicht von deren Bedeutung handeln.

Und nur dies ist der Festsetzung wesentlich, dass sie nur eine Beschreibung von Symbolen ist und nichts über das Bezeichnene ausagt.

Wie die Beschreibung der Sätze geschieht, ist unwesentlich.

3.317

Den Satz fasse ich – like Frege und Russell – als Funktion der in ihm enthaltenen Ausdrücke auf.

Das Zeichen ist das sinnlich Wahrnehmbare am Symbol.

Zwei verschiedene Symbole können also das Zeichen (Schriftzeichen oder Lautzeichen etc.) miteinander gemein haben – sie bezeichnen dann auf verschiedene Art und Weise.

Es kann nie das gemeinsame Merkmal zweier Gegenstände anzeigen, dass wir sie mit demselben Zeichen, aber durch zwei verschiedene Bezeichnungsweisen bezeichnen. Denn das Zeichen ist ja willkürlich. Man könnte also auch

3.318

What values the propositional variable can assume is determined.

The determination of the values is the variable.

The determination of the values of the propositional variable is done by indicating the propositions whose common mark the variable is.

The determination is a description of these propositions.

The determination will therefore deal only with symbols not with their meaning.

And only this is essential to the determination, that it is only a description of symbols and asserts nothing about what is symbolized.

The way in which we describe the propositions is not essential.

I conceive the proposition—like Frege and Russell—as a function of the expressions contained in it.

The sign is the part of the symbol perceptible by the senses.

Two different symbols can therefore have the sign (the written sign or the sound sign) in common—they then signify in different ways.

It can never indicate the common characteristic of two objects that we symbolize them with the same signs but by different methods of symbolizing. For the sign is arbitrary. We could therefore equally well choose two different signs and where then

3.32

Das Zeichen ist das sinnlich Wahrnehmbare am Symbol.

Zwei verschiedene Symbole können also das Zeichen (Schriftzeichen oder Lautzeichen etc.) miteinander gemein haben – sie bezeichnen dann auf verschiedene Art und Weise.

Es kann nie das gemeinsame Merkmal zweier Gegenstände anzeigen, dass wir sie mit demselben Zeichen, aber durch zwei verschiedene Bezeichnungsweisen bezeichnen. Denn das Zeichen ist ja willkürlich. Man könnte also auch

3.321

What values a propositional variable may take is something that is stipulated.

The stipulation of values is the variable.

To stipulate values for a propositional variable is to give the propositions whose common characteristic the variable is.

The stipulation is a description of those propositions.

The stipulation will therefore be concerned only with symbols, not with their meaning.

And the only thing essential to the stipulation is that it is merely a description of symbols and states nothing about what is signified.

How the description of the propositions is produced is not essential.

Like Frege and Russell I construe a proposition as a function of the expressions contained in it.

A sign is what can be perceived of a symbol.

So one and the same sign (written or spoken, etc.) can be common to two different symbols—in which case they will signify in different ways.

Our use of the same sign to signify two different objects can never indicate a common characteristic of the two, if we use it with two different modes of signification. For the sign, of course, is arbitrary. So we could choose two different signs instead,
zwei verschiedene Zeichen wählen, und wo bliebe dann das Gemeinsame in der Bezeichnung?

In der Umgangssprache kommt es ungleich häufig vor, dass dasselbe Wort auf verschiedene Art und Weise bezeichnet – also verschiedenen Symbolen angehört –, oder, dass zwei Wörter, die auf verschiedene Art und Weise bezeichnen, äußerlich in der gleichen Weise im Satz angewandt werden.

So erscheint das Wort „ist“ als Kopula, als Gleichheitszeichen und als Ausdruck der Existenz; „existieren“ als intransitives Zeitwort wie „gehen“; „identisch“ als Eigenschaftswort; wir reden von Etwas, aber auch davon, dass Etwas geschieht. (Im Satze „Grün ist grün“ – wo das erste Wort ein Personennamen, das letzte ein Eigenschaftswort ist – haben diese Worte nicht einfach verschiedene Bedeutung, sondern es sind verschiedene Symbole.)

So entstehen leicht die fundamentalsten Verwechslungen (deren die ganze Philosophie voll ist).

Um diesen Irrtümern zu entgehen, müssen wir eine Zeichensprache verwenden, welche sie ausschließt, indem sie nicht das gleiche Zeichen in verschiedenen Symbolen, und Zeichen, welche auf verschiedene Art bezeichnen, nicht äußerlich auf die gleiche Art verwenden. Eine Zeichensprache also, die der logischen Grammatik – der logischen Syntax – gehorcht. (Die Begriffsschrift Frege’s und Russell’s ist eine solche Sprache, die allerdings

3.323

würde be what was common in the symbolization?

In the language of everyday life it very often happens that the same word signifies in two different ways — and therefore belongs to two different symbols — or that two words, which signify in different ways, are apparently applied in the same way in the proposition.

Thus the word “is” appears as the copula, as the sign of equality, and as the expression of existence; “to exist” as an intransitive verb like “to go”; “identical” as an adjective; we speak of something but also of the fact of something happening.

(In the proposition “Green is green” — where the first word is a proper name as the last an adjective — these words have not merely different meanings but they are different symbols.)

Thus there easily arise the most fundamental confusions (of which the whole of philosophy is full).

In order to avoid these errors, we must employ a symbolism which excludes them, by not applying the same sign in different symbols and by not applying signs in the same way which signify in different ways. A symbolism, that is to say, which obeys the rules of logical grammar — of logical syntax.

(The logical symbolism of Frege and Russell is such a language, which, however, and then what would be left in common on the signifying side?

In everyday language it very frequently happens that the same word has different modes of signification — and so belongs to different symbols — or that two words that have different modes of signification are employed in propositions in what is superficially the same way.

Thus the word ‘is’ figures as the copula, as a sign for identity, and as an expression for existence; ‘exist’ figures as an intransitive verb like ‘go’, and ‘identical’ as an adjective; we speak of something, but also of something’s happening.

(In the proposition, ‘Green is green’ — where the first word is the proper name of a person and the last an adjective — these words do not merely have different meanings: they are different symbols.)

In this way the most fundamental confusions are easily produced (the whole of philosophy is full of them).

In order to avoid such errors we must make use of a sign-language that excludes them by not using the same sign for different symbols and by not using in a superficially similar way signs that have different modes of signification: that is to say, a sign-language that is governed by logical grammar — by logical syntax.

(The conceptual notation of Frege and Russell is such a language, though, it is
noch nicht alle Fehler ausschließt.)

Um das Symbol am Zeichen zu erkennen, muss man auf den sinnvollen Gebrauch achten.

Das Zeichen bestimmt erst mit seiner logisch-syntaktischen Verwendung eine logische Form.

Wird ein Zeichen nicht gebraucht, so ist es bedeutungslos. Das ist der Sinn der Devise Occams.

(Wenn sich alles so verhält als hätte ein Zeichen Bedeutung, dann hat es auch Bedeutung.)

In der logischen Syntax darf die Bedeutung eines Zeichens eine Rolle spielen; sie muss sich aufstellen lassen, ohne dass dabei von der Bedeutung eines Zeichens die Rede wäre, sie darf nur die Beschreibung der Ausdrücke voraussetzen.

Von dieser Bemerkung sehen wir in Russells „Theory of types“ hinüber: Der Irrtum Russells zeigt sich darin, dass er bei der Aufstellung der Zeichenregeln von der Bedeutung der Zeichen reden musste.

Kein Satz kann etwas über sich selbst aussagen, weil das Satzzeichen nicht in sich selbst enthalten sein kann (das ist die ganze „Theory of types“).

Eine Funktion kann darum nicht ihr eigenes Argument sein, weil das Funktionszeichen bereits das Urbild seines Arguments enthält und es sich nicht selbst enthalten kann.

Nehmen wir nämlich an, die Funktion $F(fx)$ könnte ihr eigenes Argument sein; dann gäbe es also einen Satz: „$F(F(fx))$“

3.326
does still not exclude all errors.)

In order to recognize the symbol in the sign we must consider the significant use.

The sign determines a logical form only together with its logical syntactic application.

If a sign is not necessary then it is meaningless. That is the meaning of Occam’s razor:

(If everything in the symbolism works as though a sign had meaning, then it has meaning.)

In logical syntax the meaning of a sign ought never to play a rôle; it must admit of being established without mention being thereby made of the meaning of a sign; it ought to presuppose only the description of the expressions.

From this observation we get a further view—into Russell’s Theory of Types. Russell’s error is shown by the fact that in drawing up his symbolic rules he has to speak about the things his signs mean.

No proposition can say anything about itself, because the propositional sign cannot be contained in itself (that is the “whole theory of types”).

A function cannot be its own argument, because the functional sign already contains the prototype of its own argument and it cannot contain itself.

If, for example, we suppose that the function $F(fx)$ could be its own argument, then there would be a proposition

3.327
true, it fails to exclude all mistakes.)

In order to recognize a symbol by its sign we must observe how it is used with a sense.

A sign does not determine a logical form unless it is taken together with its logico-syntactical employment.

If a sign is useless, it is meaningless. That is the point of Occam’s maxim.

(If everything behaves as if a sign had meaning, then it does have meaning.)

In logical syntax the meaning of a sign should never play a role. It must be possible to establish logical syntax without mentioning the meaning of a sign: only the description of expressions may be presupposed.

From this observation we turn to Russell’s ‘theory of types’. It can be seen that Russell must be wrong, because he had to mention the meaning of signs when establishing the rules for them.

No proposition can make a statement about itself, because a propositional sign cannot be contained in itself (that is the whole of the ‘theory of types’).

The reason why a function cannot be its own argument is that the sign for a function already contains the prototype of its argument, and it cannot contain itself.

3.33
ttrue, it fails to exclude all mistakes.)

For let us suppose that the function $F(fx)$ could be its own argument: in that case there would be a proposition
und in diesem müssen die äußere Funktion \( F \) und die innere Funktion \( F' \) verschiedene Bedeutungen haben, denn die innere hat die Form \( \phi(fx) \), die äußere die Form \( \psi(\phi(fx)) \). Gemeinsam ist den beiden Funktionen nur der Buchstabe „\( F \)“, der aber allein nichts bezeichnet.

Dies wird sofort klar, wenn wir statt „\( F(Fu)\)“ schreiben „\( (\exists \phi): F(\phi u) \), \( \phi u = Fu \)“.

Hiermit erledigt sich Russells Paradox. Die Regeln der logischen Syntax müssen sich von selbst verstehen, wenn man nur weiß, wie ein jedes Zeichen bezeichnet.

Der Satz besitzt wesentliche und zufällige Züge.

Zufällig sind die Züge, die von der besonderen Art der Hervorbrinigung des Satzzeichens herrühren. Wesentlich diejenigen, welche allein den Satz befähigen, seinen Sinn auszudrücken.

Das Wesentliche am Satz ist also das, was allen Sätzen, welche den gleichen Sinn ausdrücken können, gemeinsam ist.

Und ebenso ist allgemein das Wesentliche am Symbol das, was alle Symbole, die denselben Zweck erfüllen können, gemeinsam haben.

Man könnte also sagen: Der eigentliche Name ist das, was alle Symbole, die den Gegenstand bezeichnen, gemeinsam haben. Es würde sich so successiver ergeben, dass keinerlei Zusammensetzung für den Namen wesentlich ist.

An unseren Notationen ist zwar etwas willkürlich, aber das ist nicht willkürlich: Dass, wenn wir etwas willkürlich „\( F(F(fx)) \)“, und in this the outer function \( F \) and the inner function \( F' \) must have different meanings; for the inner has the form \( \phi(fx) \), the outer the form \( \psi(\phi(fx)) \). Common to both functions is only the letter “\( F' \)”, which by itself signifies nothing.

This is at once clear, if instead of „\( F(Fu)\)“ we write „\( (\exists \phi): F(\phi u) \), \( \phi u = Fu \)“.

Herewith Russell’s paradox vanishes. The rules of logical syntax must follow of themselves, if we only know how every single sign signifies.

A proposition possesses essential and accidental features.

Accidental are the features which are due to a particular way of producing the propositional sign. Essential are those which alone enable the proposition to express its sense.

The essential in a proposition is therefore that which is common to all propositions which can express the same sense.

And in the same way in general the essential in a symbol is that which all symbols which can fulfill the same purpose have in common.

One could therefore say the real name is that which all symbols, which signify an object, have in common. It would then follow, step by step, that no sort of composition was essential for a name.

In our notations there is indeed something arbitrary, but this is not arbitrary, namely that if we have determined any-

‘\( F(F(fx)) \)’, in which the outer function \( F \) and the inner function \( F' \) must have different meanings, since the inner one has the form \( \phi(fx) \) and the outer one has the form \( \psi(\phi(fx)) \). Only the letter ‘\( F' \)’ is common to the two functions, but the letter by itself signifies nothing.

This immediately becomes clear if instead of ‘\( F(Fu) \)’ we write ‘\( (\exists \phi): F(\phi u) \), \( \phi u = Fu \)’.

That disposes of Russell’s paradox.

The rules of logical syntax must go without saying, once we know how each individual sign signifies.

A proposition possesses essential and accidental features.

Accidental features are those that result from the particular way in which the propositional sign is produced. Essential features are those without which the proposition could not express its sense.

So what is essential in a proposition is what all propositions that can express the same sense have in common.

And similarly, in general, what is essential in a symbol is what all symbols that can serve the same purpose have in common.

One could therefore say that the real name of an object was what all symbols that signified it had in common. Thus, one by one, all kinds of composition would prove to be unessential to a name.

Although there is something arbitrary in our notations, this much is not arbitrary—that when we have determined one
bestimmt haben, dann etwas anderes der Fall sein muss. (Dies hängt von dem Wesen der Notation ab.)

Eine besondere Bezeichnungsweise mag unwichtig sein, aber wichtig ist es immer, dass diese eine mögliche Bezeichnungsweise ist. Und so verhält es sich in der Philosophie überhaupt: Das Einzelne erweist sich immer wieder als unwichtig, aber die Möglichkeit jedes Einzelnen gibt uns einen Aufschluss über das Wesen der Welt.

Definitionen sind Regeln der Übersetzung von einer Sprache in eine andere. Jede richtige Zeichensprache muss sich in jede andere nach solchen Regeln übersetzen lassen: Dies ist, was sie alle gemeinsam haben.

Das, was am Symbol bezeichnet, ist das Gemeinsame aller jener Symbole, durch die das erste den Regeln der logischen Syntax zufolge ersetzt werden kann.

Man kann z. B. das Gemeinsame aller Notationen für die Wahrheitsfunktionen so ausdrücken: Es ist ihnen gemeinsam, dass sich alle – z. B. – durch die Notation von „¬p“ („nicht p“) und „p ∨ q“ („p oder q“) ersetzen lassen.

(Hiermit ist die Art und Weise gekennzeichnet, wie eine spezielle mögliche Notation uns allgemeine Aufschlüsse geben kann.)

Das Zeichen des Komplexes löst sich auch bei der Analyse nicht willkürlich auf, so dass etwa seine Auflösung in jedem Satzgefüge eine andere wäre.

3.3421

A particular method of symbolizing may be unimportant, but it is always important that this is a possible method of symbolizing. And this happens as a rule in philosophy: The single thing proves over and over again to be unimportant, but the possibility of every single thing reveals something about the nature of the world.

Definitions are rules for translating one language into another. Every correct symbolism must be translatable into every other according to such rules. It is this which all have in common.

What signifies in the symbol is what is common to all those symbols by which it can be replaced according to the rules of logical syntax.

We can, for example, express what is common to all notations for the truth-functions as follows: It is common to them that they all, for example, can be replaced by the notations of “¬p” (“not p”) and “p ∨ q” (“p or q”).

(Herewith is indicated the way in which a special possible notation can give us general information.)

The sign of the complex is not arbitrarily resolved in the analysis, in such a way that its resolution would be different in every propositional structure.

Nor does analysis resolve the sign for a complex in an arbitrary way, so that it would have a different resolution every time that it was incorporated in a different proposition.
3.4 Der Satz bestimmt einen Ort im logischen Raum. Die Existenz dieses logischen Ortes ist durch die Existenz der Bestandteile allein verbürgt, durch die Existenz des sinnvollen Satzes.

3.41 Das Satzzeichen und die logischen Koordinaten: Das ist der logische Ort.

3.411 Der geometrische und der logische Ort stimmen darin überein, dass beide die Möglichkeit einer Existenz sind.

3.42 Obwohl der Satz nur einen Ort des logischen Raumes bestimmen darf, so muss doch durch ihn schon der ganze logische Raum gegeben sein.

(sonst würden durch die Verneinung, die logische Summe, das logische Produkt, etc. immer neue Elemente – in Koordination – eingeführt.)

(Das logische Gerüst um das Bild herum bestimmt den logischen Raum. Der Satz durchgreift den ganzen logischen Raum.)

3.5 Das angewandte, gedachte Satzzeichen ist der Gedanke.

4 Der Gedanke ist der sinnvolle Satz.

4.001 Die Gesamtheit der Sätze ist die Sprache.

4.002 Der Mensch besitzt die Fähigkeit Sprachen zu bauen, womit sich jeder Sinn ausdrücken lässt, ohne eine Ahnung davon zu haben, wie und was jedes Wort bedeutet. – Wie man auch spricht, ohne zu wissen, wie die einzelnen Laute hervorgebracht werden.

Die Umgangssprache ist ein Teil des menschlichen Organismus und nicht weni-
ger kompliziert als dieser.

Es ist menschenunmöglich, die Sprachlogik aus ihr unmittelbar zu entnehmen.

Die Sprache verkleidet den Gedanken. Und zwar so, dass man nach der äußeren Form des Kleides, nicht auf die Form des bekleideten Gedankens schließen kann; weil die äußere Form des Kleides nach ganz anderen Zwecken gebildet ist als danach, die Form des Körpers erkennen zu lassen.

Die stillschweigenden Abmachungen zum Verständnis der Umgangssprache sind enorm kompliziert.

Die meisten Sätze und Fragen, welche über philosophische Dinge geschrieben worden sind, sind nicht falsch, sondern unsinnig. Wir können daher Fragen dieser Art überhaupt nicht beantworten, sondern nur ihre Unsinnigkeit feststellen. Die meisten Fragen und Sätze der Philosophen beruhen darauf, dass wir unsere Sprachlogik nicht verstehen.

(Sie sind von der Art der Frage, ob das Gute mehr oder weniger identisch sei als das Schöne.)

Und es ist nicht verwunderlich, dass die tiefsten Probleme eigentlich k e i n e Probleme sind.

Alle Philosophie ist „Sprachkritik“. (Allerdings nicht im Sinne Mauthners.) Russell’s Verdienst ist es, gezeigt zu haben, dass die scheinbare logische Form des Satzes nicht seine wirkliche sein muss.

4.003

Der Satz ist ein Bild der Wirklichkeit. Der Satz ist ein Modell der Wirklichkeit.

4.0031

4.01

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4.011 keit, so wie wir sie uns denken.


Und doch erweisen sich diese ZeichenSprachen auch im gewöhnlichen Sinne als Bilder dessen, was sie darstellen.


4.013 Und wenn wir in das Wesentliche dieser Bildhaftigkeit eindringen, so sehen wir, dass dieselbe durch scheinbare Unregelmäßigkeiten (wie die Verwendung der ♯ und ♭ in der Notenschrift) nicht gestört wird.

Denn auch diese Unregelmäßigkeiten bilden das ab, was sie ausdrücken sollen; nur auf eine andere Art und Weise.

4.014 Die Grammophonplatte, der musikalische Gedanke, die Notenschrift, die Schallwellen, stehen alle in jener abbildenden internen Beziehung zu einander, die zwischen Sprache und Welt besteht.

Ihnen allen ist der logische Bau gemeinsam.

(Wie im Märchen die zwei Jünglinge, ihre zwei Pferde und ihre Lilien. Sie sind alle in gewissem Sinne Eins.)

4.0141 Dass es eine allgemeine Regel gibt, durch die der Musiker aus der Partitur die as we think it is.

At the first glance the proposition—say as it stands printed on paper—does not seem to be a picture of the reality of which it treats. But nor does the musical score appear at first sight to be a picture of a musical piece; nor does our phonetic spelling (letters) seem to be a picture of our spoken language.

And yet these symbolisms prove to be pictures—even in the ordinary sense of the word—of what they represent.

It is obvious that we perceive a proposition of the form aRb as a picture. Here the sign is obviously a likeness of the signified.

And if we penetrate to the essence of this pictorial nature we see that this is not disturbed by apparent irregularities (like the use of ♯ and ♭ in the score).

For these irregularities also picture what they are to express; only in another way.

The gramophone record, the musical thought, the score, the waves of sound, all stand to one another in that pictorial internal relation, which holds between language and the world.

To all of them the logical structure is common.

(Like the two youths, their two horses and their lilies in the story. They are all in a certain sense one.)

In the fact that there is a general rule by which the musician is able to read the we imagine it.

At first sight a proposition—one set out on the printed page, for example—does not seem to be a picture of the reality with which it is concerned. But neither do written notes seem at first sight to be a picture of a piece of music, nor our phonetic notation (the alphabet) to be a picture of our speech.

And yet these sign-languages prove to be pictures, even in the ordinary sense, of what they represent.

It is obvious that a proposition of the form ‘aRb’ strikes us as a picture. In this case the sign is obviously a likeness of what is signified.

And if we penetrate to the essence of this pictorial character, we see that it is not impaired by apparent irregularities (such as the use of ♯ and ♭ in musical notation).

For even these irregularities depict what they are intended to express; only they do it in a different way.

A gramophone record, the musical idea, the written notes, and the sound-waves, all stand to one another in the same internal relation of depicting that holds between language and the world.

They are all constructed according to a common logical pattern.

(Like the two youths in the fairy-tale, their two horses, and their lilies. They are all in a certain sense one.)

There is a general rule by means of which the musician can obtain the sym-
Symphonie entnehmen kann, durch welche
die Symphonie und nach der ersten
Regel wieder die Partitur ableiten kann,
darin besteht eben die innere Ähnlichkeit
dieser scheinbar so ganz verschiedenen Ge-
bilde. Und jene Regel ist das Gesetz der
Projektion, welches die Symphonie in die
Notensprache projiziert. Sie ist die Regel
der Übersetzung der Notensprache in die
Sprache der Grammophonplatte.

4.015
Die Möglichkeit aller Gleichnisse, der
ganzen Bildhaftigkeit unserer Ausdruck-
weise, ruht in der Logik der Abbildung.

4.016
Um das Wesen des Satzes zu verstehen,
denken wir an die Hieroglyphenschrift,
welche die Tatsachen die sie beschreibt ab-
bildet. Und aus ihr wurde die Buchstaben-
schrift, ohne das Wesentliche der Abbil-
dung zu verlieren.

4.02
Dies sehen wir daraus, dass wir den
Sinn des Satzzeichens verstehen, ohne
dass er uns erklärt wurde.

4.021
Der Satz ist ein Bild der Wirklichkeit:
Denn ich kenne die von ihm dargestellte
Sachlage, wenn ich den Satz verstehe. Und
den Satz verstehe ich, ohne dass mir sein
Sinn erklärt wurde.

4.022
Der Satz zeig t seinen Sinn.
Der Satz zeig t, wie es sich verhält,
verhält sich er wahr ist. Und er sagt, dass
es sich so verhält.

4.023
Die Wirklichkeit muss durch den Satz
auf ja oder nein fixiert sein.

---

Symphony out of the score, and that there
is a rule by which one could reconstruct the
symphony from the line on a gramophone
record and from this again—by means of
the first rule—construct the score, herein
lies the internal similarity between these
things which at first sight seem to be en-
tirely different. And the rule is the law
of projection which projects the symphony
into the language of the musical score. It is
the rule of translation of this language into
the language of the gramophone record.

The possibility of all similes, of all the
images of our language, rests on the logic
of representation.

In order to understand the essence of
the proposition, consider hieroglyphic writ-
ing, which pictures the facts it describes.

And from it came the alphabet without
the essence of the representation being lost.

This we see from the fact that we under-
stand the sense of the propositional sign,
without having had it explained to us.

The proposition is a picture of reality,
for I know the state of affairs presented
by it, if I understand the proposition. And
I understand the proposition, without its
sense having been explained to me.

The proposition shows its sense.
The proposition shows how things
stand, if it is true. And it says, that they
do so stand.

The proposition determines reality to
this extent, that one only needs to say “Yes”
phony from the score, and which makes it
possible to derive the symphony from the
groove on the gramophone record, and, us-
ning the first rule, to derive the score again.
That is what constitutes the inner similar-
ity between these things which seem to be
constructed in such entirely different ways.
And that rule is the law of projection which
projects the symphony into the language
of musical notation. It is the rule for trans-
lating this language into the language of
gramophone records.

The possibility of all imagery, of all our
pictorial modes of expression, is contained
in the logic of depiction.

In order to understand the essential
nature of a proposition, we should consider
hieroglyphic script, which depicts the facts
that it describes.

And alphabetic script developed out of
it without losing what was essential to de-
piction.

We can see this from the fact that we un-
derstand the sense of a propositional
sign without its having been explained to
us.

A proposition is a picture of reality: for
if I understand a proposition, I know the
situation that it represents. And I under-
stand the proposition without having had
its sense explained to me.

A proposition shows its sense.
A proposition shows how things stand
if it is true. And it says that they do so
stand.

A proposition must restrict reality to
two alternatives: yes or no.
Dazu muss sie durch ihn vollständig beschrieben werden.

Der Satz ist die Beschreibung eines Sachverhaltes.

Wie die Beschreibung einen Gegenstand nach seinen externen Eigenschaften, so beschreibt der Satz die Wirklichkeit nach ihren internen Eigenschaften.

Der Satz konstruiert eine Welt mit Hilfe eines logischen Gerüstes und darum kann man am Satz auch sehen, wie sich alles Logische verhält, wenn er wahr ist. Man kann aus einem falschen Satz Schlüsse ziehen.

Einen Satz verstehen, heißt, wissen was der Fall ist, wenn er wahr ist.

(Man kann ihn also verstehen, ohne zu wissen, ob er wahr ist.)

Man versteht ihn, wenn man seine Bestandteile versteht.

Die Übersetzung einer Sprache in eine andere geht nicht so vor sich, dass man jeden Satz der einen in einen Satz der anderen übersetzt, sondern nur die Satzbestandteile werden übersetzt.

(Und das Wörterbuch übersetzt nicht nur Substantiva, sondern auch Zeit-, Eigenschafts- und Bindewörter etc.; und es behandelt sie alle gleich.)

Die Bedeutung der einfachen Zeichen (der Wörter) müssen uns erklärt werden, dass wir sie verstehen.

Mit den Sätzen aber verständigen wir uns.

Es liegt im Wesen des Satzes, dass er uns einen neuen Sinn mitteilen kann.

or “No” to it to make it agree with reality.

Reality must therefore be completely described by the proposition.

A proposition is the description of a fact.

As the description of an object describes it by its external properties so propositions describe reality by its internal properties.

The proposition constructs a world with the help of a logical scaffolding, and therefore one can actually see in the proposition all the logical features possessed by reality if it is true. One can draw conclusions from a false proposition.

To understand a proposition means to know what is the case, if it is true.

(One can therefore understand it without knowing whether it is true or not.)

One understands it if one understands it constituent parts.

The translation of one language into another is not a process of translating each proposition of the one into a proposition of the other, but only the constituent parts of propositions are translated.

(And the dictionary does not only translate substantives but also adverbs and conjunctions, etc., and it treats them all alike.)

The meanings of the simple signs (the words) must be explained to us, if we are to understand them.

By means of propositions we explain ourselves.

It is essential to propositions, that they can communicate a new sense to us.

In order to do that, it must describe reality completely.

A proposition is a description of a state of affairs.

Just as a description of an object describes it by giving its external properties, so a proposition describes reality by its internal properties.

A proposition constructs a world with the help of a logical scaffolding, so that one can actually see from the proposition how everything stands logically if it is true. One can draw inferences from a false proposition.

To understand a proposition means to know what is the case if it is true.

(One can understand it, therefore, without knowing whether it is true.)

It is understood by anyone who understands its constituents.

When translating one language into another, we do not proceed by translating each proposition of the one into a proposition of the other, but merely by translating the constituents of propositions.

(And the dictionary translates not only substantives, but also verbs, adjectives, and conjunctions, etc.; and it treats them all in the same way.)

The meanings of simple signs (words) must be explained to us if we are to understand them.

With propositions, however, we make ourselves understood.

It belongs to the essence of a proposition that it should be able to communicate
Ein Satz muss mit alten Ausdrücken einen neuen Sinn mitteilen.

Der Satz teilt uns eine Sachlage mit, also muss er wesentlich mit der Sachlage zusammenhängen.

Und der Zusammenhang ist eben, dass er ihr logisches Bild ist.

Der Satz sagt nur insoweit etwas aus, als er ein Bild ist.

Im Satz wird gleichsam eine Sachlage probeweise zusammengestellt.

Man kann geradezu sagen: statt, dieser Satz hat diesen und diesen Sinn; dieser Satz stellt diese und diese Sachlage dar.

Ein Name steht für ein Ding, ein anderer für ein anderes Ding und untereinander sind sie verbunden, so stellt das Ganze – wie ein lebendes Bild – den Sachverhalt vor.

Die Möglichkeit des Satzes beruht auf dem Prinzip der Vertretung von Gegenständen durch Zeichen.

Mein Grundgedanke ist, dass die „logischen Konstanten“ nicht vertreten. Dass sich die Logik der Tatsachen nicht vertreten lässt.

Nur insoweit ist der Satz ein Bild der Sachlage, als er logisch gegliedert ist.

(Auch der Satz: „ambulo“, ist zusammengesetzt, denn sein Stamm ergibt mit einer anderen Endung, und seine Endung mit einem anderen Stamm, einen anderen Sinn.)

A proposition must communicate a new sense with old words.

The proposition communicates to us a state of affairs, therefore it must be essentially connected with the state of affairs.

And the connexion is, in fact, that it is its logical picture.

The proposition only asserts something, in so far as it is a picture.

In the proposition a state of affairs is, as it were, put together for the sake of experiment.

One can say, instead of, This proposition has such and such a sense, This proposition represents such and such a state of affairs.

One name stands for one thing, and another for another thing, and they are connected together. And so the whole, like a living picture, presents the atomic fact.

The possibility of propositions is based upon the principle of the representation of objects by signs.

My fundamental thought is that the “logical constants” do not represent. That the logic of the facts cannot be represented.

The proposition is a picture of its state of affairs, only in so far as it is logically articulated.

(A even the proposition “ambulo” is composite, for its stem gives a different sense with another termination, or its termination with another stem.)

4.031

A proposition must use old expressions to communicate a new sense.

A proposition communicates a situation to us, and so it must be essentially connected with the situation.

And the connexion is precisely that it is its logical picture.

A proposition states something only in so far as it is a picture.

In a proposition a situation is, as it were, constructed by way of experiment.

Instead of, ‘This proposition has such and such a sense’, we can simply say, ‘This proposition has such a sense’.

One name stands for one thing, another for another thing, and they are combined with one another. In this way the whole group—like a tableau vivant—presents a state of affairs.

The possibility of propositions is based on the principle that objects have signs as their representatives.

My fundamental idea is that the ‘logical constants’ are not representatives; that there can be no representatives of the logic of facts.

It is only in so far as a proposition is logically articulated that it is a picture of a situation.

(Even the proposition, Ambulo, is composite: for its stem with a different ending yields a different sense, and so does its ending with a different stem.)
4.04 Am Satz muss gerade soviel zu unterscheiden sein, als an der Sachlage, die er darstellt.

Die beiden müssen die gleiche logische (mathematische) Mannigfaltigkeit besitzen. (Vergleiche Hertz’s „Mechanik,“ über Dynamische Modelle.)

Diese mathematische Mannigfaltigkeit kann man natürlich nicht selbst wieder abbilden. Aus ihr kann man beim Abbilden nicht heraus.

4.041 Wollten wir z. B. das, was wir durch 
\((x). f x\)

ausdrücken, durch Vorsetzen eines Indexes vor 
\(f x\)
ausdrücken – etwa so: 
\(\text{Alg}. f x\) – es würde nicht genügen – wir wüssten nicht, was verallgemeinert wurde. Wollten wir es durch einen Index 
\(x a\)
anzeigen – etwa so: 
\(f(x a)\) – es würde auch nicht genügen – wir wüssten nicht den Bereich der Allgemeinheitsbezeichnung.

Wollten wir es durch Einführung einer Marke in die Argumentstellen versuchen – etwa so: 

Alle diese Bezeichnungsweisen genügen nicht, weil sie nicht die notwendige mathematische Mannigfaltigkeit haben.

Aus derselben Grunde genügt die idealistische Erklärung des Sehens der räumlichen Beziehungen durch die „Raumbrille“ nicht, weil sie nicht die Mannigfaltigkeit dieser Beziehungen erklären kann.

Die Wirklichkeit wird mit dem Satz verglichen.

4.0411 In the proposition there must be exactly as many things distinguishable as there are in the state of affairs, which it represents.

They must both possess the same logical (mathematical) multiplicity (cf. Hertz’s Mechanics, on Dynamic Models).

This mathematical multiplicity naturally cannot in its turn be represented. One cannot get outside it in the representation.

If we tried, for example, to express what is expressed by “\((x). f x\)” by putting an index before \(f x\), like: “Gen. \(f x\)”, it would not do, we should not know what was generalized. If we tried to show it by an index \(g\), like: “\(f(x g)\)” it would not do—we should not know the scope of the generalization.

If we were to try it by introducing a mark in the argument places, like “\((G, G). F(G, G)\)”, it would not do—we could not determine the identity of the variables, etc.

All these ways of symbolizing are inadequate because they have not the necessary mathematical multiplicity.

For the same reason the idealist explanation of the seeing of spatial relations through “spatial spectacles” does not do, because it cannot explain the multiplicity of these relations.

Reality is compared with the proposition.

4.0412 In a proposition there must be exactly as many distinguishable parts as in the situation that it represents.

The two must possess the same logical (mathematical) multiplicity. (Compare Hertz’s Mechanics on dynamical models.)

This mathematical multiplicity, of course, cannot itself be the subject of depiction. One cannot get away from it when depicting.

If, for example, we wanted to express what we now write as ‘\((x). f x\)’ by putting an affix in front of ‘\(f x\)’—for instance by writing ‘Gen. \(f x\)’—it would not be adequate: we should not know what was being generalized. If we wanted to signalize it with an affix ‘\(g\)’—for instance by writing ‘\(f(x g)\)’—that would not be adequate either: we should not know the scope of the generality-sign.

If we were to try it by introducing a mark into the argument-places—for instance by writing ‘\((G, G). F(G, G)\)’—it would not be adequate: we should not be able to establish the identity of the variables. And so on.

All these modes of signifying are inadequate because they lack the necessary mathematical multiplicity.

For the same reason the idealist’s appeal to ‘spatial spectacles’ is inadequate to explain the seeing of spatial relations, because it cannot explain the multiplicity of these relations.

Reality is compared with propositions.
Nur dadurch kann der Satz wahr oder falsch sein, indem er ein Bild der Wirklichkeit ist.

Beachtet man nicht, dass der Satz einen von den Tatsachen unabhängigen Sinn hat, so kann man leicht glauben, dass wahr und falsch gleichberechtigte Beziehungen von Zeichen und Bezeichnetem sind.

Man könnte dann z. B. sagen, dass „p“ auf die wahre Art bezeichnet, was „¬p“ auf die falsche Art, etc.


Dass in einem Satz die Verneinung vorkommt, ist noch kein Merkmal seines Sinnes (¬¬p = p).

Die Sätze „p“ und „¬p“ haben entgegengesetzten Sinn, aber es entspricht ihnen eine und dieselbe Wirklichkeit.

Ein Bild zur Erklärung des Wahrheitsbegriffes: Schwarzer Fleck auf weißem Papier; die Form des Fleckes kann man beschreiben, indem man für jeden Punkt der Fläche angibt, ob er weiß oder schwarz ist. Propositions can be true or false only by being pictures of the reality.

If one does not observe that propositions have a sense independent of the facts, one can easily believe that true and false are two relations between signs and things signified with equal rights.

One could, then, for example, say that “p” signifies in the true way what “¬p” signifies in the false way, etc.

Can we not make ourselves understood with false propositions just as we have done up till now with true ones?—So long as it is known that they are meant to be false.—No! For a proposition is true, if what we assert by means of it is the case; and if by “p” we mean ¬p, and what we mean is the case, then “p” in the new conception is true and not false.

That, however, the signs “p” and “¬p” can say the same thing is important, for it shows that the sign “¬” corresponds to nothing in reality.

That negation occurs in a proposition, is no characteristic of its sense (¬¬p = p).

The propositions “p” and “¬p” have opposite senses, but to them corresponds one and the same reality.

An illustration to explain the concept of truth. A black spot on white paper; the form of the spot can be described by saying of each point of the plane whether it is white or black. To the fact that a point
Der Tatsache, dass ein Punkt schwarz ist, entspricht eine positive – der, dass ein Punkt weiß (nicht schwarz) ist, eine negative Tatsache. Bezeichne ich einen Punkt der Fläche (einen Fregeschen Wahrheitswert), so entspricht dies der Annahme, die zur Beurteilung aufgestellt wird, etc. etc.


Der Punkt, an dem das Gleichnis hinkt ist nun der: Wir können auf einen Punkt des Papiers zeigen, auch ohne zu wissen, was weiß und schwarz ist; einem Satz ohne Sinn aber entspricht gar nichts, denn er bezeichnet kein Ding (Wahrheitswert) dessen Eigenschaften etwa „falsch“ oder „wahr“ hießen; das Verbum eines Satzes ist nicht „ist wahr“ oder „ist falsch“ – wie Frege glaubte, sondern das, was „wahr“ ist, muss das Verbum schon enthalten.

Jeder Satz muss schon einen Sinn haben; die Bejahung kann ihn ihm nicht geben, denn sie bejaht ja gerade den Sinn. Und dasselbe gilt von der Verneinung, etc.

One could say: Die Verneinung bezieht sich schon auf den logischen Ort, den der verneinte Satz bestimmt.

Der verneinende Satz bestimmt einen anderen logischen Ort als der verneinte.

4.064

Man könnte sagen: Die Verneinung bezieht sich schon auf den logischen Ort, den der verneinte Satz bestimmt.

4.0641

Es ist schwarz entspricht eine positive Tatsache; zu der Tatsache, dass ein Punkt weiß (nicht schwarz) ist, eine negative Tatsache. Bezeichne ich einen Punkt der Fläche (einen Fregeschen Wahrheitswert), so entspricht dies der Annahme, die zur Beurteilung aufgestellt wird, etc. etc.

But to be able to say that a point is black or white, I must first know under what conditions a point is called white or black; in order to be able to say “p” is true (or false) I must have determined under what conditions I call “p” true, and thereby I determine the sense of the proposition.

But in order to be able to say that a point is black or white, I must first know when a point is called black, and when white: in order to be able to say, “p” is true (or false), I must have determined in what circumstances I call ‘p’ true, and in so doing I determine the sense of the proposition.

Now the point where the simile breaks down is this: we can indicate a point on the paper even if we do not know what black and white are, but if a proposition has no sense, nothing corresponds to it, since it does not designate a thing (a truth-value) which might have properties called ‘false’ or ‘true’. The verb of a proposition is not ‘is true’ or ‘is false’, as Frege thought: rather, that which ‘is true’ must already contain the verb.

Every proposition must already have a sense: assertion cannot give it a sense, for what it asserts is the sense itself. And the same holds of denial, etc.

One could say, the denial is already related to the logical place determined by the proposition that is denied.

The denying proposition determines a logical place different from that of the negated proposition.
Der verneinende Satz bestimmt einen logischen Ort mit Hilfe des logischen Ortes des verneinten Satzes, indem er jenen als außerhalb diesem liegend beschreibt.

Dass man den verneinten Satz wieder verneinen kann, zeigt schon, dass das, was verneint wird, schon ein Satz und nicht erst die Vorbereitung zu einem Satze ist.

4.1 Der Satz stellt das Bestehen und Nichtbestehen der Sachverhalte dar.

4.11 Die Gesamtheit der wahren Sätze ist die gesamte Naturwissenschaft (oder die Gesamtheit der Naturwissenschaften).

4.111 Die Philosophie ist keine der Naturwissenschaften.

    (Das Wort „Philosophie“ muss etwas bedeuten, was über oder unter, aber nicht neben den Naturwissenschaften steht.)

4.112 Der Zweck der Philosophie ist die logische Klärung der Gedanken.

    Die Philosophie ist keine Lehre, sondern eine Tätigkeit.

    Ein philosophisches Werk besteht wesentlich aus Erläuterungen.

    Das Resultat der Philosophie sind nicht „philosophische Sätze“, sondern das Klarwerden von Sätzen.

    Die Philosophie soll die Gedanken, die sonst, gleichsam, trübe und verschwommen sind, klar machen und scharf abgrenzen.

4.1121 Die Psychologie ist der Philosophie nicht verwandter als irgend eine andere Naturwissenschaft.

    Erkenntnistheorie ist die Philosophie

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The denying proposition determines a logical place, with the help of the logical place of the proposition denied, by saying that it lies outside the latter place.

That one can deny again the denied proposition, shows that what is denied is already a proposition and not merely the preliminary to a proposition.

A proposition presents the existence and non-existence of atomic facts.

The totality of true propositions is the total natural science (or the totality of the natural sciences).

Philosophy is not one of the natural sciences.

(The word “philosophy” must mean something which stands above or below, but not beside the natural sciences.)

The object of philosophy is the logical clarification of thoughts.

Philosophy is not a theory but an activity.

A philosophical work consists essentially of elucidations.

The result of philosophy is not a number of “philosophical propositions”, but to make propositions clear.

Philosophy should make clear and delimit sharply the thoughts which otherwise are, as it were, opaque and blurred.

Psychology is no nearer related to philosophy than is any other natural science.

The theory of knowledge is the philosophy

The negating proposition determines a logical place with the help of the logical place of the negated proposition. For it describes it as lying outside the latter’s logical place.

The negated proposition can be negated again, and this in itself shows that what is negated is already a proposition, and not merely something that is preliminary to a proposition.

Propositions represent the existence and non-existence of states of affairs.

The totality of true propositions is the whole of natural science (or the whole corpus of the natural sciences).

Philosophy is not one of the natural sciences.

(The word ‘philosophy’ must mean something whose place is above or below the natural sciences, not beside them.)

Philosophy aims at the logical clarification of thoughts.

Philosophy is not a body of doctrine but an activity.

A philosophical work consists essentially of elucidations.

Philosophy does not result in ‘philosophical propositions’, but rather in the clarification of propositions.

Without philosophy thoughts are, as it were, cloudy and indistinct: its task is to make them clear and to give them sharp boundaries.

Psychology is no more closely related to philosophy than any other natural science.

Theory of knowledge is the philosophy
Entspricht nicht mein Studium der Zeichensprache dem Studium der Denkprozesse, welches die Philosophen für die Philosophie der Logik für so wesentlich hielten? Nur verwinkelten sie sich meistens in unwesentliche psychologische Untersuchungen und eine analoge Gefahr gibt es auch bei meiner Methode.

Die Darwinische Theorie hat mit der Philosophie nicht mehr zu schaffen als irgendeine andere Hypothese der Naturwissenschaft.

Die Philosophie begrenzt das bestreitbare Gebiet der Naturwissenschaft.

Sie soll das Denkbare abgrenzen und damit das Undenkbare.

Sie soll das Undenkbare von innen durch das Denkbare begrenzen.

Sie wird das Unsagbare bedeuten, indem sie das Sagbare klar darstellt.

Alles was überhaupt gedacht werden kann, kann klar gedacht werden. Alles, was sich aussprechen lässt, lässt sich klar aussprechen.

Der Satz kann die gesamte Wirklichkeit darstellen, aber er kann nicht das darstellen, was er mit der Wirklichkeit gemein haben muss, um sie darstellen zu können – die logische Form.

Um die logische Form darstellen zu können, müssten wir uns mit dem Satze außerhalb der Logik aufstellen können, das heißt außerhalb der Welt.

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Philosophy limits the disputable sphere of natural science.

It should limit the thinkable and thereby the unthinkable.

It should limit the unthinkable from within through the thinkable.

It will mean the unspeakable by clearly displaying the speakable.

Everything that can be thought at all can be thought clearly. Everything that can be said can be said clearly.

Propositions can represent the whole reality, but they cannot represent what they must have in common with reality in order to be able to represent it—the logical form.

To be able to represent the logical form, we should have to be able to put ourselves with the propositions outside logic, that is outside the world.

Propositions can represent the whole reality, but they cannot represent what they must have in common with reality in order to be able to represent it—logical form.

In order to be able to represent logical form, we should have to be able to station ourselves with propositions somewhere outside logic, that is to say outside the world.
4.121 Der Satz kann die logische Form nicht darstellen, sie spiegelt sich in ihm.  

Propositions cannot represent the logical form: this mirrors itself in the propositions.

Was sich in der Sprache spiegelt, kann sie nicht darstellen.  

That which mirrors itself in language, language cannot represent.

Was sich in der Sprache ausdrückt, können wir nicht durch sie ausdrücken.  

The propositions show the logical form of reality.

4.1211 So zeigt ein Satz „fa“, dass in seinem Sinn der Gegenstand a vorkommt, zwei Sätze „fa“ und „ga“, dass in ihnen beiden von demselben Gegenstand die Rede ist.  

Thus a proposition “fa” shows that in its sense the object a occurs, two propositions “fa” and “ga” that they are both about the same object.

Wenn zwei Sätze einander widersprechen, so zeigt dies ihre Struktur; ebenso, wenn einer aus dem anderen folgt. U.s.w.  

If two propositions contradict one another, this is shown by their structure; similarly if one follows from another, etc.

4.1212 Was gezeigt werden kann, kann nicht gesagt werden.  

What can be shown cannot be said.

4.1213 Jetzt verstehen wir auch unser Gefühl: dass wir im Besitz einer richtigen logischen Auffassung seien, wenn nur einmal alles in unserer Zeichensprache stimmt.  

Now we understand our feeling that we are in possession of the right logical conception, if only all is right in our symbolism.


We can speak in a certain sense of formal properties of objects and atomic facts, or of properties of the structure of facts, and in the same sense of formal relations and relations of structures.


I also say “internal property”; instead of relation of structures “internal relation”.


(I instead of ‘structural property’ I say ‘internal property’; instead of relation of structures ‘internal relation’.


Eine interne Eigenschaft einer Tatsache können wir auch einen Zug dieser Tatsache nennen. (In dem Sinn, in welchem wir etwa von Gesichtszügen sprechen.)

Eine Eigenschaft ist intern, wenn es undenkbar ist, dass ihr Gegenstand sie nicht besitzt. (Diese blaue Farbe und jene stehen in der internen Relation von heller und dunkler eo ipso. Es ist undenkbar, dass diese beiden Gegenstände nicht in dieser Relation stünden.)

(Die internen Eigenschaften einer Tatsache und Relationen kann aber nicht durch Sätze behauptet werden, sondern es zeigt sich in den Sätzen, welche jene Sachverhalte darstellen und von jenen Gegenständen handeln.)

Das Bestehen solcher interner Eigenschaften und Relationen kann aber nicht durch Sätze behauptet werden, sondern es zeigt sich in den Sätzen, welche jene Sachverhalte darstellen und von jenen Gegenständen handeln.

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(Hier entspricht dem schwankenden Gebrauch der Worte „Eigenschaft“ und „Relation“ der schwankende Gebrauch des Wortes „Gegenstand“.)

Das Bestehen einer internen Eigenschaft einer möglichen Sachlage wird nicht durch einen Satz ausgedrückt, sondern es drückt sich in dem Satz, den sie darstellenden Satz durch eine interne Eigenschaft dieses Satzes aus.

Es wäre ebenso unsinnig, dem Satze eine formale Eigenschaft zuzusprechen, als sie ihm abzusprechen.

Formen kann man nicht dadurch voneinander unterscheiden, dass man sagt, die eine habe diese, die andere aber jene Eigenschaft; denn dies setzt voraus, dass lations.)

The holding of such internal properties and relations cannot, however, be asserted by propositions, but it shows itself in the propositions, which present the facts and treat of the objects in question.

An internal property of a fact we also call a feature of this fact. (In the sense in which we speak of facial features.)

A property is internal if it is unthinkable that its object does not possess it.

(This bright blue colour and that stand in the internal relation of bright and darker eo ipso. It is unthinkable that these two objects should not stand in this relation.)

(Here to the shifting use of the words “property” and “relation” there corresponds the shifting use of the word “object”.)

The existence of an internal property of a possible state of affairs is not expressed by a proposition, but it expresses itself in the proposition which presents that state of affairs, by an internal property of this proposition. It would be as senseless to ascribe a formal property to a proposition as to deny it the formal property.

One cannot distinguish forms from one another by saying that one has this property, the other that: for this assumes that there is a sense in asserting either property widespread among philosophers.)

It is impossible, however, to assert by means of propositions that such internal properties and relations obtain: rather, this makes itself manifest in the propositions that represent the relevant states of affairs and are concerned with the relevant objects. An internal property of a fact can also be called a feature of that fact (in the sense in which we speak of facial features, for example).

A property is internal if it is unthinkable that its object should not possess it.

(This shade of blue and that one stand, eo ipso, in the internal relation of lighter to darker. It is unthinkable that these two objects should not stand in this relation.)

(Here the shifting use of the word ‘object’ corresponds to the shifting use of the words ‘property’ and ‘relation’.)

The existence of an internal property of a possible situation is not expressed by means of a proposition: rather, it expresses itself in the proposition representing the situation, by means of an internal property of that proposition. It would be just as nonsensical to assert that a proposition had a formal property as to deny it.

It is impossible to distinguish forms from one another by saying that one has this property and another that property: for this presupposes that it makes sense to
es einen Sinn habe, beide Eigenschaften von beiden Formen auszusagen.

The existence of an internal relation between possible states of affairs expresses itself in language by an internal relation between the propositions presenting them.

Hier erledigt sich nun die Streitfrage, „ob alle Relationen intern oder extern seien“.

Now this settles the disputed question “whether all relations are internal or external”.

Reihen, welche durch interne Relationen geordnet sind, nenne ich Formenreihen.

Series which are ordered by internal relations I call formal series.

Die Zahlenreihe ist nicht nach einer externen, sondern nach einer internen Relation geordnet.

The order of the number-series is not governed by an external relation but by an internal relation.

Ebenso die Reihe der Sätze „$aRb$“, „$(\exists x): aRx . xRb$“, „$(\exists x, y): aRx . xRy . yRb$“, usw.

Similarly the series of propositions “$aRb$”, “$(\exists x): aRx . xRb$”, “$(\exists x, y): aRx . xRy . yRb$”, etc.

(Stehen $b$ in einer dieser Beziehungen zu $a$, so nenne ich $b$ einen Nachfolger von $a$.)

In the sense in which we speak of formal properties we can now speak also of formal concepts.

We can now talk about formal concepts, in the same sense that we speak of formal properties.

(Ich führe diesen Ausdruck ein, um den Grund der Verwechslung der formalen Griffe mit den eigentlichen Griffe, welche die ganze alte Logik durchzieht, klar zu machen.)

(I introduce this expression in order to make clear the confusion of formal concepts with proper concepts which pervades the whole of traditional logic.)

Dass etwas unter einen formalen Begriff als dessen Gegenstand fällt, kann nicht durch einen Satz ausgedrückt werden. Sondern es zeigt sich an dem Zeichen dieses Gegenstandes selbst. (Der Name zeigt, dass er einen Gegenstand bezeichnet, das Zahlenzeichen, dass es eine Zahl von einer Form erfasst.)

That anything falls under a formal concept as an object belonging to it, cannot be expressed by a proposition. But it is shown in the symbol for the object itself. (The name shows that it signifies an object, the numerical sign that it signifies a number, etc.)

ascribe either property to either form.

The existence of an internal relation between possible situations expresses itself in language by means of an internal relation between the propositions representing them.

Here we have the answer to the vexed question 'whether all relations are internal or external'.

I call a series that is ordered by an internal relation a series of forms.

The same is true of the series of propositions ‘$aRb$’, ‘$(\exists x): aRx . xRb$’, ‘$(\exists x, y): aRx . xRy . yRb$’, and so forth.

(If $b$ stands in one of these relations to $a$, I call $b$ a successor of $a$.)

We can now talk about formal concepts, in the same sense that we speak of formal properties.

(I introduce this expression in order to exhibit the source of the confusion between formal concepts and concepts proper, which pervades the whole of traditional logic.)

When something falls under a formal concept as one of its objects, this cannot be expressed by means of a proposition. Instead it is shown in the very sign for this object. (A name shows that it signifies an object, a sign for a number that it signifies a number, etc.)
bezeichnet etc.)

Die formalen Begriffe können ja nicht, wie die eigentlichen Begriffe, durch eine Funktion dargestellt werden.

Denn ihre Merkmale, die formalen Eigenschaften, werden nicht durch Funktionen ausgedrückt.

Der Ausdruck der formalen Eigenschaft ist ein Zug gewisser Symbole.

Das Zeichen der Merkmale eines formalen Begriffes ist also ein charakteristischer Zug aller Symbole, deren Bedeutungen unter den Begriff fallen.

Der Ausdruck des formalen Begriffes also, eine Satzvariable, in welcher nur dieser charakteristische Zug konstant ist.

Die Satzvariable bezeichnet den formalen Begriff und ihre Werte die Gegenstände, welche unter diesen Begriff fallen.

Jede Variable ist das Zeichen eines formalen Begriffes.

Denn jede Variable stellt eine konstante Form dar, welche alle ihre Werte besitzen, und die als formale Eigenschaft dieser Werte aufgefasst werden kann.

So ist der variable Name „x“ das eigentliche Zeichen des Scheinbegriffes G e - n s t a n d.

Wo immer das Wort „Gegenstand“ („Ding“, „Sache“, etc.) richtig gebraucht wird, wird es in der Begriffsschrift durch den variablen Namen ausgedrückt.

Zum Beispiel in dem Satz „es gibt 2 Gegenstände, welche . . . “ durch „(∃x, y) . . . “.

Wo immer es anders, also als eigentlicher Begriffswort gebraucht wird, entste-

4.127

Formal concepts, cannot, like proper concepts, be presented by a function.

For their characteristics, the formal properties, are not expressed by the functions.

The expression for a formal property is a feature of certain symbols.

So the sign for the characteristics of a formal concept is a distinctive feature of all symbols whose meanings fall under the concept.

4.1271

Every variable is the sign for a formal concept.

For every variable represents a constant form, which all its values possess, and which can be regarded as a formal property of these values.

Thus the variable name ‘x’ is the proper sign for the pseudo-concept object.

4.1272

Wherever the word ‘object’ (‘thing’, ‘entity’, etc.) is rightly used, it is expressed in conceptual notation by a variable name.

For example in the proposition “there are 2 objects which . . . ”, it is expressed by ‘(∃x, y) . . . ’.

Wherever it is used otherwise, i.e. as a proper concept word, there arise senseless
hen unsinnige Scheinsätze.

So kann man z. B. nicht sagen „Es gibt Gegenstände“, wie man etwa sagt: „Es gibt Bücher“. Und ebenso wenig: „Es gibt 100 Gegenstände“, oder „Es gibt $\aleph_0$ Gegenstände“.

Und es ist unsinnig, von der Anzahl aller Gegenstände zu sprechen.

Dasselbe gilt von den Worten „Komplex“, „Tatsache“, „Funktion“, „Zahl“, etc.

Sie alle bezeichnen formale Begriffe und werden in der Begriffsschrift durch Variable, nicht durch Funktionen oder Klassen dargestellt. (Wie Frege und Russell glaubten.)

Ausdrücke wie „1 ist eine Zahl“, „es gibt nur Eine Null“ und alle ähnlichen sind unsinnig.

(Es ist ebenso unsinnig zu sagen: „es gibt nur eine 1“, als es unsinnig wäre, zu sagen: 2 + 2 ist um 3 Uhr gleich 4.)

4.12721 Der formale Begriff ist mit einem Gegenstand, der unter ihn fällt, bereits gegeben. Man kann also nicht Gegenstände eines formalen Begriffes und den formalen Begriff selbst als Grundbegriffe einführen. Man kann also z. B. nicht den Begriff der Funktion, und auch spezielle Funktionen (wie Russell) als Grundbegriffe einführen; oder den Begriff der Zahl und bestimmte Zahlen.

Wollen wir den allgemeinen Satz: „b ist ein Nachfolger von a“ in der Begriffsschrift ausdrücken, so brauchen wir hierzu einen Ausdruck für das allgemeine Glied der Formenreihe:

pseudo-propositions.

So one cannot, e.g. say “There are objects” as one says “There are books”. Nor “There are 100 objects” or “There are $\aleph_0$ objects”.

And it is senseless to speak of the number of all objects.

The same holds of the words “Complex”, “Fact”, “Function”, “Number”, etc.

They all signify formal concepts and are represented in logical symbolism by variables, not by functions or classes (as Frege and Russell thought).

Expressions like “1 is a number”, “there is only one number nought”, and all like them are senseless.

(It is as senseless to say, “there is only one 1” as it would be to say: 2 + 2 is at 3 o’clock equal to 4.)

The formal concept is already given with an object, which falls under it. One cannot, therefore, introduce both, the objects which fall under a formal concept and the formal concept itself, as primitive ideas. One cannot, therefore, e.g. introduce (as Russell does) the concept of function and also special functions as primitive ideas; or the concept of number and definite numbers.

If we want to express in logical symbolism the general proposition “b is a successor of a” we need for this an expression for the general term of the formal series:
Das allgemeine Glied einer Formenreihe kann man nur durch eine Variable ausdrücken, denn der Begriff: Glied dieser Formenreihe, ist ein formaler Begriff. (Dies haben Frege und Russell übersehen; die Art und Weise, wie sie allgemeine Sätze wie den obigen ausdrücken wollen, ist daher falsch; sie enthält einen circulus vitiosus.)

Wir können das allgemeine Glied der Formenreihe bestimmen, indem wir ihr erstes Glied angeben und die allgemeine Form der Operation, welche das folgende Glied aus dem vorhergehenden Satz erzeugt.

The general term of a formal series can only be expressed by a variable, for the concept symbolized by “term of this formal series” is a formal concept. (This Frege and Russell overlooked; the way in which they express general propositions like the above is, therefore, false; it contains a vicious circle.)

We can determine the general term of the formal series by giving its first term and the general form of the operation, which generates the following term out of the preceding proposition.

The question about the existence of a formal concept is senseless. For no proposition can answer such a question.

(For example, one cannot ask: “Are there unanalysable subject-predicate propositions?”)

The logical forms are anumerical. Therefore there are in logic no pre-eminent numbers, and therefore there is no philosophical monism or dualism, etc.

The sense of a proposition is its agreement and disagreement with the possibilities of the existence and non-existence of the atomic facts.

The simplest proposition, the elementary proposition, asserts the existence of

In order to express the general term of a series of forms, we must use a variable, because the concept ‘term of that series of forms’ is a formal concept. (This is what Frege and Russell overlooked: consequently the way in which they want to express general propositions like the one above is incorrect; it contains a vicious circle.)

We can determine the general term of a series of forms by giving its first term and the general form of the operation that produces the next term out of the proposition that precedes it.

To ask whether a formal concept exists is nonsensical. For no proposition can be the answer to such a question.

(So, for example, the question, ‘Are there unanalysable subject-predicate propositions?’ cannot be asked.)

Logical forms are without number.

Hence there are no pre-eminent numbers in logic, and hence there is no possibility of philosophical monism or dualism, etc.

The sense of a proposition is its agreement and disagreement with possibilities of existence and non-existence of states of affairs.

The simplest kind of proposition, an elementary proposition, asserts the existence
4.211 Ein Zeichen des Elementarsatzes ist es, dass kein Elementarsatz mit ihm in Widerspruch stehen kann.

4.22 Der Elementarsatz besteht aus Namen. Er ist ein Zusammenhang, eine Verkettung, von Namen.

4.221 Es ist offenbar, dass wir bei der Analyse der Sätze auf Elementarsätze kommen müssen, die aus Namen in unmittelbarer Verbindung bestehen.

...Es frägt sich hier, wie kommt der Satzverband zustande.

4.2211 Auch wenn die Welt unendlich komplex ist, so dass jede Tatsache aus unendlich vielen Sachverhalten besteht und jeder Sachverhalt aus unendlich vielen Gegenständen zusammengesetzt ist, auch dann müsste es Gegenstände und Sachverhalte geben.

...Der Name kommt im Satz nur im Zusammenhange des Elementarsatzes vor.

4.23 Den Elementarsatz schreibe ich als Funktion der Namen in der Form: "fx", "φ(x,y)", etc.

4.24 Gebrauche ich zwei Zeichen in ein und derselben Bedeutung, so drücke ich dies aus, indem ich zwischen beide das Zeichen "=" setze.

..."a = b" heißt also: das Zeichen "a" ist durch das Zeichen "b" ersetzbar.

an atomic fact.

It is a sign of an elementary proposition, that no elementary proposition can contradict it.

The elementary proposition consists of names. It is a connexion, a concatenation, of names.

It is obvious that in the analysis of propositions we must come to elementary propositions, which consist of names in immediate combination.

...The question arises here, how the propositional connexion comes to be.

...Even if the world is infinitely complex, so that every fact consists of an infinite number of atomic facts and every atomic fact is composed of an infinite number of objects, even then there must be objects and atomic facts.

The name occurs in the proposition only in the context of the elementary proposition.

...The names are the simple symbols, I indicate them by single letters (x, y, z).

The elementary proposition I write as function of the names, in the form “fx”, “φ(x,y)”, etc.

Or I indicate it by the letters p, q, r.

If I use two signs with one and the same meaning, I express this by putting between them the sign “=”.

"a = b" means then, that the sign “a” is replaceable by the sign “b”.

of a state of affairs.

It is a sign of a proposition’s being elementary that there can be no elementary proposition contradicting it.

An elementary proposition consists of names. It is a nexus, a concatenation, of names.

It is obvious that the analysis of propositions must bring us to elementary propositions which consist of names in immediate combination.

...This raises the question how such combination into propositions comes about.

...Even if the world is infinitely complex, so that every fact consists of infinitely many states of affairs and every state of affairs is composed of infinitely many objects, there would still have to be objects and states of affairs.

...It is only in the nexus of an elementary proposition that a name occurs in a proposition.

...Names are the simple symbols: I indicate them by single letters (‘x’, ‘y’, ‘z’).

...I write elementary propositions as functions of names, so that they have the form ‘fx’, ‘φ(x,y)’, etc.

Or I indicate them by the letters ‘p’, ‘q’, ‘r’.

...When I use two signs with one and the same meaning, I express this by putting the sign ‘=’ between them.

...So ‘a = b’ means that the sign ‘b’ can be substituted for the sign ‘a’.
Führe ich durch eine Gleichung ein neues Zeichen „b“ ein, indem ich bestimme, es solle ein bereits bekanntes Zeichen „a“ ersetzen, so schreibe ich die Gleichung – Definition – (wie Russell) in der Form „a = b Def.“. Die Definition ist eine Zeichenregel.

Ausdrücke von der Form „a = b“ sind also nur Behelfe der Darstellung; sie sagen nichts über die Bedeutung der Zeichen „a“, „b“ aus.

Können wir zwei Namen verstehen, ohne zu wissen, ob sie dasselbe Ding oder zwei verschiedene Dinge bezeichnen? – Können wir einen Satz, worin zwei Namen vorkommen, verstehen, ohne zu wissen, ob sie Dasselbe oder Verschiedenes bedeuten?

Kenne ich etwa die Bedeutung eines englischen und eines gleichbedeutenden deutschen Wortes, so ist es unmöglich, dass ich nicht weiß, dass die beiden gleichbedeutend sind; es ist unmöglich, dass ich sie nicht ineinander übersetzen kann.

Ausdrücke wie „a = a“, oder von diesen abgeleitete, sind weder Elementarsätze, noch sonst sinnvolle Zeichen. (Dies wird sich später zeigen.)

4.242

If I introduce by an equation a new sign “b”, by determining that it shall replace a previously known sign “a”, I write the equation—definition—(like Russell) in the form “a = b Def.”. A definition is a symbolic rule.

Expressions of the form “a = b” are therefore only expediants in presentation: They assert nothing about the meaning of the signs “a” and “b”.

Can we understand two names without knowing whether they signify the same thing or two different things? Can we understand a proposition in which two names occur, without knowing if they mean the same or different things?

If I know the meaning of an English and a synonymous German word, it is impossible for me not to know that they are synonymous, it is impossible for me not to be able to translate them into one another.

Expressions like “a = a”, or expressions deduced from these are neither elementary propositions nor otherwise significant signs. (This will be shown later.)

If the elementary proposition is true, the atomic fact exists; if it is false the atomic fact does not exist.

The specification of all true elementary propositions describes the world completely. The world is completely described by the specification of all elementary propositions plus the specification, which of

(If I use an equation to introduce a new sign ‘b’, laying down that it shall serve as a substitute for a sign ‘a’ that is already known, then, like Russell, I write the equation—definition—in the form ‘a = b Def.’ A definition is a rule dealing with signs.)

Expressions of the form ‘a = b’ are, therefore, mere representational devices. They state nothing about the meaning of the signs ‘a’ and ‘b’.

4.243

Can we understand two names without knowing whether they signify the same thing or two different things?—Can we understand a proposition in which two names occur without knowing whether their meaning is the same or different?

Suppose I know the meaning of an English word and of a German word that means the same: then it is impossible for me to be unaware that they do mean the same; I must be capable of translating each into the other.

Expressions like ‘a = a’, and those derived from them, are neither elementary propositions nor is there any other way in which they have sense. (This will become evident later.)

4.25

If an elementary proposition is true, the state of affairs exists: if an elementary proposition is false, the state of affairs does not exist.

4.26

Die Angabe aller wahren Elementarsätze beschreibt die Welt vollständig. Die Welt ist vollständig beschrieben durch die Angaben aller Elementarsätze plus der Angabe, welche von ihnen wahr und welche falsch.

The specification of all true elementary propositions is a complete description of the world. The world is completely described by giving all elementary propositions, and adding which of them are true...
### 4.27

Bezüglich des Bestehens und Nichtbestehens von $n$ Sachverhalten gibt es $K_n = \sum_{v=0}^{n} \binom{n}{v}$ Möglichkeiten.

Es können alle Kombinationen der Sachverhalte bestehen, die andern nicht bestehen.

### 4.28

Diesen Kombinationen entsprechen ebenso viele Möglichkeiten der Wahrheit – und Falschheit – von $n$ Elementarsätzen.

### 4.3

Die Wahrheitsmöglichkeiten der Elementarsätze bedeuten die Möglichkeiten des Bestehens und Nichtbestehens der Sachverhalte.

### 4.31


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### 4.4

Der Satz ist der Ausdruck der Übereinstimmung und Nichtübereinstimmung mit den Wahrheitsmöglichkeiten der Elementarsätze, die sind.

#### 4.27

With regard to the existence of $n$ atomic facts there are $K_n = \sum_{v=0}^{n} \binom{n}{v}$ possibilities.

It is possible for all combinations of atomic facts to exist, and the others not to exist.

To these combinations correspond the same number of possibilities of the truth—and falsehood—of $n$ elementary propositions.

The truth-possibilities of the elementary propositions mean the possibilities of the existence and non-existence of the atomic facts.

The truth-possibilities can be presented by schemata of the following kind ("T" means “true”, "F" “false”. The rows of T's and F's under the row of the elementary propositions mean their truth-possibilities in an easily intelligible symbolism):
4.41 Die Wahrheitsmöglichkeiten der Elementarsätze sind die Bedingungen der Wahrheit und Falschheit der Sätze.


4.42 Bezüglich der Übereinstimmung und Nichtübereinstimmung eines Satzes mit den Wahrheitsmöglichkeiten von \( n \) Elementarsätzen gibt es \( \sum_{\kappa=0}^{K_n} L_n \) Möglichkeiten.


4.431 Der Ausdruck der Übereinstimmung und Nichtübereinstimmung mit den Wahrheitsmöglichkeiten der Elementarsätze drückt die Wahrheitsbedingungen des Satzes aus.

Die Übereinstimmung mit der Wahrheitsbedingung des Satzes drückt die Wahrheitsbedingungen des Satzes aus.

Der Satz ist der Ausdruck seiner Wahrheitsbedingungen.

(Frege hat sie daher ganz richtig als Erklärung der Zeichen seiner Begriffsschrift vorausgeschickt. Nur ist die Erklärung des Wahrheitsbegriffes bei Frege falsch: Wären „das Wahre“ und „das Falsche“ wirklich Ge-

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Das Zeichen, welches durch die Zuordnung jener Abzeichen „W“ und der Wahrheitsmög-
klichkeiten entsteht, ist ein Satzzeichen.


Analoges gilt natürlich für alle Zeichen, die dasselbe ausdrücken wie die Schemata der „W“ und „F“.

Es ist z. B.:

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ein Satzzeichen.

(Frege’s „Urteilsstrich“ „⊢“ ist logisch ganz bedeutungslos; er zeigt bei Frege (und Russell) nur an, dass diese Autoren die so bezeichneten Sätze für wahr halten. „⊢“ gehört daher ebenso wenig zum Satzgefüge, wie etwa die Nummer des Satzes. Ein Satz kann unmöglich von sich selbst aussagen, dass er wahr ist.)

Ist die Reihenfolge der Wahrheitsmög-
klichkeit im Schema durch eine Kombina-

tion der arguments in \(~p\) etc., then the sense of \(~p\) would by no means be determined by Frege’s determination.

The sign which arises from the coordination of that mark “T” with the truth-possibilities is a propositional sign.

It is clear that to the complex of the signs “F” and “T” no object (or complex of objects) corresponds; any more than to horizontal and vertical lines or to brackets. There are no “logical objects”.

Something analogous holds of course for all signs, which express the same as the schemata of “T” and “F”.

Thus e. g.

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is a propositional sign.

(Frege’s assertion sign “⊢” is logically altogether meaningless; in Frege (and Russell) it only shows that these authors hold as true the propositions marked in this way. “⊢” belongs therefore to the propositions no more than does the number of the proposition. A proposition cannot possibly assert of itself that it is true.)

If the sequence of the truth-possibilities in the schema is once for all determined by

‘the false’ were really objects, and were the arguments in \(~p\) etc., then Frege’s method of determining the sense of \(~p\) would leave it absolutely undetermined.

The sign that results from correlating the mark ‘T’ with truth-possibilities is a propositional sign.

It is clear that a complex of the signs ‘F’ and ‘T’ has no object (or complex of objects) corresponding to it, just as there is none corresponding to the horizontal and vertical lines or to the brackets.—There are no ‘logical objects’.

Of course the same applies to all signs that express what the schemata of ‘T’s’ and ‘F’s’ express.

For example, the following is a propositional sign:

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</table>

(Frege’s ‘judgement stroke’ ‘⊢’ is logically quite meaningless: in the works of Frege (and Russell) it simply indicates that these authors hold the propositions marked with this sign to be true. Thus ‘⊢’ is no more a component part of a proposition than is, for instance, the proposition’s number. It is quite impossible for a proposition to state that it itself is true.)

If the order or the truth-possibilities in a schema is fixed once and for all by a
tionsregel ein für allemal festgesetzt, dann ist die letzte Kolonne allein schon ein Ausdruck der Wahrheitsbedingungen. Schreiben wir diese Kolonne als Reihe hin, so wird das Satzzeichen zu

or deutlicher

oder deutlicher

(Die Anzahl der Stellen in der linken Klammer ist durch die Anzahl der Glieder in der rechten bestimmt.)

Für \( n \) Elementarsätze gibt es \( L_n \) mögliche Gruppen von Wahrheitsbedingungen.

Die Gruppen von Wahrheitsbedingungen, welche zu den Wahrheitsmöglichkeiten einer Anzahl von Elementarsätzen gehören, lassen sich in eine Reihe ordnen.

Unter den möglichen Gruppen von Wahrheitsbedingungen gibt es zwei extreme Fälle.

In dem einen Fall ist der Satz für sämtliche Wahrheitsmöglichkeiten der Elementarsätze wahr. Wir sagen, die Wahrheitsbedingungen sind tautologisch.

Im zweiten Fall ist der Satz für sämtliche Wahrheitsmöglichkeiten falsch: Die Wahrheitsbedingungen sind kontradiktorisch.

Im ersten Fall nennen wir den Satz eine Tautologie, im zweiten Fall eine Kontradiktion.

Der Satz zeigt was er sagt, die Tautologie und die Kontradiktion, dass sie nichts sagen.

Die Tautologie hat keine Wahrheitsbedingungen, denn sie ist bedingungslos oder

a rule of combination, then the last column by itself will be an expression of the truth-conditions. If we write this column as a row the propositional sign becomes:

or more plainly:

or more explicitly

(The number of places in the left-hand bracket is determined by the number of terms in the right-hand bracket.)

For \( n \) elementary propositions there are \( L_n \) possible groups of truth-conditions.

The groups of truth-conditions which belong to the truth-possibilities of a number of elementary propositions can be ordered in a series.

Among the possible groups of truth-conditions there are two extreme cases.

In the one case the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are tautological.

In the second case the proposition is false for all the truth-possibilities. The truth-conditions are self-contradictory.

In the first case we call the proposition a tautology, in the second case a contradiction.

The proposition shows what it says, the tautology and the contradiction that they say nothing.

The tautology has no truth-conditions, for it is unconditionally true; and the combinatory rule, then the last column by itself will be an expression of the truth-conditions. If we now write this column as a row, the propositional sign will become

or

The number of places in the left-hand pair of brackets is determined by the number of terms in the right-hand pair.

For \( n \) elementary propositions there are \( L_n \) possible groups of truth-conditions.

The groups of truth-conditions that are obtainable from the truth-possibilities of a given number of elementary propositions can be arranged in a series.

Among the possible groups of truth-conditions there are two extreme cases.

In one of these cases the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are tautological.

In the second case the proposition is false for all the truth-possibilities: the truth-conditions are contradictory.

In the first case we call the proposition a tautology; in the second, a contradiction.

Propositions show what they say: tautologies and contradictions show that they say nothing.

A tautology has no truth-conditions, since it is unconditionally true: and a con-
wahr; und die Kontradiktion ist unter kei-
ner Bedingung wahr.

Tautologie und Kontradiktion sind sinnlos.

(Wie der Punkt, von dem zwei Pfeile in entgegengesetzter Richtung auseinander-
gehen.)

(Ich weiß z. B. nichts über das Wetter, wenn ich weiß, dass es regnet oder nicht regnet.)

Tautologie und Kontradiktion sind aber nicht unsinnig; sie gehören zum Symbolis-
mus, und zwar ähnlich wie die „0“ zum Symbolismus der Arithmetik.

Tautologie und Kontradiktion sind nicht Bilder der Wirklichkeit. Sie stellen keine mögliche Sachlage dar. Denn jene lässt jede mögliche Sachlage zu, diese keine.

In der Tautologie heben die Bedingun-
gen der Übereinstimmung mit der Welt – die darstellenden Beziehungen – einander auf, so dass sie in keiner darstellenden Be-
ziehung zur Wirklichkeit steht.

Die Wahrheitsbedingungen bestimmen den Spielraum, der den Tatsachen durch den Satz gelassen wird.

(Der Satz, das Bild, das Modell, sind im negativen Sinne wie ein fester Körper, der die Bewegungsfreiheit der anderen be-
schränkt; im positiven Sinne, wie der von fester Substanz begrenzte Raum, worin ein Körper Platz hat.)

Die Tautologie lässt der Wirklichkeit den ganzen – unendlichen – logischen Raum; die Kontradiktion erfüllt den gan-
zen logischen Raum und lässt der Wirklich-
tradiction is on no condition true.

Tautology and contradiction are without sense.

(Like the point from which two arrows go out in opposite directions.)

(Tautology and contradiction are, however, not nonsensical; they are part of the symbolism, in the same way that “0” is part of the symbolism of Arithmetic.

Tautology and contradiction are not pictures of the reality. They present no possible state of affairs. For the one allows every possible state of affairs, the other none.

In the tautology the conditions of agreement with the world—the presenting relations—cancel one another, so that it stands in no presenting relation to reality.

The truth-conditions determine the range, which is left to the facts by the proposition.

(The proposition, the picture, the model, are in a negative sense like a solid body, which restricts the free movement of another; in a positive sense, like the space limited by solid substance, in which a body may be placed.)

Tautology leaves to reality the whole infinite logical space; contradiction fills the whole logical space and leaves no point to reality. Neither of them, therefore, can in

Tautologies and contradictions lack sense.

(For example, I know nothing about the weather when I know that it is either raining or not raining.)

Tautologies and contradictions are not, however, nonsensical. They are part of the symbolism, much as ‘0’ is part of the symbol-ism of arithmetic.

Tautologies and contradictions are not pictures of reality. They do not represent any possible situations. For the former admit all possible situations, and latter none.

In a tautology the conditions of agreement with the world—the representational relations—cancel one another, so that it does not stand in any representational relation to reality.

The truth-conditions of a proposition determine the range that it leaves open to the facts.

(A proposition, a picture, or a model is, in the negative sense, like a solid body that restricts the freedom of movement of others, and, in the positive sense, like a space bounded by solid substance in which there is room for a body.)

A tautology leaves open to reality the whole—the infinite whole—of logical space: a contradiction fills the whole of logical space leaving no point of it for reality. Thus
2.464 Die Wahrheit der Tautologie ist gewiss, des Satzes möglich, der Kontradiktion unmöglich.
(Gewiss, möglich, unmöglich: Hier haben wir das Anzeichen jener Gradation, die wir in der Wahrscheinlichkeitslehre brauchen.)

2.465 Das logische Produkt einer Tautologie und eines Satzes sagt dasselbe, wie der Satz. Denn man kann das Wesentliche des Symbols nicht ändern, ohne seinen Sinn zu ändern.

2.466 Einer bestimmten logischen Verbindung von Zeichen entspricht eine bestimmte logische Verbindung ihrer Bedeutungen; jede beliebige Verbindung entspricht nur den unverbundenen Zeichen.

Das heißt, Sätze, die für jede Sachlage wahr sind, können überhaupt keine Zeichenverbindungen sein, denn sonst könnten ihnen nur bestimmte Verbindungen von Gegenständen entsprechen.
(Und keiner logischen Verbindung entspricht keine Verbindung der Gegenstände.)

Tautologie und Kontradiktion sind die Grenzfälle der Zeichenverbindung, nämlich ihre Auflösung.

2.4661 Freilich sind auch in der Tautologie und Kontradiktion die Zeichen noch mit einander verbunden, d. h. sie stehen in Beziehungen zu einander, aber diese Beziehungen sind bedeutungslos, dem Symbol — any way determine reality.

The truth of tautology is certain, of propositions possible, of contradiction impossible.

(Certain, possible, impossible: here we have an indication of that gradation which we need in the theory of probability.)

The logical product of a tautology and a proposition says the same thing as the proposition. Therefore that product is identical with the proposition. For the essence of the symbol cannot be altered without altering its sense.

To a definite logical combination of signs corresponds a definite logical combination of their meanings; every arbitrary combination only corresponds to the unconnected signs.

That is, propositions which are true for every state of affairs cannot be combinations of signs at all, for otherwise there could only correspond to them definite combinations of objects.

(And to no logical combination corresponds no combination of the objects.)

Tautology and contradiction are the limiting cases of the combination of symbols, namely their dissolution.

Of course the signs are also combined with one another in the tautology and contradiction, i.e. they stand in relations to one another, but these relations are meaningless, unessential to the symbol.

A tautology's truth is certain, a proposition's possible, a contradiction's impossible.

(Certain, possible, impossible: here we have the first indication of the scale that we need in the theory of probability.)

The logical product of a tautology and a proposition says the same thing as the proposition. This product, therefore, is identical with the proposition. For it is impossible to alter what is essential to a symbol without altering its sense.

What corresponds to a determinate logical combination of signs is a determinate logical combination of their meanings. It is only to the uncombined signs that absolutely any combination corresponds.

In other words, propositions that are true for every situation cannot be combinations of signs at all, since, if they were, only determinate combinations of objects could correspond to them.

(And what is not a logical combination has no combination of objects corresponding to it.)

Tautology and contradiction are the limiting cases—indeed the disintegration—of the combination of signs.

Admittedly the signs are still combined with one another even in tautologies and contradictions—i.e. they stand in certain relations to one another: but these relations have no meaning, they are not essen-
| 4.5 | Nun scheint es möglich zu sein, die allgemeinste Satzform anzugeben: das heißt, eine Beschreibung der Sätze irgendeiner Zeichensprache zu geben, so dass jeder mögliche Sinn durch ein Symbol, auf welches die Beschreibung passt, ausgedrückt werden kann, und jedes Symbol, worauf die Beschreibung passt, einen Sinn ausdrücken kann, wenn die Bedeutungen der Namen entsprechend gewählt werden.

Es ist klar, dass bei der Beschreibung der allgemeinste Satzform nur ihr Wesentliches beschrieben werden darf, – sonst wäre sie nämlich nicht die allgemeinste.

Dass es eine allgemeine Satzform gibt, wird dadurch bewiesen, dass es keinen Satz geben darf, dessen Form man nicht hätte voraussehen (d. h. konstruieren) können. Die allgemeine Form des Satzes ist: Es verhält sich so und so.


Die Sätze sind Alles, was aus der Gesamtheit aller Elementarsätze folgt (natürlich auch daraus, dass es die Gesamtheit aller ist). (So könnte man in gewissem Sinne sagen, dass alle Sätze Verallgemeinerungen der Elementarsätze sind.)

Die allgemeine Satzform ist eine Variable.

| 4.51 | Now it appears to be possible to give the most general form of proposition; i.e. to give a description of the propositions of some one sign language, so that every possible sense can be expressed by a symbol, which falls under the description, and so that every symbol which falls under the description can express a sense, if the meanings of the names are chosen accordingly.

It is clear that in the description of the most general form of proposition only what is essential to it may be described—otherwise it would not be the most general form.

That there is a general form is proved by the fact that there cannot be a proposition whose form could not have been foreseen (i.e. constructed). The general form of proposition is: Such and such is the case.

Suppose all elementary propositions were given me: then we can simply ask: what propositions I can build out of them. And these are all propositions and so are they limited.

The propositions are everything which follows from the totality of all elementary propositions (of course also from the fact that it is the totality of them all). (So, in some sense, one could say, that all propositions are generalizations of the elementary propositions.)

The general proposition form is a variable.

| 4.52 | It now seems possible to give the most general propositional form: that is, to give a description of the propositions of any sign-language whatsoever in such a way that every possible sense can be expressed by a symbol satisfying the description, and every symbol satisfying the description can express a sense, provided that the meanings of the names are suitably chosen.

The existence of a general propositional form is proved by the fact that there cannot be a proposition whose form could not have been foreseen (i.e. constructed). The general form of a proposition is: This is how things stand.

Suppose that I am given all elementary propositions: then I can simply ask what propositions I can construct out of them. And there I have all propositions, and that fixes their limits.

Propositions comprise all that follows from the totality of all elementary propositions (and, of course, from its being the totality of them all). (Thus, in a certain sense, it could be said that all propositions were generalizations of elementary propositions.)

The general propositional form is a variable.
Der Satz ist eine Wahrheitsfunktion der Elementarsätze.

(Der Elementarsatz ist eine Wahrheitsfunktion seiner selbst.)

Die Elementarsätze sind die Wahrheitsargumente des Satzes.

Es liegt nahe, die Argumente von Funktionen mit den Indices von Namen zu verwechseln. Ich erkenne nämlich sowohl am Argument wie am Index die Bedeutung des sie enthaltenden Zeichens.

In Russells „$+c$“ ist z. B. „$c$“ ein Index, der darauf hinweist, dass das ganze Zeichen das Additionszeichen für Kardinalzahlen ist. Aber diese Bezeichnung beruht auf willkürlicher Übereinkunft und man könnte statt „$+c$“ auch ein einfaches Zeichen wählen; in „$\neg p$“ aber ist „$p$“ kein Index, sondern ein Argument: der Sinn von „$\neg p$“ k a n n n i c h t verstanden werden, ohne dass vorher der Sinn von „$p$“ verstanden worden wäre. (Im Namen Julius Cäsar ist „Julius“ ein Index. Der Index ist immer ein Teil einer Beschreibung des Gegenstandes, dessen Namen wir ihn anhängen. Z. B. Der Cäsar aus dem Geschlechte der Julier.)


Die Wahrheitsfunktionen lassen sich in Reihen ordnen.

Das ist die Grundlage der Wahrscheinlichkeit.

Propositions are truth-functions of elementary propositions.

(An elementary proposition is a truth-function of itself.)

The elementary propositions are the truth-arguments of propositions.

It is natural to confuse the arguments of functions with the indices of names. For I recognize the meaning of the sign containing it from the argument just as much as from the index.

In Russell's "$+c$", for example, "$c$" is an index which indicates that the whole sign is the addition sign for cardinal numbers. But this way of symbolizing depends on arbitrary agreement, and one could choose a simple sign instead of "$+c$": but in "$\neg p$" "$p$" is not an index but an argument; the sense of "$\neg p$" cannot be understood, unless the sense of "$p$" has previously been understood. (In the name Julius Caesar, Julius is an index. The index is always part of a description of the object to whose name we attach it, e.g. The Caesar of the Julian gens.)

The confusion of argument and index is, if I am not mistaken, at the root of Frege's theory of the meaning of propositions and functions. For Frege the propositions of logic were names and their arguments the indices of these names.

The truth-functions can be ordered in series.

That is the foundation of the theory of

A proposition is a truth-function of elementary propositions.

(An elementary proposition is a truth-function of itself.)

Elementary propositions are the truth-arguments of propositions.

The arguments of functions are readily confused with the affixes of names. For both arguments and affixes enable me to recognize the meaning of the signs containing them.

For example, when Russell writes ‘$+c$’, the ‘$c$’ is an affix which indicates that the sign as a whole is the addition-sign for cardinal numbers. But the use of this sign is the result of arbitrary convention and it would be quite possible to choose a simple sign instead of ‘$+c$’; in ‘$\neg p$’, however, ‘$p$’ is not an affix but an argument: the sense of ‘$\neg p$’ cannot be understood unless the sense of ‘$p$’ has been understood already. (In the name Julius Caesar ‘Julius’ is an affix. An affix is always part of a description of the object to whose name we attach it: e.g. the Caesar of the Julian gens.)

If I am not mistaken, Frege’s theory about the meaning of propositions and functions is based on the confusion between an argument and an affix. Frege regarded the propositions of logic as names, and their arguments as the affixes of those names.

Truth-functions can be arranged in series.

That is the foundation of the theory of
lichkeitslehre.

Die Wahrheitsfunktionen jeder Anzahl von Elementarsätzen lassen sich in einem Schema folgender Art hinschreiben:

(WWWW)(p,q) Tautologie (Wenn p, so p; und wenn q, so q.)
(F F W W)(p,q) in Worten: Nicht beides p und q. (¬(p . q))
(W W F W)(p,q) " " Wenn q, so p. (q → p)
(W W W F)(p,q) " " Wenn p, so q. (p → q)
(F F W W)(p,q) " " Nicht q. (¬ q)
(F F F W)(p,q) " " Nicht p. (¬ p)
(F W W F)(p,q) " " p, oder q, aber nicht beide.
(W F W F)(p,q) " " Wenn p, so q; und wenn q, so p.
(F W W F)(p,q) " " p oder q, aber nicht beide.
(W F W F)(p,q) " " Wenn p, so q; und wenn q, so p.
(W W F F)(p,q) " " Weder p noch q.
(F F W F)(p,q) " " p und nicht q. (p . ¬ q)
(F W F F)(p,q) " " q und nicht p. (q . ¬ p)
(W F F F)(p,q) Kontradiktion (p und nicht q; und q und nicht p.) (¬ p . ¬ q)

Diejenigen Wahrheitsmöglichkeiten seiner Wahrheitsargumente, welche den Satz bewahren, will ich seine Wahrheitsgründe nennen.

Die Wahrheitsgründe, die einer Anzahl von Sätzen gemeinsam sind, sämtlich auch Wahrheitsgründe eines bestimmten Satzes, so sagen wir, die Wahrheit dieses Satzes folge aus der Wahrheit jener Sätze.

Insbesondere folgt die Wahrheit eines Satzes „p“ aus der Wahrheit eines anderen „q“, wenn alle Wahrheitsgründe des zweiten Wahrheitsgrades des ersten sind.

Die Wahrheitsgründe des einen sind in denen des anderen enthalten; p folgt aus q.

Folgt p aus q, so ist der Sinn von „p“ im Sinne von „q“ enthalten.

probability.

The truth-functions of every number of elementary propositions can be written in a schema of the following kind:

(TTTT)(p,q) Tautology (if p then p; and if q then q)
(TTTF)(p,q) " " If q then p. (q → p)
(TTTF)(p,q) " " If p then q. (p → q)
(TFFF)(p,q) " " Neither p nor q.
(F TFTF)(p,q) " " p and not q. (p . ¬ q)
(F FTFF)(p,q) " " q and not p. (q . ¬ p)
(F F F F)(p,q) Contradiction (p and not q; and q and not p.) (¬ p . ¬ q)

Those truth-possibilities of its truth-arguments, which verify the proposition, I shall call its truth-grounds.

If the truth-grounds which are common to a number of propositions are also truth-grounds of some one proposition, we say that the truth of this proposition follows from the truth of those propositions.

In particular the truth of a proposition p follows from that of a proposition q, if all the truth-grounds of the second are truth-grounds of the first.

The truth-grounds of q are contained in those of p; p follows from q.

If p follows from q, the sense of “p” is contained in that of “q”.

probability.

The truth-functions of a given number of elementary propositions can always be set out in a schema of the following kind:

(TTTT)(p,q) Tautology (If p then p; and if q then q)
(TTTF)(p,q) " " If q then p. (q → p)
(TTTF)(p,q) " " If p then q. (p → q)
(TFFF)(p,q) " " Neither p nor q.
(F TFTF)(p,q) " " p and not q. (p . ¬ q)
(F FTFF)(p,q) " " q and not p. (q . ¬ p)
(F F F F)(p,q) Contradiction (p and not q; and q and not p.) (¬ p . ¬ q)

I will give the name truth-grounds of a proposition to those truth-possibilities of its truth-arguments that make it true.

If all the truth-grounds that are common to a number of propositions are at the same time truth-grounds of a certain proposition, then we say that the truth of that proposition follows from the truth of the others.

In particular, the truth of a proposition ‘p’ follows from the truth of another proposition ‘q’ if all the truth-grounds of the latter are truth-grounds of the former.

The truth-grounds of the one are contained in those of the other: p follows from q.

If p follows from q, the sense of ‘p’ is contained in the sense of ‘q’.
Wenn ein Gott eine Welt erschafft, warin gewisse Sätze wahr sind, so schafft er damit auch schon eine Welt, in welcher alle ihre Folgesätze stimmen. Und ähnlich könnte er keine Welt schaffen, worin der Satz „p“ wahr ist, ohne seine sämtlichen Gegenstände zu schaffen.

Der Satz bejaht jeden Satz, der aus ihm folgt.

„p . q“ ist einer der Sätze, welche „p“ bejahren, und zugleich einer der Sätze, welche „q“ bejahren.

Zwei Sätze sind einander entgegengesetzt, wenn es keinen sinnvollen Satz gibt, der sie beide bejaht.

Jeder Satz der einem anderen widerspricht, verneint ihn.

Dass die Wahrheit eines Satzes aus der Wahrheit anderer Sätze folgt, ersehen wir aus der Struktur der Sätze.

Folgt die Wahrheit eines Satzes aus der Wahrheit anderer, so drückt sich dies durch Beziehungen aus, in welchen die Formen jener Sätze zu einander stehen; und zwar brauchen wir sie nicht erst in jene Beziehungen zu setzen, indem wir sie in einem Satz miteinander verbinden, sondern diese Beziehungen sind intern und bestehen, sobald, und dadurch dass, jene Sätze bestehen.

Wenn wir von \( p \lor q \) und \( \neg p \) auf q schließen, so ist hier durch die Bezeichnungsweise die Beziehung der Satzformen von „p \lor q“ und „\neg p“ verhüllt. Schreiben wir aber z.B. statt „p \lor q“, „p | q . p | q“ und statt „\neg p“ „p | p“ (\( p | q \) = weder p, noch q), wenn wir von \( p \lor q \) und \( \neg p \) auf q schließen, so ist hier durch die Bezeichnungsweise die Beziehung der Satzformen von „p \lor q“ und „\neg p“ verhüllt. Schreiben wir aber z.B. statt „p \lor q“, „p | q . p | q“ und statt „\neg p“ „p | p“ (\( p | q \) = weder p, noch q),

If a god creates a world in which certain propositions are true, he creates thereby also a world in which all propositions consequent on them are true. And similarly he could not create a world in which the proposition “p” is true without creating all its objects.

A proposition asserts every proposition which follows from it.

“p . q” is one of the propositions which assert “p” and at the same time one of the propositions which assert “q”.

Two propositions are opposed to one another if there is no significant proposition which asserts them both.

Every proposition which contradicts another, denies it.

That the truth of one proposition follows from the truth of other propositions, we perceive from the structure of the propositions.

If the truth of one proposition follows from the truth of others, this expresses itself in relations in which the forms of these propositions stand to one another, and we do not need to put them in these relations first by connecting them with one another in a proposition; for these relations are internal, and exist as soon as, and by the very fact that, the propositions exist.

When we conclude from \( p \lor q \) and \( \neg p \) to q the relation between the forms of the propositions “p \lor q” and “\neg p” is here concealed by the method of symbolizing. But if we write, e.g. instead of “p \lor q” “p | q . p | q” and instead of “\neg p” “p | p” (\( p | q \) = neither p

When we infer q from \( p \lor q \) and \( \neg p \), the relation between the propositional forms of ‘p \lor q’ and ‘\neg p’ is masked, in this case, by our mode of signifying. But if instead of ‘p \lor q’ we write, for example, ‘p | q . p | q’, and instead of ‘\neg p’, ‘p | p’ (\( p | q \) = neither p
so wird der innere Zusammenhang offen-
p nor q), then the inner connexion becomes
bar.

5.132

(Dass man aus \((x).fx\) auf \(fa\) schließen

\(\text{Folgt } p \text{ aus } q, \text{ so kann ich von } q \text{ auf } p\)
schließen; \(p\) aus \(q\) folgen.

\(\text{Die Art des Schlusses ist allein aus den}

\(\text{Nur sie selbst können den Schluss}

\(\text{„Schlussgesetze“, welche – wie bei Fre-

\(\text{Alles Folgern geschieht a priori.}

\(\text{Aus einem Elementarsatz lässt sich}

\(\text{Auf keine Weise kann aus dem Beste-

\(\text{Einen Kausalnexus, der einen solchen}

\(\text{Die Ereignisse der Zukunft kön nen wir}

\(\text{Der Glaube an den Kausalnexus ist der}

\(\text{Die Willensfreiheit besteht darin, dass}

\(\text{Der Zusammenhang von Wissen und Gewusst}

\(\text{p nor q), then the inner connexion becomes}

\(\text{obvious.}

\(\text{The possibility of inference from } (x).fx \text{ to } fa \text{ shows that the symbol } (x).fx \text{ itself}

\(\text{If } p \text{ follows from } q, \text{ I can conclude from }

\(\text{The method of inference is to be under-

\(\text{Only they themselves can justify the}

\(\text{Laws of inference, which—as in Frege}

\(\text{All inference takes place a priori.}

\(\text{One elementary proposition cannot be}

\(\text{All deductions are made } a \text{ priori.}

\(\text{All inference takes place a priori.}

\(\text{One elementary proposition cannot be}

\(\text{There is no possible way of making an}

\(\text{There is no causal nexus to justify such an}

\(\text{Superstition is nothing but belief in the}

\(\text{Superstition is the belief in the causal}

\(\text{The freedom of the will consists in the}

\(\text{The freedom of the will consists in the}

\(\text{The possibility of inference from } (x).fx \text{ to } fa \text{ shows that the symbol } (x).fx \text{ itself}

\(\text{If } p \text{ follows from } q, \text{ I can make an in-

\(\text{The nature of the inference can be gath-

\(\text{They themselves are the only possible}

\(\text{‘Laws of inference’, which are supposed}

\(\text{Frege and Russell, have no sense, and}

\(\text{would be superfluous.}

\(\text{would be superfluous.}

\(\text{would be superfluous.}

\(\text{would be superfluous.}

\(\text{would be superfluous.}

\)
(„A weiß, dass p der Fall ist“ ist sinnlos, wenn p eine Tautologie ist.)

5.1363

Wenn daraus, dass ein Satz uns einleuchtet, nicht folgt, dass er wahr ist, so ist das Einleuchten auch keine Rechtfertigung für unseren Glauben an seine Wahrheit.

Folgt ein Satz aus einem anderen, so sagt dieser mehr als jener, jener weniger als dieser.

Folgt p aus q und q aus p, so sind sie ein und derselbe Satz.

Die Tautologie folgt aus allen Sätzen: sie sagt nichts.

Die Kontradiktion folgt aus einem Schema wie dem obigen in 5.101 W.

Die Kontradiktion ist das Gemeinsame der Sätze, was k e i n Satz mit einem anderen gemein hat. Die Tautologie ist das Gemeinsame aller Sätze, welche nichts miteinander gemein haben.

Die Kontradiktion verschwindet sozusagen außerhalb, die Tautologie innerhalb aller Sätze.

Die Kontradiktion ist die äußere Grenze der Sätze, die Tautologie ihr substanzloser Mittelpunkt.

5.15

Ist Wr die Anzahl der Wahrheitsgründe des Satzes „r“, Wrs die Anzahl derjenigen Wahrheitsgründe des Satzes „p“, die zugleich Wahrheitsgründe von „r“ sind, dann nennen wir das Verhältnis: Wrs : Wr das Maß der W a h r s c h e i n l i c h k e i t, welche der Satz „r“ dem Satz „s“ gibt.

5.151

Sei in einem Schema wie dem obigen in No. 5.101 Wr die Anzahl der „W“ im Satze r; Wrs die Anzahl derjenigen „W“ im Satze s, die in gleichen Kolonnen mit „W“ des Satzes r stehen. Der Satz r gibt dann dem

(„A knows that p is the case“ is senseless if p is a tautology.)

If from the fact that a proposition is obvious to us it does not follow that it is true, then obviousness is no justification for our belief in its truth.

If a proposition follows from another, then the latter says more than the former, the former less than the latter.

If p follows from q and q from p then they are one and the same proposition.

A tautology follows from all propositions: it says nothing.

Contradiction is something shared by propositions which no proposition has in common with another. Tautology is that which is shared by all propositions, which have nothing in common with one another.

Contradiction vanishes so to speak outside, tautology inside all propositions.

Contradiction is the external limit of the propositions, tautology their substanceless centre.

If Tr is the number of the truth-grounds of the proposition „r“, Trs the number of those truth-grounds of the proposition „s“ which are at the same time truth-grounds of „r“, then we call the ratio Trs : Tr the measure of the probability which the proposition „r“ gives to the proposition „s“.

Suppose in a schema like that above in No. 5.101 Trr is the number of the “T”’s in the proposition r, Trs the number of those “T”’s in the proposition s, which stand in the same columns as “T”’s of the propesi

(‘A knows that p is the case’, has no sense if p is a tautology.)

If the truth of a proposition does not follow from the fact that it is self-evident to us, then its self-evidence in no way justifies our belief in its truth.

If one proposition follows from another, then the latter says more than the former, and the former less than the latter.

If p follows from q and q from p, then they are one and the same proposition.

A tautology follows from all propositions: it says nothing.

Contradiction is that common factor of propositions which no proposition has in common with another. Tautology is the common factor of all propositions that have nothing in common with one another.

Contradiction, one might say, vanishes outside all propositions: tautology vanishes inside them.

Contradiction is the outer limit of propositions: tautology is the unsubstantial point at their centre.

If Tr is the number of the truth-grounds of a proposition ‘r’, and if Trs is the number of the truth-grounds of a proposition ‘s’ that are at the same time truth-grounds of ‘r’, then we call the ratio Trs : Tr the degree of probability that the proposition ‘r’ gives to the proposition ‘s’.

In a schema like the one above in 5.101, let Tr be the number of ‘T’s’ in the proposition r, and let Trs, be the number of ‘T’s’ in the proposition s that stand in columns in which the proposition r has ‘T’s’. Then the
Satze $s$ die Wahrscheinlichkeit: $W_{rs} : W_r$.

5.1511

Es gibt keinen besonderen Gegenstand, der den Wahrscheinlichkeitssätzen eigen wäre.

5.152

Sätze, welche keine Wahrheitsargumente mit einander gemein haben, nennen wir von einander unabhängig.

Zwei Elementarsätze geben einander die Wahrscheinlichkeit $\frac{1}{2}$.

 Folgt $p$ aus $q$, so gibt der Satz „$q$“ dem Satz „$p$“ die Wahrscheinlichkeit 1. Die Gewissheit des logischen Schlusses ist ein Grenzfall der Wahrscheinlichkeit.

(Anwendung auf Tautologie und Kontradiktion.)

5.153


In einer Urne seien gleichviel weiße und schwarze Kugeln (und keine anderen). Ich ziehe eine Kugel nach der anderen und lege sie wieder in die Urne zurück. Dann kann ich durch den Versuch feststellen, dass sich die Zahlen der gezogenen schwarzen und weißen Kugeln bei fortgesetztem Ziehen einander nähern.

Das ist also kein mathematisches Faktum.

Wenn ich nun sage: Es ist gleich wahrscheinlich, dass ich eine weiße Kugel wie eine schwarze ziehen werde, so heißt das: Alle mir bekannten Umstände (die hypothetisch angenommenen Naturgesetze mitbegriffen) geben dem Eintreffen eines der proposition $r$; then the proposition $r$ gives to the proposition $s$ the probability $T_{rs} : T_r$.

There is no special object peculiar to probability propositions.

5.153

Propositions which have no truth-arguments in common with one another we call independent.

Independent propositions (e.g. any two elementary propositions) give to one another the probability $\frac{1}{2}$.

If $p$ follows from $q$, the proposition $q$ gives to the proposition $p$ the probability 1. The certainty of logical conclusion is a limiting case of probability.

(Application to tautology and contradiction.)

A proposition is in itself neither probable nor improbable. An event occurs or does not occur, there is no middle course.

In an urn there are equal numbers of white and black balls (and no others). I draw one ball after another and put them back in the urn. Then I can determine by the experiment that the numbers of the black and white balls which are drawn approximate as the drawing continues.

So this is not a mathematical fact.

If then, I say, It is equally probable that I should draw a white and a black ball, this means, All the circumstances known to me (including the natural laws hypothetically assumed) give to the occurrence of the one event no more probability than to the occurrence of the other event.

There is no special object peculiar to probability propositions.

When propositions have no truth-arguments in common with one another, we call them independent of one another.

Two elementary propositions give one another the probability $\frac{1}{2}$.

If $p$ follows from $q$, then the proposition ‘$q$’ gives to the proposition ‘$p$’ the probability 1. The certainty of logical inference is a limiting case of probability.

(Application of this to tautology and contradiction.)

In itself, a proposition is neither probable nor improbable. Either an event occurs or it does not: there is no middle way.

Suppose that an urn contains black and white balls in equal numbers (and none of any other kind). I draw one ball after another, putting them back into the urn. By this experiment I can establish that the number of black balls drawn and the number of white balls drawn approximate to one another as the draw continues.

So this is not a mathematical truth.

Now, if I say, ‘The probability of my drawing a white ball is equal to the probability of my drawing a black one’, this means that all the circumstances that I know of (including the laws of nature assumed as hypotheses) give no more prob-
Ereignisse nicht mehr Wahrscheinlichkeit als dem Eintreffen des anderen. Das heißt, sie geben — wie aus den obigen Erklärungen leicht zu entnehmen ist — jedem die Wahrscheinlichkeit \( \frac{1}{2} \).

Was ich durch den Versuch bestätige ist, dass das Eintreffen der beiden Ereignisse von den Umständen, die ich nicht näher kenne, unabhängig ist.


So ist die Wahrscheinlichkeit eine Ver allgemeinerung.

Sie involviert eine allgemeine Beschreibung einer Satzform.

Nur in Ermangelung der Gewissheit gebrauchen wir die Wahrscheinlichkeit. — Wenn wir zwar eine Tatsache nicht vollkommen kennen, wohl aber etwas über ihre Form wissen.

(Ein Satz kann zwar ein unvollständiges Bild einer gewissen Sachlage sein, aber er ist immer ein vollständiges Bild.)

Der Wahrscheinlichkeits satz ist gleich sam ein Auszug aus anderen Sätzen.

Die Strukturen der Sätze stehen in internen Beziehungen zu einander.

Wir können diese internen Beziehungen dadurch in unserer Ausdrucksweise hervorheben, dass wir einen Satz als Resultat einer Operation darstellen, die ihn aus anderen Sätzen (den Basen der Operation) hervorbringt.

Die Operation ist der Ausdruck einer occurrence of the other. That is they give—as can easily be understood from the above explanations—to each the probability \( \frac{1}{2} \).

What I can verify by the experiment is that the occurrence of the two events is independent of the circumstances with which I have no closer acquaintance.

The unit of the probability proposition is: The circumstances—which I am not further acquainted—give to the occurrence of a definite event such and such a degree of probability.

Probability is a generalization.

It involves a general description of a propositional form.

Only in default of certainty do we need probability. If we are not completely acquainted with a fact, but know something about its form.

(A proposition can, indeed, be an incomplete picture of a certain state of affairs, but it is always a complete picture.)

The probability proposition is, as it were, an extract from other propositions.

The structures of propositions stand to one another in internal relations.

We can bring out these internal relations in our manner of expression, by presenting a proposition as the result of an operation which produces it from other propositions (the bases of the operation).

The operation is the expression of a re-
Beziehung zwischen den Strukturen ihres Resultats und ihrer Basen.

5.23
Die Operation ist das, was mit dem einen Satz geschehen muss, um aus ihm den anderen zu machen.

5.231
Und das wird natürlich von ihren formalen Eigenschaften, von der internen Ähnlichkeit ihrer Formen abhängen.

5.232
Die interne Relation, die eine Reihe ordnet, ist äquivalent mit der Operation, durch welche ein Glied aus dem anderen entsteht.

Die Operation kann erst dort auftreten, wo ein Satz auf logisch bedeutungsvolle Weise aus einem anderen entsteht. Also dort, wo die logische Konstruktion des Satzes anfängt.

5.233
Die Wahrheitsfunktionen der Elementarsätze sind Resultate von Operationen, die die Elementarsätze als Basen haben. (Ich nenne diese Operationen Wahrheitsoperationen.)

5.234
Der Sinn einer Wahrheitsfunktion von \( p \) ist eine Funktion des Sinnes von \( p \).

Verneinung, logische Addition, logische Multiplikation, etc., etc. sind Operationen. (Die Verneinung verkehrt den Sinn des Satzes.)

Die Operation zeigt sich in einer Variablen; sie zeigt, wie man von einer Form von Sätzen zu einer anderen gelangen kann.

Sie bringt den Unterschied der Formen zum Ausdruck.

(Und das Gemeinsame zwischen den Basen und dem Resultat der Operation sind eben die Basen.)

5.24
An operation shows itself in a variable; it shows how we can proceed from one form of proposition to another.

It gives expression to the difference between the forms.

(And that which is common to the bases, and the result of an operation, is the bases themselves.)

Die Operation ist das, was mit dem einem Satz geschehen muss, um aus ihm den anderen zu machen.

And that will naturally depend on their formal properties, on the internal similarity of their forms.

The internal relation which orders a series is equivalent to the operation by which one term arises from another.

The first place in which an operation can occur is where a proposition arises from another in a logically significant way; \( i.e. \) where the logical construction of the proposition begins.

The truth-functions of elementary proposition, are results of operations which have the elementary propositions as bases. (I call these operations, truth-operations.)

The sense of a truth-function of \( p \) is a function of the sense of \( p \).

Denial, logical addition, logical multiplication, etc., etc., are operations. (Denial reverses the sense of a proposition.)

An operation shows itself in a variable; it shows how we can proceed from one form of proposition to another.

It gives expression to the difference between the forms.

(And that which is common to the bases, and the result of an operation, is the bases themselves.)

The operation is what has to be done to the one proposition in order to make the other out of it.

And that will, of course, depend on their formal properties, on the internal similarity of their forms.

The internal relation by which a series is ordered is equivalent to the operation that produces one term from another.

Operations cannot make their appearance before the point at which one proposition is generated out of another in a logically meaningful way; \( i.e. \) the point at which the logical construction of propositions begins.

Truth-functions of elementary propositions are results of operations with elementary propositions as bases. (These operations I call truth-operations.)

The sense of a truth-function of \( p \) is a function of the sense of \( p \).

Negation, logical addition, logical multiplication, etc. etc. are operations. (Negation reverses the sense of a proposition.)

An operation manifests itself in a variable; it shows how we can get from one form of proposition to another.

It gives expression to the difference between the forms.

(And what the bases of an operation and its result have in common is just the bases themselves.)
5.241 Die Operation kennzeichnet keine Form, sondern nur den Unterschied der Formen.

5.242 Dieselbe Operation, die „q“ aus „p“ macht, macht aus „q“, „r“ u. s. f. Dies kann nur darin ausgedrückt sein, dass „p“, „q“, „r“, etc. Variable sind, die gewisse formale Relationen allgemein zum Ausdruck bringen.

5.25 Das Vorkommen der Operation charakterisiert den Sinn des Satzes nicht. Die Operation sagt ja nichts aus, nur ihr Resultat, und dies hängt von den Basen der Operation ab. (Operation und Funktion dürfen nicht miteinander verwechselt werden.)

5.251 Eine Funktion kann nicht ihr eigenes Argument sein, wohl aber kann das Resultat einer Operation ihre eigene Basis werden.

5.252 Nur so ist das Fortschreiten von Glied zu Glied in einer Formenreihe (von Type zu Type in den Hierarchien Russells und Whiteheads) möglich. (Russell und Whitehead haben die Möglichkeit dieses Fortschreitens nicht zugegeben, aber immer wieder von ihr Gebrauch gemacht.)

5.2521 Die fortgesetzte Anwendung einer Operation auf ihr eigenes Resultat nennt ich ihre successive Anwendung („O’O’O’a“ ist das Resultat der dreimaligen successive Anwendung von „O’ξ“ auf „a“).


Das allgemeine Glied einer Formenreihe a, O’a, O’O’a, … schreibe ich daher so:

The operation does not characterize a form but only the difference between forms.

The same operation which makes “q” from “p”, makes “r” from “q”, and so on. This can only be expressed by the fact that “p”, “q”, “r”, etc., are variables which give general expression to certain formal relations.

The occurrence of an operation does not characterize the sense of a proposition. For an operation does not assert anything; only its result does, and this depends on the bases of the operation. (Operation and function must not be confused with one another.)

A function cannot be its own argument, but the result of an operation can be its own basis.

Only in this way is the progress from term to term in a formal series possible (from type to type in the hierarchies of Russell and Whitehead). (Russell and Whitehead have not admitted the possibility of this progress but have made use of it all the same.)

The repeated application of an operation to its own result I call its successive application (“O’O’O’a” is the result of the threefold successive application of “O’ξ” to “a”).

In a similar sense I speak of the successive application of several operations to a number of propositions.

The general term of the formal series a, O’a, O’O’a, … I write thus:

An operation is not the mark of a form, but only of a difference between forms.

The operation that produces ‘q’ from ‘p’ also produces ‘r’ from ‘q’, and so on. There is only one way of expressing this: ‘p’, ‘q’, ‘r’, etc. have to be variables that give expression in a general way to certain formal relations.

The occurrence of an operation does not characterize the sense of a proposition. Indeed, no statement is made by an operation, but only by its result, and this depends on the bases of the operation. (Operations and functions must not be confused with each other.)

A function cannot be its own argument, whereas an operation can take one of its own results as its base.

It is only in this way that the step from one term of a series of forms to another is possible (from one type to another in the hierarchies of Russell and Whitehead). (Russell and Whitehead did not admit the possibility of such steps, but repeatedly availed themselves of it.)

If an operation is applied repeatedly to its own results, I speak of successive applications of it. (‘O’O’O’a’ is the result of three successive applications of the operation ‘O’ξ’ to ‘a’.)

In a similar sense I speak of successive applications of more than one operation to a number of propositions.

Accordingly I use the sign ‘[a, x, O’x]’ for the general term of the series of forms.
...

5.2523
der Begriff der successiven Anwendung der Operation ist äquivalent mit dem Begriff „und so weiter“.

Eine Operation kann die Wirkung einer anderen rückgängig machen. Operationen können einander aufheben.

5.253

Die Operation kann verschwinden (z. B. die Verneinung in „\sim\sim p“: \sim\sim p \equiv p).

5.254

Alle Sätze sind Resultate von Wahrheitsoperationen mit den Elementarsätzen.

Die Wahrheitsoperation ist die Art und Weise, wie aus den Elementarsätzen die Wahrheitsfunktion entsteht.


Jeder Satz ist das Resultat von Wahrheitsoperationen mit Elementarsätzen.

5.3

Alle Sätze sind Resultate von Wahrheitsoperationen mit den Elementarsätzen.

Die Wahrheitsoperation ist die Art und Weise, wie aus den Elementarsätzen die Wahrheitsfunktion entsteht.


Jeder Satz ist das Resultat von Wahrheitsoperationen mit Elementarsätzen.

The concept of successive applications of an operation is equivalent to the concept ‘and so on’.

One operation can counteract the effect of another. Operations can cancel one another.

An operation can vanish (e.g. denial in ‘\sim\sim p’: \sim\sim p \equiv p).

All propositions are results of truth-operations on the elementary propositions.

5.3

A truth-operation is the way in which a truth-function is produced out of elementary propositions.

A truth-operation is the way in which a truth-function is produced out of elementary propositions.

It is of the essence of truth-operations that, just as elementary propositions yield a truth-function of themselves, so too in the same way truth-functions yield a further truth-function. When a truth-operation is applied to truth-functions of elementary propositions, it always generates another truth-function of elementary propositions, another proposition. When a truth-operation is applied to the results of truth-operations on elementary propositions, there is always a single operation on elementary propositions that has the same result.

Every proposition is the result of truth-operations on elementary propositions.
Die Schemata No. 4.31 haben auch dann eine Bedeutung, wenn „p“, „q“, „r“, etc. nicht Elementarsätze sind.

Und es ist leicht zu sehen, dass das Satzzeichen in No. 4.442, auch wenn „p“ und „q“ Wahrheitsfunktionen von Elementarsätzen sind, Eine Wahrheitsfunktion von Elementarsätzen ausdrückt.

Alle Wahrheitsfunktionen sind Resultate der successiven Anwendung einer endlichen Anzahl von Wahrheitsoperationen auf die Elementarsätze.

Hier zeigt es sich, dass es „logische Gegenstände“, „logische Konstante“ (im Sinne Freges und Russells) nicht gibt.

Denn: Alle Resultate von Wahrheitsoperationen mit Wahrheitsfunktionen sind identisch, welche eine und dieselbe Wahrheitsfunktion von Elementarsätzen sind.

Dass ∨, ⊃, etc. nicht Beziehungen im Sinne von rechts und links etc. sind, leuchtet ein.

Die Möglichkeit des kreuzweisen Definierens der logischen „Urzeichen“ Freges und Russells zeigt schon, dass diese keine Urzeichen sind, und schon erst recht, dass sie keine Relationen bezeichnen.


Dass aus einer Tatsache p unaendlich viele andere folgen sollten, nämlich ∼p, ∼∼p, etc., ist doch von vornherein

The Schemata No. 4.31 are also significant, if “p”, “q”, “r”, etc. are not elementary propositions.

And it is easy to see that the propositional sign in No. 4.442 expresses one truth-function of elementary propositions even when “p” and “q” are truth-functions of elementary propositions.

All truth-functions are results of the successive application of a finite number of truth-operations to elementary propositions.

At this point it becomes manifest that there are no ‘logical objects’ or ‘logical constants’ (in Frege’s and Russell’s sense).

The reason is that the results of truth-operations on truth-functions are always identical whenever they are one and the same truth-function of elementary propositions.

At this point it becomes manifest that there are no ‘logical objects’ or ‘logical constants’ (in Frege’s and Russell’s sense).

The possibility of crosswise definition of the logical “primitive signs” of Frege and Russell shows by itself that these are not primitive signs and that they signify no relations.

The interdefinability of Frege’s and Russell’s ‘primitive signs’ of logic is enough to show that they are not primitive signs, still less signs for relations.

And it is obvious that the ‘⊃’ defined by means of ‘∼’ and ‘∨’ is identical with the one that figures with ‘∼’ in the definition of ‘∨’; and that the second ‘∨’ is identical with the first one; and so on.

Even at first sight it seems scarcely credible that there should follow from one fact p infinitely many others, namely ∼p,
kaum zu glauben. Und nicht weniger merkwürdig ist, dass die unendliche Anzahl der Sätze der Logik (der Mathematik) aus einem halben Dutzend „Grundgesetzen“ folgen.

Alle Sätze der Logik sagen aber dasselbe. Nämlich nichts.

Die Wahrheitsfunktionen sind keine materiellen Funktionen.

Wenn man z. B. eine Bejahung durch doppelte Verneinung erzeugen kann, ist dann die Verneinung – in irgend einem Sinn – in der Bejahung enthalten? Verneint „¬¬p“ ~p, oder bejaht es p; oder beides?

Der Satz „¬¬p“ handelt nicht von der Verneinung wie von einem Gegenstand; wohl aber ist die Möglichkeit der Verneinung in der Bejahung bereits präjudiziert.


Dieses Verschwinden der scheinbaren logischen Konstanten tritt auch ein, wenn „¬¬(∃x).¬fx“ dasselbe sagt wie „(x).fx“, oder „(∃x).fx . x = a“ dasselbe wie „fa“.

Wenn uns ein Satz gegeben ist, so sind mit ihm auch schon die Resultate aller Wahrheitsoperationen, die ihn zur Basis haben, gegeben.

Gibt es logische Urzeichen, so muss eine richtige Logik ihre Stellung zueinander klar machen und ihr Dasein rechtfertigen. Der Bau der Logik aus ihren Urzeichen is no less wonderful that the infinite number of propositions of logic (of mathematics) should follow from half a dozen “primitive propositions”.

But the propositions of logic say the same thing. That is, nothing.

Truth-functions are not material functions.

If e.g. an affirmation can be produced by repeated denial, is the denial—in any sense—contained in the affirmation? Does “¬¬p” deny ~p, or does it affirm p; or both?

The proposition “¬¬p” does not treat of denial as an object, but the possibility of denial is already prejudged in affirmation.

And if there was an object called “~”, then “¬¬p” would want to say something other than “p”. For the one proposition would then treat of ~, the other would not.

This disappearance of the apparent logical constants also occurs if “¬¬(∃x).¬fx” says the same as “(x).fx”, or “(∃x).fx . x = a” the same as “fa”.

If a proposition is given to us then the results of all truth-operations which have it as their basis are given with it.

If there are logical primitive signs a correct logic must make clear their position relative to one another and justify their existence. The construction of logic out of ~~~p, etc. And it is no less remarkable that the infinite number of propositions of logic (mathematics) follow from half a dozen ‘primitive propositions’.

But in fact all the propositions of logic say the same thing, to wit nothing.

Truth-functions are not material functions.

For example, an affirmation can be produced by double negation: in such a case does it follow that in some sense negation is contained in affirmation? Does ‘¬¬p’ negate ~p, or does it affirm p—or both?

The proposition ‘¬¬p’ is not about negation, as if negation were an object: on the other hand, the possibility of negation is already written into affirmation.

And if there were an object called ‘~, it would follow that ‘¬¬p’ said something different from what ‘p’ said, just because the one proposition would then be about ~ and the other would not.

This vanishing of the apparent logical constants also occurs in the case of ‘¬¬(∃x).¬fx’, which says the same as ‘(x).fx’, and in the case of ‘(∃x).fx . x = a’, which says the same as ‘fa’.

If we are given a proposition, then with it we are also given the results of all truth-operations that have it as their base.

If there are primitive logical signs, then any logic that fails to show clearly how they are placed relatively to one another and to justify their existence will be incor-
muss klar werden.

Hat die Logik Grundbegriffe, so müssen sie von einander unabhängig sein. Ist ein Grundbegriff eingeführt, so muss er in allen Verbindungen eingeführt sein, worin er überhaupt vorkommt. Man kann ihn also nicht zuerst für eine Verbindung, dann noch einmal für eine andere einführen. Z. B.: Ist die Verneinung eingeführt, so müssen wir sie jetzt in Sätzen von der Form \( \sim p \) ebenso verstehen, wie in Sätzen wie \( \sim (p \lor q) \), \( \sim \exists x \). \( 
abla f x \) u. a. Wir dürfen sie nicht erst für die eine Klasse von Fällen, dann für die andere einführen, denn es bliebe dann zweifelhaft, ob ihre Bedeutung in beiden Fällen die gleiche wäre und es wäre kein Grund vorhanden, in beiden Fällen dieselbe Art der Zeichenverbindung zu benützen.

(Kurz, für die Einführung der Urzeichern gilt, mutatis mutandis, dasselbe, was Frege („Grundgesetze der Arithmetik“) has said about the introduction of signs by definitions holds.)


If logic has primitive ideas these must be independent of one another. If a primitive idea is introduced it must be introduced in all contexts in which it occurs at all. One cannot therefore introduce it for one context and then again for another. For example, if denial is introduced, we must understand it in propositions of the form \( \sim p \), just as in propositions like \( \sim (p \lor q) \), \( \sim (p \rightarrow q) \), \( \sim (p \land q) \) and others. We may not first introduce it for one class of cases and then again for another. For example, once negation has been introduced, we must understand it both in propositions of the form \( \sim p \) and in propositions like \( \sim (p \lor q) \), \( \sim (p \rightarrow q) \), \( \sim (p \land q) \), etc. We must not introduce it at all for the one class of cases and then for the other, since it would then be left in doubt whether its meaning were the same in both cases, and no reason would have been given for combining the signs in the same way in both cases.

(In short, what Frege ("Grundgesetze der Arithmetik") has said about the introduction of signs by definitions holds, mutatis mutandis, for the introduction of primitive signs also.)

The introduction of a new expedient in the symbolism of logic must always be an event full of consequences. No new symbol may be introduced in logic in brackets or in the margin—with, so to speak, an entirely innocent face.

(Thus in the “Principia Mathematica” of Russell and Whitehead there occur definitions and primitive propositions expressed in words. Why suddenly words here? This would need a justification. There was none, and can be none for the process is actually not rect. The construction of logic out of its primitive signs must be made clear.

If logic has primitive ideas, they must be independent of one another. If a primitive idea has been introduced, it must have been introduced in all the combinations in which it ever occurs. It cannot, therefore, be introduced first for one combination and later reintroduced for another. For example, once negation has been introduced, we must understand it in propositions of the form \( \sim p \) and in propositions like \( \sim (p \lor q) \), \( \sim (p \rightarrow q) \), \( \sim (p \land q) \), etc. We must not introduce it at all for the one class of cases and then for the other, since it would then be left in doubt whether its meaning were the same in both cases, and no reason would have been given for combining the signs in the same way in both cases.

(In short, Frege’s remarks about introducing signs by means of definitions (in The Fundamental Laws of Arithmetic) also apply, mutatis mutandis, to the introduction of primitive signs.)

The introduction of any new device into the symbolism of logic is necessarily a momentous event. In logic a new device should not be introduced in brackets or in a footnote with what one might call a completely innocent air.

(Thus in Russell and Whitehead’s Principia Mathematica there occur definitions and primitive propositions expressed in words. Why this sudden appearance of words? It would require a justification, but none is given, or could be given, since the
da das Vorgehen tatsächlich unerlaubt ist.)

Hat sich aber die Einführung eines neuen Behelfes an einer Stelle als nötig erwiesen, so muss man sich nun sofort fragen: Wo muss dieser Behelf nun immer angebracht werden? Seine Stellung in der Logik muss nun erklärt werden.

5.453
Alle Zahlen der Logik müssen sich rechtfertigen lassen.

Oder vielmehr: Es muss sich herausstellen, dass es in der Logik keine Zahlen gibt.

Es gibt keine ausgezeichneten Zahlen.

5.454
In der Logik gibt es kein Nebeneinander, kann es keine Klassifikation geben.

In der Logik kann es nicht Allgemeines und Spezielles geben.

Die Lösungen der logischen Probleme müssen einfach sein, denn sie setzen den Standard der Einfachheit.


Ein Gebiet, in dem der Satz gilt: simplex sigillum veri.

Wenn man die logischen Zeichen richtig einführte, so hätte man damit auch schon den Sinn aller ihrer Kombinationen eingeführt; also nicht nur „p ∨ q“ sondern auch schon „¬(p ∨ ¬q)“ etc. etc. Man hätte damit auch schon die Wirkung aller nur möglichen Kombinationen von Klammern eingeführt. Und damit wäre es klar geworden, dass die eigentlichen allgemeinen Zeichen nicht die „p ∨ q“, „(∃x). f(x)“, etc. sind, allowed.)

But if the introduction of a new expedient has proved necessary in one place, we must immediately ask: Where is this expedient always to be used? Its position in logic must be made clear.

All numbers in logic must be capable of justification.

Or rather it must become plain that there are no numbers in logic.

There are no pre-eminent numbers.

In logic there is no side by side, there can be no classification.

In logic there cannot be a more general and a more special.

The solution of logical problems must be neat for they set the standard of neatness.

Men have always thought that there must be a sphere of questions whose answers—a priori—are symmetrical and united into a closed regular structure.

A sphere in which the proposition, simplex sigillum veri, is valid.

When we have rightly introduced the logical signs, the sense of all their combinations has been already introduced with them: therefore not only “p ∨ q” but also “¬(p ∨ ¬q)”, etc. etc. We should then already have introduced the effect of all possible combinations of brackets; and it would then have become clear that the proper general primitive signs are not “p ∨ q”, “(∃x) f(x)”, etc., but the most general procedure is in fact illicit.)

But if the introduction of a new device has proved necessary at a certain point, we must immediately ask ourselves, ‘At what points is the employment of this device now unavoidable?’ and its place in logic must be made clear.

All numbers in logic stand in need of justification.

Or rather, it must become evident that there are no numbers in logic.

There are no privileged numbers.

In logic there is no co-ordinate status, and there can be no classification.

In logic there can be no distinction between the general and the specific.

The solutions of the problems of logic must be simple, since they set the standard of simplicity.

Men have always had a presentiment that there must be a realm in which the answers to questions are symmetrically combined—a priori—to form a self-contained system.

A realm subject to the law: Simplex sigillum veri.

If we introduced logical signs properly, then we should also have introduced at the same time the sense of all combinations of them; i.e. not only ‘p ∨ q’ but ‘¬(p ∨ ¬q)’ as well, etc. etc. We should also have introduced at the same time the effect of all possible combinations of brackets. And thus it would have been made clear that the real general primitive signs are not ‘p ∨ q’, ‘(∃x) f(x)’, etc. but the most general form of
sondern die allgemeinste Form ihrer Kom-

5.461

binationen.

Bedeutungsvoll ist die scheinbar un-

5.4611

wichtige Tatsache, dass die logischen Scheinbeziehungen, wie ∨ und ⊃, der Klammern bedürfen – im Gegensatz zu den wirklichen Beziehungen.

Die Benützung der Klammern mit je-

5.47

nen scheinbaren Urzeichen deutet ja schon darauf hin, dass diese nicht die wirklichen Urzeichen sind. Und es wird doch wohl niemand glauben, dass die Klammern eine selbständige Bedeutung haben.

Die logischen Operationszeichen sind Interpunktionen.

Es ist klar, dass alles, was sich über-

5.471

haupt von vornherein über die Form aller Sätze sagen läs-

5.4711

st, sich a u f e i n m a l sagen lassen muss.

Sind ja schon im Elementarsatze alle logischen Operationen enthalten. Denn „fa“ sagt dasselbe wie

„(∃x). fx . x = a“.

Wo Zusammengesetztheit ist, da ist Ar-

5.4612

gument und Funktion, und wo diese sind, sind bereits alle logischen Konstanten.

Man könnte sagen: Die Eine logische Konstante ist das, was a l l e Sätze, ihrer Natur nach, mit einander gemein haben.

Das aber ist die allgemeine Satzform.

Die allgemeine Satzform ist das Wesen des Satzes.

Das Wesen des Satzes angeben, heißt, das Wesen aller Beschreibung angeben, al-

form of their combinations.

The apparently unimportant fact that the apparent relations like ∨ and ⊃ need brackets—unlike real relations—is of great importance.

The use of brackets with these apparent primitive signs shows that these are not the real primitive signs; and nobody of course would believe that the brackets have meaning by themselves.

Logical operation signs are punctua-

5.4712

tions.

It is clear that everything which can be said beforehand about the form of all propositions at all can be said on one occa-

5.4713

sion.

An elementary proposition really contains all logical operations in itself. For ‘fa’ says the same as

‘(∃x). fx . x = a’.

Wherever there is compositeness, argument and function are present, and where these are present, we already have all the logical constants.

One could say: that the sole logical con-

5.4714

stant was what all propositions, by their very nature, had in common with one another.

That however is the general form of proposition.

The general form of proposition is the essence of proposition.

To give the essence of proposition means to give the essence of all description,

their combinations.

Though it seems unimportant, it is in fact significant that the pseudo-relations of logic, such as ∨ and ⊃, need brackets—unlike real relations.

Indeed, the use of brackets with these apparently primitive signs is itself an indication that they are not primitive signs. And surely no one is going to believe brackets have an independent meaning.

Signs for logical operations are punctuation-marks.

It is clear that whatever we can say in advance about the form of all propositions, we must be able to say all at once.

An elementary proposition really con-

5.4715

tains all logical operations in itself. For ‘fa’ says the same as

‘(∃x). fx . x = a’.

Wherever there is compositeness, argument and function are present, and where these are present, we already have all the logical constants.

One could say: that the sole logical con-

stant was what all propositions, by their very nature, had in common with one another.

But that is the general propositional form.

The general propositional form is the essence of a proposition.

To give the essence of a proposition means to give the essence of all description,
so das Wesen der Welt. 

Die Beschreibung der allgemeinsten Satzform ist die Beschreibung des einen und einzigen allgemeinen Urzeichens der Logik.

Die Logik muss für sich selber sorgen. Ein mögliches Zeichen muss auch bezeichnen können. Alles was in der Logik möglich ist, ist auch erlaubt. („Sokrates ist identisch“ heißt darum nichts, weil es keine Eigenschaft gibt, die „identisch“ heißt. Der Satz ist unsinnig, weil wir eine willkürlich Bestimmung nicht getroffen haben, aber nicht darum, weil das Symbol an und für sich unerlaubt wäre.)

Wir können uns, in gewissem Sinne, nicht in der Logik irren.

Das Einleuchten, von dem Russell so viel sprach, kann nur dadurch in der Logik entbehrlieh werden, dass die Sprache selbst jeden logischen Fehler verhindert. – Dass die Logik a priori ist, besteht darin, dass nicht unlogisch gedacht werden kann.

Wir können einem Zeichen nicht den unrechten Sinn geben.

Occam’s Devise ist natürlich keine willkürliche, oder durch ihren praktischen Erfolg gerechtfertigte Regel: Sie besagt, dass unnötige Zeicheneinheiten nichts bedeuten.

Zeichen, die Einen Zweck erfüllen, sind logisch äquivalent, Zeichen, die keinen Zweck erfüllen, logisch bedeutungslos.

Frege sagt: Jeder rechtmäßig gebildete Satz muss einen Sinn haben; und ich sage: therefore the essence of the world.

The description of the most general propositional form is the description of the one and only general primitive sign in logic.

Logic must take care of itself.

A possible sign must also be able to signify. Everything which is possible in logic is also permitted. (“Socrates is identical” means nothing because there is no property which is called “identical”. The proposition is senseless because we have not made some arbitrary determination, not because the symbol is in itself unpermissible.)

In a certain sense we cannot make mistakes in logic.

Self-evidence, of which Russell has said so much, can only be discarded in logic by language itself preventing every logical mistake. That logic is a priori consists in the fact that we cannot think illogically.

We cannot give a sign the wrong sense.

Occam’s razor is, of course, not an arbitrary rule nor one justified by its practical success. It simply says that unnecessary elements in a symbolism mean nothing.

Signs which serve one purpose are logically equivalent, signs which serve no purpose are logically meaningless.

Frege says: Every legitimately constructed proposition must have a sense; and thus the essence of the world.

The description of the most general propositional form is the description of the one and only general primitive sign in logic.

Logic must look after itself.

If a sign is possible, then it is also capable of signifying. Whatever is possible in logic is also permitted. (The reason why ‘Socrates is identical’ means nothing is that there is no property called ‘identical’. The proposition is nonsensical because we have failed to make an arbitrary determination, and not because the symbol, in itself, would be illegitimate.)

In a certain sense, we cannot make mistakes in logic.

Self-evidence, which Russell talked about so much, can become dispensable in logic, only because language itself prevents every logical mistake.—What makes logic a priori is the impossibility of illogical thought.

We cannot give a sign the wrong sense.

Occam’s maxim is, of course, not an arbitrary rule, nor one that is justified by its success in practice: its point is that unnecessary units in a sign-language mean nothing.

Signs that serve one purpose are logically equivalent, and signs that serve none are logically meaningless.

Frege says that any legitimately constructed proposition must have a sense.
Jeder mögliche Satz ist rechtmäßig gebildet, und wenn er keinen Sinn hat, so kann das nur daran liegen, dass wir einigen seiner Bestandteile keine Bedeutung geben haben.

(Wenn wir auch glauben, es getan zu haben.)

So sagt „Sokrates ist identisch“ darum nichts, weil wir dem Wort „identisch“ als Eigenschaftswort keine Bedeutung geben haben. Denn, wenn es als Gleichheitszeichen auftritt, so symbolisiert es auf ganz andere Art und Weise – die bezeichnende Beziehung ist eine andere, – also ist auch das Symbol in beiden Fällen ganz verschieden; die beiden Symbole haben nur das Zeichen zufällig miteinander gemein.

5.474

Die Anzahl der nötigen Grundoperationen hängt nur von unserer Notation ab.

5.475


Es ist klar, dass es sich hier nicht um eine Anzahl von Grundbegriffen handelt, die bezeichnet werden müssen, sondern um den Ausdruck einer Regel.

5.476

Jede Wahrheitsfunktion ist ein Resultat der successiven Anwendung der Operation (−−−−−W)(ξ, . . . . .) auf Elementarsätze.

Diese Operation verneint sämtliche Sätze in der rechten Klammer, und ich nenne sie die Negation dieser Sätze.

and I say: Every possible proposition is legitimately constructed, and if it has no sense this can only be because we have given no meaning to some of its constituent parts.

(Even if we believe that we have done so.)

Thus “Socrates is identical” says nothing, because we have given no meaning to the word “identical” as adjective. For when it occurs as the sign of equality it symbolizes in an entirely different way—the symbolizing relation is another—therefore the symbol is in the two cases entirely different; the two symbols have the sign in common with one another only by accident.

The number of necessary fundamental operations depends only on our notation.

It is only a question of constructing a system of signs of a definite number of dimensions—of a definite mathematical multiplicity.

It is clear that we are not concerned here with a number of primitive ideas which must be signified but with the expression of a rule.

Every truth-function is a result of the successive application of the operation (−−−−−T)(ξ, . . . . .) to elementary propositions.

This operation denies all the propositions in the right-hand bracket and I call it the negation of those propositions.

And I say that any possible proposition is legitimately constructed, and, if it has no sense, that can only be because we have failed to give a meaning to some of its constituents.

(Even if we think that we have done so.)

Thus the reason why ‘Socrates is identical’ says nothing is that we have not given any adjectival meaning to the word ‘identical’. For when it appears as a sign for identity, it symbolizes in an entirely different way—the signifying relation is a different one—therefore the symbols also are entirely different in the two cases: the two symbols have only the sign in common, and that is an accident.

The number of fundamental operations that are necessary depends solely on our notation.

All that is required is that we should construct a system of signs with a particular number of dimensions—with a particular mathematical multiplicity.

It is clear that this is not a question of a number of primitive ideas that have to be signified, but rather of the expression of a rule.

Every truth-function is a result of successive applications to elementary propositions of the operation ‘(−−−−−T) (ξ, . . . . .)’.

This operation negates all the propositions in the right-hand pair of brackets, and I call it the negation of those propositions.

(Hat also ξ etwa die 3 Werte P, Q, R, so ist (ξ) = (P, Q, R).)

Die Werte der Variablen werden festgesetzt.

Die Festsetzung ist die Beschreibung der Sätze, welche die Variable vertritt.

Wie die Beschreibung der Glieder des Klammerausdruckes geschieht, ist unwe-sentlich.


Ich schreibe also statt „(−−−−W) (ξ, . . . . .)“ „N(ξ)“.

N(ξ) ist die Negation sämtlicher Werte der Satzvariablen ξ.

Da sich offenbar leicht ausdrücken

An expression in brackets whose terms are propositions I indicate—if the order of the terms in the bracket is indifferent—by a sign of the form “(ξ)”. “ξ” is a variable whose values are the terms of the expression in brackets, and the line over the variable indicates that it stands for all its values in the bracket.

(Thus if ξ has the 3 values P, Q, R, then (ξ) = (P, Q, R).)

The values of the variables must be determined.

The determination is the description of the propositions which the variable stands for.

How the description of the terms of the expression in brackets takes place is unessential.

We may distinguish 3 kinds of description: 1. Direct enumeration. In this case we can place simply its constant values instead of the variable. 2. Giving a functionfx, whose values for all values of x are the propositions to be described. 3. Giving a formal law, according to which those propositions are constructed. In this case the terms of the expression in brackets are all the terms of a formal series.

Therefore I write instead of “(−−−−T) (ξ, . . . . .)”, “N(ξ)“.

N(ξ) is the negation of all the values of the propositional variable ξ.

As it is obviously easy to express how

When a bracketed expression has propositions as its terms—and the order of the terms inside the brackets is indifferent—then I indicate it by a sign of the form ‘(ξ)’. ‘ξ’ is a variable whose values are terms of the bracketed expression and the bar over the variable indicates that it is the representative of all its values in the brackets.

(E.g. if ξ has the three values P, Q, R, then (ξ) = (P, Q, R).)

What the values of the variable are is something that is stipulated.

The stipulation is a description of the propositions that have the variable as their representative.

How the description of the terms of the bracketed expression is produced is not essential.

We can distinguish three kinds of description: 1. direct enumeration, in which case we can simply substitute for the variable the constants that are its values; 2. giving a functionfx whose values for all values of x are the propositions to be described; 3. giving a formal law that governs the construction of the propositions, in which case the bracketed expression has as its members all the terms of a series of forms.

So instead of ‘(−−−−T) (ξ, . . . . .)’, I write ‘N(ξ)’.

N(ξ) is the negation of all the values of the propositional variable ξ.

It is obvious that we can easily express
propositions can be constructed by means of this operation and how propositions are not to be constructed by means of it, this must be capable of exact expression.

If $\xi$ has only one value, then $N(\xi) = \sim p$ (not $p$); if it has two values then $N(\xi) = \sim p \lor \sim q$ (neither $p$ nor $q$).

How can the all-embracing logic which mirrors the world use such special catches and manipulations? Only because all these are connected into an infinitely fine network, to the great mirror.

"$\sim p$" is true if "$p$" is false. Therefore in the true proposition "$\sim p$" "$p$" is a false proposition. How then can the stroke "$\sim$" bring it into agreement with reality?

That which denies in "$\sim p$" is however not "$\sim$", but that which all signs of this notation, which deny $p$, have in common.

Hence the common rule according to which "$\sim p$", "$\sim \sim p$", "$\sim p \lor \sim p$", "$\sim p \sim p$", etc. etc. (to infinity) are constructed. And this which is common to them all mirrors denial.

We could say: What is common to all symbols, which assert both $p$ and $q$, is the proposition "$p \land q$". What is common to all symbols, which asserts either $p$ or $q$, is the proposition "$p \lor q$".

And similarly we can say: Two propositions are opposed to one another when they have nothing in common with one another; and every proposition has only one negative, because there is only one proposition how propositions may be constructed with this operation, and how they may not be constructed with it; so it must be possible to find an exact expression for this.

If $\xi$ has only one value, then $N(\xi) = \sim p$ (not $p$); if it has two values, then $N(\xi) = \sim p \lor \sim q$ (neither $p$ nor $q$).

How can logic—all-embracing logic, which mirrors the world—use such peculiar crotchets and contrivances? Only because they are all connected with one another in an infinitely fine network, the great mirror.

"$\sim p$" is true if "$p$" is false. Therefore, in the proposition "$\sim p$", when it is true, "$p$" is a false proposition. How then can the stroke "$\sim$" make it agree with reality?

But in "$\sim p$" it is not "$\sim$" that negates, it is rather what is common to all the signs of this notation that negate $p$.

That is to say the common rule that governs the construction of "$\sim p$", "$\sim \sim p$", "$\sim p \lor \sim p$", "$\sim p \sim p$", etc. etc. (ad inf.). And this common factor mirrors negation.

We might say that what is common to all symbols that affirm both $p$ and $q$ is the proposition "$p \land q$"; and that what is common to all symbols that affirm either $p$ or $q$ is the proposition "$p \lor q$".

And similarly we can say that two propositions are opposed to one another if they have nothing in common with one another, and that every proposition has only one negative, since there is only one
liegt.
Es zeigt sich so auch in Russells Notation, dass „q : p ∨ ¬p“ dasselbe sagt wie „q“; dass „p ∨ ¬p“ nichts sagt.

5.514

Ist eine Notation festgelegt, so gibt es in ihr eine Regel, nach der alle p verneinenden Sätze gebildet werden, eine Regel, nach der alle p bejahenden Sätze gebildet werden, eine Regel, nach der alle p oder q bejahenden Sätze gebildet werden, u. s. f. Diese Regeln sind den Symbolen äquivalent und in ihnen spiegelt sich ihr Sinn wieder.

5.515

Es muss sich an unseren Symbolen zeigen, dass das, was durch „∨“, „¬“, etc. miteinander verbunden ist, Sätze sein müssen.


5.5151

Muss das Zeichen des negativen Satzes mit dem Zeichen des positiven gebildet werden? Warum sollte man den negativen Satz nicht durch eine negative Tatsache ausdrücken können. (Etwa: Wenn „a“ nicht in einer bestimmten Beziehung zu „b“ steht, könnte das ausdrücken, dass aRb nicht der Fall ist.)

Aber auch hier ist ja der negative Satz

which lies altogether outside it.
Thus in Russell’s notation also it appears evident that “q : p ∨ ¬p” says the same thing as “q”; that “p ∨ ¬p” says nothing.

If a notation is fixed, there is in it a rule according to which all the propositions denying p are constructed, a rule according to which all the propositions asserting p are constructed, a rule according to which all the propositions asserting p or q are constructed, and so on. These rules are equivalent to the symbols and in them their sense is mirrored.

It must be recognized in our symbols that what is connected by “∨“, “¬“, etc., must be propositions.

And this is the case, for the symbols “p” and “q” presuppose “∨“, “¬”, etc. If the sign “p” in “p ∨ q” does not stand for a complex sign, then by itself it cannot have sense; but then also the signs “p ∨ p”, “p . p”, etc., which have the same sense as “p” have no sense. If, however, “p ∨ p” has no sense, then also “p ∨ q” can have no sense.

Must the sign of the negative proposition be constructed by means of the sign of the positive? Why should one not be able to express the negative proposition by means of a negative fact? (Like: if “a” does not stand in a certain relation to “b”, it could express that aRb is not the case.)

But here also the negative proposition

proposition that lies completely outside it.
Thus in Russell’s notation too it is manifest that ‘q : p ∨ ¬p’ says the same thing as ‘q’, that ‘p ∨ ¬p’ says nothing.

Once a notation has been established, there will be in it a rule governing the construction of all propositions that negate p, a rule governing the construction of all propositions that affirm p, and a rule governing the construction of all propositions that affirm p or q; and so on. These rules are equivalent to the symbols; and in them their sense is mirrored.

It must be manifest in our symbols that it can only be propositions that are combined with one another by ‘∨’, ‘¬’, etc.

And this is indeed the case, since the symbol in ‘p’ and ‘q’ itself presupposes ‘∨’, ‘¬’, etc. If the sign ‘p’ in ‘p ∨ q’ does not stand for a complex sign, then it cannot have sense by itself: but in that case the signs ‘p ∨ p’, ‘p . p’, etc., which have the same sense as p, must also lack sense. But if ‘p ∨ p’ has no sense, then ‘p ∨ q’ cannot have a sense either.

Must the sign of a negative proposition be constructed with that of the positive proposition? Why should it not be possible to express a negative proposition by means of a negative fact? (E.g. suppose that ‘a’ does not stand in a certain relation to ‘b’; then this might be used to say that aRb was not the case.)

But really even in this case the nega-
indirekt durch den positiven gebildet.

Der positive Satz muss die Existenz des negativen voraussetzen und umgekehrt.

Sind die Werte von \( \xi \) sämtliche Werte einer Funktion \( fx \) für alle Werte von \( x \), so wird \( N(\vec{\xi}) = \sim (\exists x). fx \).

Ich trenne den Begriff All von der Wahrheitsfunktion.

Frege und Russell haben die Allgemeinheit in Verbindung mit dem logischen Produkt oder der logischen Summe eingeführt. So wurde es schwer, die Sätze „\( \exists x \). fx“ und „\( (x). fx \)“, in welchen beide Ideen be- schlossen liegen, zu verstehen.

Das Eigentümliche der Allgemeinheitsbezeichnung ist erstens, dass sie auf ein logisches Urbild hinweist, und zweitens, dass sie Konstante hervorhebt.

Die Allgemeinheitsbezeichnung tritt als Argument auf.

Wenn die Gegenstände gegeben sind, so sind uns damit auch schon alle Gegenstände gegeben.

Wenn die Elementarsätze gegeben sind, so sind damit auch alle Elementarsätze gegeben.

Es ist unrichtig, den Satz „\( \exists x \). fx“ – wie Russell dies tut – in Worten durch „\( fx \) ist möglich“ wiedergegeben.

Gewissheit, Möglichkeit oder Unmöglichkeit einer Sachlage wird nicht durch einen Satz ausgedrückt, sondern dadurch, dass ein Ausdruck eine Tautologie, ein sinnvoller Satz oder eine Kontradiktion ist.

Jener Präzedenzfall, auf den man sich is indirectly constructed with the positive.

The positive proposition must presuppose the existence of the negative proposition and conversely.

If the values of \( \xi \) are the total values of a function \( fx \) for all values of \( x \), then \( N(\vec{\xi}) = \sim (\exists x). fx \).

I separate the concept all from the truth-function.

Frege and Russell have introduced generality in connexion with the logical product or the logical sum. Then it would be difficult to understand the propositions “\( \exists x \). fx” and “\( (x). fx \)” in which both ideas lie concealed.

That which is peculiar to the “symbolism of generality” is firstly, that it refers to a logical prototype, and secondly, that it makes constants prominent.

The generality symbol occurs as an argument.

If the objects are given, therewith are all objects also given.

If the elementary propositions are given, then therewith all elementary propositions are also given.

It is not correct to render the proposition “\( \exists x \). fx” — as Russell does — in the words “\( fx \) is possible”.

Certainty, possibility or impossibility of a state of affairs are not expressed by a proposition but by the fact that an expression is a tautology, a significant proposition or a contradiction.

That precedent to which one would al-}

The positive proposition necessarily presupposes the existence of the negative proposition and vice versa.

If \( \xi \) has as its values all the values of a function \( fx \) for all values of \( x \), then \( N(\vec{\xi}) = \sim (\exists x). fx \).

I dissociate the concept all from truth-functions.

Frege and Russell introduced generality in association with logical product or logical sum. This made it difficult to understand the propositions ‘\( \exists x \). \( fx \)' and ‘\( (x). \( fx \)' in which both ideas are embedded.

What is peculiar to the generality-sign is first, that it indicates a logical prototype, and secondly, that it gives prominence to constants.

The generality-sign occurs as an argument.

If objects are given, then at the same time we are given all objects.

If elementary propositions are given, then at the same time all elementary propositions are given.

It is incorrect to render the proposition ‘\( \exists x \). \( fx \)' in the words, ‘\( fx \) is possible’ as Russell does.

The certainty, possibility, or impossibility of a situation is not expressed by a proposition, but by an expression’s being a tautology, a proposition with a sense, or a contradiction.

The precedent to which we are con-
immer berufen möchte, muss schon im Symbol selber liegen.
Man kann die Welt vollständig durch vollkommen verallgemeinerte Sätze beschreiben, das heißt also, ohne irgendeinen Namen von vornherein einem bestimmten Gegenstand zuzuordnen.
Um dann auf die gewöhnliche Ausdrucksweise zu kommen, muss man einfach nach einem Ausdruck: „Es gibt ein und nur ein $x$, welches …“ sagen: Und dies $x$ ist $a$.

Ein vollkommen verallgemeinerter Satz ist, wie jeder andere Satz, zusammengesetzt. (Dies zeigt sich daran, dass wir in „$(\exists x, \phi) \cdot \phi x“ „\phi“ und „$x“ getrennt erwähnen müssen. Beide stehen unabhängig in bezeichnenden Beziehungen zur Welt, wie im unverallgemeinerten Satz.)

Kennzeichen des zusammengesetzten Symbols: Es hat etwas mit anderen Symbolen gemeinsam.
Es verändert ja die Wahr- oder Falschheit jedes Satzes etwas am allgemeinen Bau der Welt. Und der Spielraum, welcher ihrem Bau durch die Gesamtheit der Elementarsätze gelassen wird, ist eben derjenige, welchen die ganz allgemeinen Sätze begreifen.
(Wenn ein Elementarsatz wahr ist, so ist damit doch jedenfalls Ein Elementarsatz mehr wahr.)


ways appeal, must be present in the symbol itself.
One can describe the world completely by completely generalized propositions, i.e. without from the outset co-ordinating any name with a definite object.

In order then to arrive at the customary way of expression we need simply say after an expression “there is one and only one $x$, which …“: and this $x$ is $a$.

A completely generalized proposition is like every other proposition composite. (This is shown by the fact that in “$(\exists x, \phi) \cdot \phi x“ we must mention “$\phi“ and “$x“ separately. Both stand independently in signifying relations to the world as in the ungeneralized proposition.)

A characteristic of a composite symbol: it has something in common with other symbols.
The truth or falsehood of every proposition alters something in the general structure of the world. And the range which is allowed to its structure by the totality of elementary propositions is exactly that which the completely general propositions delimit.

(If an elementary proposition is true, then, at any rate, there is one more elementary proposition true.)

Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs.

constantly inclined to appeal must reside in the symbol itself.
We can describe the world completely by means of fully generalized propositions, i.e. without first correlating any name with a particular object.

Then, in order to arrive at the customary mode of expression, we simply need to add, after an expression like, ‘There is one and only one $x$ such that …‘, the words, ‘and that $x$ is $a$‘.

A fully generalized proposition, like every other proposition, is composite. (This is shown by the fact that in ‘$(\exists x, \phi) \cdot \phi x“ we have to mention ‘$\phi“ and ‘$x“ separately. They both, independently, stand in signifying relations to the world, just as is the case in ungeneralized propositions.)

It is a mark of a composite symbol that it has something in common with other symbols.
The truth or falsity of every proposition does make some alteration in the general construction of the world. And the range that the totality of elementary propositions leaves open for its construction is exactly the same as that which is delimited by entirely general propositions.

(If an elementary proposition is true, that means, at any rate, one more true elementary proposition.)

Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs.
Dass die Identität keine Relation zwischen Gegenständen ist, leuchtet ein. Dies wird sehr klar, wenn man z. B. den Satz \( (x): f x \ni x = a \) betrachtet. Was dieser Satz sagt, ist einfach, dass \( n u r \ a \) der Funktion \( f \) genügt, und nicht, dass nur solche Dinge der Funktion \( f \) genügen, welche eine gewisse Beziehung zu \( a \) haben.

Man könnte nun freilich sagen, dass eben \( n u r \ a \) diese Beziehung zu \( a \) habe, aber, um dies auszudrücken, brauchten wir das Gleichheitszeichen selber.

Russels Definition von \( = \) genügt nicht; weil man nach ihr nicht sagen kann, dass zwei Gegenstände alle Eigenschaften gemeinsam haben. (Selbst wenn dieser Satz nie richtig ist, hat er doch \( S i n n \).)

Beiläufig gesprochen: Von \( z w e i \) Ding zu sagen, sie seien identisch, ist ein Unsinn, und von \( E i n \) \( e m \) zu sagen, es sei identisch mit sich selbst, sagt gar nichts.

Ich schreibe also nicht \( f(a, b) \ni a = b \), sondern \( f(a, a) \) (oder \( f(b, b) \)). Und nicht \( f(a, b) \ni a = b \), sondern \( f(a, b) \).

Und analog: Nicht \( \exists (x, y). f(x, y) \ni x = y \), sondern \( \exists (x, y). f(x, x) \); und nicht \( \exists (x, y). f(x, y) \ni x = y \), sondern \( \exists (x, y). f(x, y) \).

(Also statt des Russellschen \( \exists (x, y). f(x, y) : (\exists x, y). f(x, x) \cdot (\exists x, y). f(x, x) \).

Statt \( (x): f x \ni x = a \) schreiben wir also z. B. \( \exists (x). f(x) \ni f a \ni (\exists x, y). f(x) \cdot f y \).

Und der Satz: \( n u r \ \text{Ein} \ x \ \text{befriedigt} \ f() \) lautet: \( \exists (x). f(x) \ni (\exists y, y). f(x) \cdot f y \).

Das Gleichheitszeichen ist also kein we-

That identity is not a relation between objects. This becomes very clear if, for example, one considers the proposition \( "(x): f x \ni x = a" \). What this proposition says is simply that \( o n l y \ a \) satisfies the function \( f \), and not that only such things satisfy the function \( f \) which have a certain relation to \( a \).

And analogously I do not write \( f(a, a) \ni a = b \), but \( "(a, a)" \) (or \( "(b, b)" \)). And not \( f(a, b) \ni a = b \), but \( "(a, b)" \).

And analogously: not \( (\exists x, y). f(x, y) \ni x = y \), but \( (\exists x, y). f(x, x) \); and not \( (\exists x, y). f(x, y) \ni x = y \), but \( (\exists x, y). f(x, y) \).

(Therefore instead of Russell’s \( "(\exists x, y). f(x, y) : (\exists x, y). f(x, x) \cdot (\exists x, y). f(x, x) \)."

Instead of \( "(x): f x \ni x = a" \) we therefore write e.g. \( "(\exists x). f(x) \ni f a \ni (\exists x, y). f(x) \cdot f y \)."

And if the proposition \( \text{"Only one} x \ \text{satisfies} \ f() \) \) reads: \( (\exists x). f(x) \ni (\exists x, y). f(x) \cdot f y \).

It is self-evident that identity is not a relation between objects. This becomes very clear if one considers, for example, the proposition \( "(x): f x \ni x = a' \). What this proposition says is simply that \( o n l y \ a \) satisfies the function \( f \), and not that only things that have a certain relation to \( a \) satisfy the function \( f \).

Of course, it might then be said that \( o n l y \ a \) did have this relation to \( a \); but in order to express that, we should need the identity-sign itself.

Russell’s definition of ‘=’ is inadequate, because according to it we cannot say that two objects have all their properties in common. (Even if this proposition is never correct, it still has \( \text{S i n n} \).)

Roughly speaking, to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing at all.

Thus I do not write \( f(a, b) \ni a = b \), but \( "(a, a)" \) (or \( "(b, b)" \)); and not \( f(a, b) \ni a = b \), but \( "(a, b)" \).

And analogously I do not write \( (\exists x, y). f(x, y) \ni x = y \), but \( (\exists x, y). f(x, x) \); and not \( (\exists x, y). f(x, y) \ni x = y \), but \( (\exists x, y). f(x, y) \).

(So Russell’s \( "(\exists x, y). f(x, y) : (\exists x, y). f(x, x) \cdot (\exists x, y). f(x, x) \)."

Thus, for example, instead of \( "(x): f x \ni x = a' \) we write \( "(\exists x, y). f(x) \ni f a \ni (\exists x, y). f(x) \cdot f y \)."

And the proposition, ‘\( \text{Only one} x \ \text{satisfies} \ f() \) \) will read: \( (\exists x). f(x) \ni (\exists x, y). f(x) \cdot f y \).

The identity-sign, therefore, is not an
sentlicher Bestandteil der Begriffsschrift.

5.534

Und nun sehen wir, dass Scheinsätze wie: \( a = a' \), \( a = b \cdot b = c \supset a = c' \), \( (\exists x).x = a' \), etc. sich in einer richtigen Begriffsschrift gar nicht hinschreiben lassen.

Damit erledigen sich auch alle Probleme, die an solche Scheinsätze geknüpft waren.

Alle Probleme, die Russells „Axiom of Infinity“ mit sich bringt, sind schon hier zu lösen.

Das, was das Axiom of Infinity sagen soll, würde sich in der Sprache dadurch ausdrücken, dass es unendlich viele Namen mit verschiedener Bedeutung gäbe.


(Es ist schon darum Unsinn, die Hypothese \( p \supset p' \) vor einen Satz zu stellen, um ihm Argumente der richtigen Form zu sichern, weil die Hypothese für einen Nicht-Satz als Argument nicht falsch, sondern unsinnig wird, und weil der Satz selbst durch die unrichtige Gattung von Argumenten unsinnig wird, also sich selbst)

essential constituent of logical notation.

And we see that the apparent propositions like: \( a = a' \), \( a = b \cdot b = c \supset a = c' \), \( (\exists x).x = a' \), etc. cannot be written in a correct logical notation at all.

So all problems disappear which are connected with such pseudo-propositions.

This is the place to solve all the problems with arise through Russell’s “Axiom of Infinity”.

What the axiom of infinity is meant to say would be expressed in language by the fact that there is an infinite number of names with different meanings.

There are certain cases in which one is tempted to use expressions of the form \( a = a' \) or \( p \supset p' \) and the like. In fact, this happens when one wants to talk about prototypes, e.g. about proposition, thing, etc. Thus in Russell’s Principles of Mathematics ‘\( p \) is a proposition’—which is nonsense—was given the symbolic rendering \( \supset p \) and placed as an hypothesis in front of certain propositions in order to exclude from their argument-places everything but propositions.

(It is nonsense to place the hypothesis \( p \supset p' \) in front of a proposition, in order to ensure that its arguments shall have the right form, because the hypotheses for a non-proposition as argument becomes not false but meaningless, and because the proposition itself becomes senseless for arguments of the wrong kind, and therefore it sur-

essential constituent of conceptual notation.

And now we see that in a correct conceptual notation pseudo-propositions like \( a = a' \), \( a = b \cdot b = c \supset a = c' \), \( (\exists x).x = \alpha' \), etc. cannot even be written down.

This also disposes of all the problems that were connected with such pseudo-propositions.

All the problems that Russell’s ‘axiom of infinity’ brings with it can be solved at this point.

What the axiom of infinity is intended to say would express itself in language through the existence of infinitely many names with different meanings.

There are certain cases in which one is tempted to use expressions of the form \( a = a' \) or \( p \supset p' \) and the like. In fact, this happens when one wants to talk about prototypes, e.g. about proposition, thing, etc. Thus in Russell’s Principles of Mathematics ‘\( p \) is a proposition’—which is nonsense—was given the symbolic rendering \( \supset p \) and placed as an hypothesis in front of certain propositions in order to exclude from their argument-places everything but propositions.

(It is nonsense to place the hypothesis \( p \supset p' \) in front of a proposition, in order to ensure that its arguments shall have the right form, because the hypotheses for a non-proposition as argument becomes not false but meaningless, and because the proposition itself becomes senseless for arguments of the wrong kind, and therefore it sur-

essential constituent of conceptual notation.
ebenso gut, oder so schlecht, vor den unrechten Argumenten bewahrt wie die zu diesem Zweck angehängte sinnlose Hypothese.)

Ebenso wollte man „Es gibt keine Dinge“ ausdrücken durch „¬(∃x).x = x“. Aber selbst wenn dies ein Satz wäre – wäre er nicht auch wahr, wenn es zwar „Dinge gäbe“, aber diese nicht mit sich selbst identisch wären?

In der allgemeinen Satzform kommt der Satz im Satze nur als Basis der Wahrheitsoperationen vor.

Auf den ersten Blick scheint es, als könne ein Satz in einem anderen auch auf andere Weise vorkommen.

Besonders in gewissen Satzformen der Psychologie, wie „A glaubt, dass p der Fall ist“, oder „A denkt p“, etc.

Hier scheint es nämlich oberflächlich, als stünde der Satz p zu einem Gegenstand A in einer Art von Relation.

(Und in der modernen Erkenntnistheorie (Russell, Moore, etc.) sind jene Sätze auch so aufgefasst worden.)


Dies zeigt auch, dass die Seele – das Subjekt etc. – wie sie in der heutigen oberflächlichen Psychologie aufgefasst wird, ein Unding ist.

5.5352

vives the wrong arguments no better and no worse than the senseless hypothesis attached for this purpose.)

Similarly it was proposed to express
“There are no things” by “¬(∃x).x = x”. But even if this were a proposition—would it not be true if indeed “There were things”, but these were not identical with themselves?

In the general propositional form, propositions occur in a proposition only as bases of the truth-operations.

At first sight it appears as if there were also a different way in which one proposition could occur in another.

Especially in certain propositional forms of psychology, like “A thinks, that p is the case”, or “A thinks p”, etc.

Here it appears superficially as if the proposition p stood to the object A in a kind of relation.

(And in modern epistemology (Russell, Moore, etc.) those propositions have been conceived in this way.)

But it is clear that “A believes that p”, “A thinks p”, “A says p”, are of the form “p says p”: and here we have no co-ordination of a fact and an object, but a co-ordination of facts by means of a co-ordination of their objects.

This shows that there is no such thing as the soul—the subject, etc.—as it is conceived in superficial psychology.

5.541

it preserves itself from wrong arguments just as well, or as badly, as the hypothesis without sense that was appended for that purpose.)

In the same way people have wanted to express, ‘There are no things’, by writing ‘¬(∃x).x = x’. But even if this were a proposition, would it not be equally true if in fact ‘there were things’ but they were not identical with themselves?

In the general propositional form propositions occur in other propositions only as bases of truth-operations.

At first sight it looks as if it were also possible for one proposition to occur in another in a different way.

Particularly with certain forms of proposition in psychology, such as ‘A believes that p is the case’ and A has the thought p’, etc.

For if these are considered superficially, it looks as if the proposition p stood in some kind of relation to an object A.

(And in modern theory of knowledge (Russell, Moore, etc.) these propositions have actually been construed in this way.)

It is clear, however, that ‘A believes that p’, ‘A has the thought p’, and ‘A says p’ are of the form “p says p”: and this does not involve a correlation of a fact with an object, but rather the correlation of facts by means of the correlation of their objects.

This shows too that there is no such thing as the soul—the subject, etc.—as it is conceived in the superficial psychology of the present day.
Eine zusammengesetzte Seele wäre nämlich keine Seele mehr.

5.5422
Die richtige Erklärung der Form des Satzes „A urteilt p“ muss zeigen, dass es unmöglich ist, einen Unsinn zu urteilen. (Russells Theorie genügt dieser Bedingung nicht.)

Einen Komplex wahrnehmen heißt wahrnehmen, dass sich seine Bestandteile so und so zu einander verhalten.

Dies erklärt wohl auch, dass man die Figur

![Figur](image)

auf zweierlei Art als Würfel sehen kann; und alle ähnlichen Erscheinungen. Denn wir sehen eben wirklich zwei verschiedene Tatsachen.

(Sehe ich erst auf die Ecken $a$ und nur flüchtig auf $b$, so erscheint $a$ vorne; und umgekehrt.)

5.5423
Wir müssen nun die Frage nach allen möglichen Formen der Elementarsätze a priori beantworten.

5.55
Der Elementarsatz besteht aus Namen. Da wir aber die Anzahl der Namen von verschiedener Bedeutung nicht angeben können, so können wir auch nicht die Zusammensetzung des Elementarsatzes an-

A composite soul would not be a soul any longer.

The correct explanation of the form of the proposition “A judges $p$” must show that it is impossible to judge a nonsense. (Russell’s theory does not satisfy this condition.)

To perceive a complex means to perceive that its constituents are combined in such and such a way.

This perhaps explains that the figure can be seen in two ways as a cube; and all similar phenomena. For we really see two different facts.

(If I fix my eyes first on the corners $a$ and only glance at $b$, $a$ appears in front and $b$ behind, and vice versa.)

We must now answer a priori the question as to all possible forms of the elementary propositions.

The elementary proposition consists of names. Since we cannot give the number of names with different meanings, we cannot give the composition of the elementary proposition.

Indeed a composite soul would no longer be a soul.

The correct explanation of the form of the proposition, ‘A makes the judgement $p$’, must show that it is impossible for a judgement to be a piece of nonsense. (Russell’s theory does not satisfy this requirement.)

To perceive a complex means to perceive that its constituents are related to one another in such and such a way.

This no doubt also explains why there are two possible ways of seeing the figure as a cube; and all similar phenomena. For we really see two different facts.

(If I look in the first place at the corners marked $a$ and only glance at the $b$’s, then the $a$’s appear to be in front, and vice versa).

We now have to answer a priori the question about all the possible forms of elementary propositions.

Elementary propositions consist of names. Since, however, we are unable to give the number of names with different meanings, we are also unable to give the composition of elementary propositions.
geben.

Unser Grundsatz ist, dass jede Frage, die sich überhaupt durch die Logik entscheiden lässt, sich ohne weiteres entscheiden lassen muss.

(Und wenn wir in die Lage kommen, ein solches Problem durch Ansehen der Welt beantworten zu müssen, so zeigt dies, dass wir auf grundfalscher Fährte sind.)

Die „Erfahrung“, die wir zum Verstehen der Logik brauchen, ist nicht die, dass sich etwas so und so verhält, sondern, dass etwas ist: aber das ist eben keine Erfahrung.

Die Logik ist vor jeder Erfahrung – dass etwas so ist.

Sie ist vor dem Wie, nicht vor dem Was.

Und wenn dies nicht so wäre, wie könnten wir die Logik anwenden? Man könnte sagen: Wenn es eine Logik gäbe, auch wenn es keine Welt gäbe, wie könnte es dann eine Logik geben, da es eine Welt gibt.


(Eine ausgezeichnete Zahl gibt es nicht.)

Die Angabe jeder speziellen Form wäre vollkommen willkürlich.

Es soll sich a priori angeben lassen, ob ich z. B. in die Lage kommen kann, etwas mit dem Zeichen einer 27-stelligen Relation bezeichnen zu müssen.

Our fundamental principle is that every question which can be decided at all by logic can be decided off-hand.

(And if we get into a situation where we need to answer such a problem by looking at the world, this shows that we are on a fundamentally wrong track.)

The “experience” which we need to understand logic is not that such and such is the case, but that something is; but that is no experience.

Logic precedes every experience—that something is so.

It is before the How, not before the What.

And if this were not the case, how could we apply logic? We could say: if there were a logic, even if there were no world, how then could there be a logic, since there is a world?

Russell said that there were simple relations between different numbers of things (individuals). But between what numbers? And how should this be decided—by experience?

(There is no pre-eminent number.)

The enumeration of any special forms would be entirely arbitrary.

How could we decide a priori whether, for example, I can get into a situation in which I need to symbolize with a sign of a 27-termed relation?

Our fundamental principle is that whenever a question can be decided by logic at all it must be possible to decide it without more ado.

(And if we get into a position where we have to look at the world for an answer to such a problem, that shows that we are on a completely wrong track.)

The ‘experience’ that we need in order to understand logic is not that something or other is the state of things, but that something is: that, however, is not an experience.

Logic is prior to every experience—that something is so.

It is prior to the question ‘How?’, not prior to the question ‘What?’

And if this were not so, how could we apply logic? We might put it in this way: if there would be a logic even if there were no world, how then could there be a logic given that there is a world?

Russell said that there were simple relations between different numbers of things (individuals). But between what numbers? And how is this supposed to be decided?—By experience?

(There is no privileged number.)

It would be completely arbitrary to give any specific form.

It is supposed to be possible to answer a priori the question whether I can get into a position in which I need the sign for a 27-termed relation in order to signify something.
<table>
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| 5.552 | Dürfen wir denn aber überhaupt so fragen? Können wir eine Zeichenform aufstellen und nicht wissen, ob ihr etwas entsprechen könne? 
Hat die Frage einen Sinn: Was muss sein, damit etwas der-Fall-sein kann? |
| 5.555 | Es ist klar, wir haben vom Elementarsatz einen Begriff, abgesehen von seiner besonderen logischen Form. 
Wo man aber Symbole nach einem System bilden kann, dort ist dieses System das logisch wichtige und nicht die einzelnen Symbole. 
Und wie wäre es auch möglich, dass ich es in der Logik mit Formen zu tun hätte, die ich erfinden kann; sondern mit dem muss ich es zu tun haben, was es mir möglich macht, sie zu erfinden. |
| 5.556 | Eine Hierarchie der Formen der Elementarsätze kann es nicht geben. Nur was wir selbst konstruieren, können wir vornehmen. |
Die Hierarchien sind, und müssen unabhängig von der Realität sein. |
| 5.5562 | Wissen wir aus rein logischen Gründen, dass es Elementarsätze geben muss, dann muss es jeder wissen, der die Sätze in ihrer unanalysierten Form versteht. |
| 5.5563 | Alle Sätze unserer Umgangssprache sind tatsächlich, so wie sie sind, logisch vollkommen geordnet. – Jenes Einfachste, May we then ask this at all? Can we set out a sign form and not know whether anything can correspond to it? 
Has the question sense: what must there be in order that anything can be the case? 
It is clear that we have a concept of the elementary proposition apart from its special logical form. 
Where, however, we can build symbols according to a system, there this system is the logically important thing and not the single symbols. 
And how would it be possible that I should have to deal with forms in logic which I can invent: but I must have to deal with that which makes it possible for me to invent them. 
There cannot be a hierarchy of the forms of the elementary propositions. Only that which we ourselves construct can we foresee. 
Empirical reality is limited by the totality of objects. The boundary appears again in the totality of elementary propositions. 
The hierarchies are and must be independent of reality. 
If we know on purely logical grounds, that there must be elementary propositions, then this must be known by everyone who understands propositions in their unanalyzed form. 
All propositions of our colloquial language are actually, just as they are, logically completely in order. That simple |
| 5.5564 | But is it really legitimate even to ask such a question? Can we set up a form of sign without knowing whether anything can correspond to it? 
Does it make sense to ask what there must be in order that something can be the case? 
Clearly we have some concept of elementary propositions quite apart from their particular logical forms. 
But when there is a system by which we can create symbols, the system is what is important for logic and not the individual symbols. 
And anyway, is it really possible that in logic I should have to deal with forms that I can invent? What I have to deal with must be that which makes it possible for me to invent them. 
There cannot be a hierarchy of the forms of elementary propositions. We can foresee only what we ourselves construct. 
Empirical reality is limited by the totality of objects. The limit also makes itself manifest in the totality of elementary propositions. 
Hierarchies are and must be independent of reality. 
If we know on purely logical grounds that there must be elementary propositions, then everyone who understands propositions in their unanalyzed form must know it. 
In fact, all the propositions of our everyday language, just as they stand, are in perfect logical order.—That utterly simple |
was wir hier angeben sollen, ist nicht ein Gleichnis der Wahrheit, sondern die volle Wahrheit selbst. (Unsere Probleme sind nicht abstrakt, sondern vielleicht die konkretesten, die es gibt.)

Die Anwendung der Logik entscheidet darüber, welche Elementarsätze es gibt. Was in der Anwendung liegt, kann die Logik nicht vorausnehmen.

Aber die Logik muss sich mit ihrer Anwendung berühren. Also dürfen die Logik und ihre Anwendung einander nicht übergreifen.

Wenn ich die Elementarsätze nicht a priori angeben kann, dann muss es zu offenbarem Unsinn führen, sie angeben zu wollen.

5.557

Die Grenzen meiner Sprache bedeuten die Grenzen meiner Welt. Die Logik erfüllt die Welt; die Grenzen der Welt sind auch ihre Grenzen. Wir können also in der Logik nicht sagen: Das und das gibt es in der Welt, jenes nicht.

Das würde nämlich scheinbar voraussetzen, dass wir gewisse Möglichkeiten ausschließen, und dies kann nicht der Fall sein, da sonst die Logik über die Grenzen der Welt hinaus müsste; wenn sie nämlich diese Grenzen auch von der anderen Seite betrachten könnte.

Was wir nicht denken können, das können wir nicht denken; wir können also thing which we ought to give here is not a model of the truth but the complete truth itself.

(Our problems are not abstract but perhaps the most concrete that there are.)

The application of logic decides what elementary propositions there are.

What lies in its application logic cannot anticipate. It is clear that logic may not conflict with its application. But logic must have contact with its application. Therefore logic and its application may not overlap one another. If I cannot give elementary propositions a priori then it must lead to obvious nonsense to try to give them.

The limits of my language mean the limits of my world. Logic fills the world: the limits of the world are also its limits. We cannot therefore say in logic: This and this there is in the world, that there is not.

For that would apparently presuppose that we exclude certain possibilities, and this cannot be the case since otherwise logic must get outside the limits of the world: that is, if it could consider these limits from the other side also.

What we cannot think, that we cannot think: we cannot therefore say what we thing, which we have to formulate here, is not an image of the truth, but the truth itself in its entirety. (Our problems are not abstract, but perhaps the most concrete that there are.)

The application of logic decides what elementary propositions there are.

What belongs to its application, logic cannot anticipate. It is clear that logic must not clash with its application. But logic has to be in contact with its application. Therefore logic and its application must not overlap. If I cannot say a priori what elementary propositions there are, then the attempt to do so must lead to obvious nonsense.

The limits of my language mean the limits of my world. Logic pervades the world: the limits of the world are also its limits. So we cannot say in logic, ‘The world has this in it, and this, but not that.’

For that would appear to presuppose that we were excluding certain possibilities, and this cannot be the case, since it would require that logic should go beyond the limits of the world; for only in that way could it view those limits from the other side as well.

We cannot think what we cannot think; so what we cannot think we cannot say
5.62

Diese Bemerkung gibt den Schlüssel zur Entscheidung der Frage, inwieweit der Solipsismus eine Wahrheit ist.

Was der Solipsismus nämlich meint, ist ganz richtig, nur lässt es sich nicht sagen, sondern es zeigt sich.

Dass die Welt meine Welt ist, das zeigt sich darin, dass die Grenzen der Sprache (der Sprache, die allein ich verstehe) die Grenzen meiner Welt bedeuten.

Die Welt und das Leben sind Eins.

5.621

Die Welt und das Leben sind Eins. (Der Mikrokosmos.)

5.63

Ich bin meine Welt. (Der Mikrokosmos.)

5.631

Das denkende, vorstellende, Subjekt gibt es nicht.

Wenn ich ein Buch schribe „Die Welt, wie ich sie vorfand“, so wäre darin auch über meinen Leib zu berichten und zu sagen, welche Glieder meinem Willen unterstehen und welche nicht, etc., dies ist nämlich eine Methode, das Subjekt zu isolieren, oder vielmehr zu zeigen, dass es in einem wichtigen Sinne kein Subjekt gibt: Von ihm allein nämlich könnte in diesem Buch nicht die Rede sein.—

5.632

Das Subjekt gehört nicht zur Welt, sondern es ist eine Grenze der Welt.

5.633

Wo in der Welt ist ein metaphysisches Subjekt zu merken?


Und nichts am Gesichtsfeld lässt darauf schließen, dass es von einem Auge gesehen wird.

5.6331

Das Gesichtsfeld hat nämlich nicht et-

cannot think.

This remark provides a key to the question, to what extent solipsism is a truth.

In fact what solipsism means, is quite correct, only it cannot be said, but it shows itself.

That the world is my world, shows itself in the fact that the limits of the language (the language which I understand) mean the limits of my world.

The world and life are one.

If I wrote a book “The world as I found it”, I should also have therein to report on my body and say which members obey my will and which do not, etc. This then would be a method of isolating the subject or rather of showing that in an important sense there is no subject: that is to say, of it alone in this book mention could not be made.

The subject does not belong to the world but it is a limit of the world.

Where in the world is a metaphysical subject to be noted?

You say that this case is altogether like that of the eye and the field of sight. But you do not really see the eye.

And from nothing in the field of sight can it be concluded that it is seen from an eye.

For the field of sight has not a form like either.

This remark provides the key to the problem, how much truth there is in solipsism.

For what the solipsist means is quite correct; only it cannot be said, but makes itself manifest.

The world is my world: this is manifest in the fact that the limits of language (of that language which alone I understand) mean the limits of my world.

The world and life are one.

There is no such thing as the subject that thinks or entertains ideas.

If I wrote a book called The World as I found it, I should have to include a report on my body, and should have to say which parts were subordinate to my will, and which were not, etc., this being a method of isolating the subject, or rather of showing that in an important sense there is no subject; for it alone could not be mentioned in that book.—

The subject does not belong to the world: rather, it is a limit of the world.

Where in the world is a metaphysical subject to be found?

You will say that this is exactly like the case of the eye and the visual field. But really you do not see the eye.

And nothing in the visual field allows you to infer that it is seen by an eye.

For the form of the visual field is surely
wa eine solche Form:

Auge —

5.634

Das hängt damit zusammen, dass kein Teil unserer Erfahrung auch a priori ist.

Alles, was wir sehen, könnte auch anders sein.

Alles, was wir überhaupt beschreiben können, könnte auch anders sein.

Es gibt keine Ordnung der Dinge a priori.

Hier sieht man, dass der Solipsismus, streng durchgeführt, mit dem reinen Realismus zusammenfällt. Das Ich des Solipsismus schrumpft zum ausdehnungslosen Punkt zusammen, und es bleibt die ihm koordinierte Realität.

Es gibt also wirklich einen Sinn, in welchem in der Philosophie nichtspsychologisch vom Ich die Rede sein kann.

Das Ich tritt in die Philosophie dadurch ein, dass „die Welt meine Welt ist“.

Das philosophische Ich ist nicht der Mensch, nicht der menschliche Körper, oder die menschliche Seele, von der die Psychologie handelt, sondern das metaphysische Subjekt, die Grenze – nicht ein Teil – der Welt.

Die allgemeine Form der Wahrheitsfunktion ist: \([\overline{p}, \bar{\xi}, N(\bar{\xi})]\).

Dies sagt nichts anderes, als dass jeder

5.64

This is connected with the fact that no part of our experience is also a priori.

Everything we see could also be otherwise.

Everything we describe at all could also be otherwise.

There is no order of things a priori.

Here we see that solipsism strictly carried out coincides with pure realism. The I in solipsism shrinks to an extensionless point and there remains the reality co-ordinated with it.

There is therefore really a sense in which the philosophy we can talk of a non-psychological I.

The I occurs in philosophy through the fact that the “world is my world”.

The philosophical I is not the man, not the human body or the human soul of which psychology treats, but the metaphysical subject, the limit—not a part of the world.

The general form of truth-function is: \([\overline{p}, \bar{\xi}, N(\bar{\xi})]\).

This is the general form of proposition.

This says nothing else than that every

5.641

This is connected with the fact that no part of our experience is at the same time a priori.

Whatever we see could be other than it is.

Whatever we can describe at all could be other than it is.

There is no a priori order of things.

Here it can be seen that solipsism, when its implications are followed out strictly, coincides with pure realism. The self of solipsism shrinks to a point without extension, and there remains the reality co-ordinated with it.

Thus there really is a sense in which philosophy can talk about the self in a non-psychological way.

What brings the self into philosophy is the fact that ‘the world is my world’.

The philosophical self is not the human being, not the human body, or the human soul, with which psychology deals, but rather the metaphysical subject, the limit of the world—not a part of it.

The general form of a truth-function is \([\overline{p}, \bar{\xi}, N(\bar{\xi})]\).

This is the general form of a proposition.

What this says is just that every propo-
6.002

A proposition is the result of successive applications of the operation $N(\xi)$ to the elementary propositions.

If we are given the general form of the way in which a proposition is constructed, then thereby we are also given the general form of the way in which by an operation out of one proposition another can be created.

The general form of the operation $\Omega(\eta)$ is therefore: $[\xi, N(\xi)](\eta) = [\eta, \xi, N(\xi)]$.

This is the most general form of transition from one proposition to another.

And thus we come to numbers: I define:

$x = \Omega^0 x$ Def.

$\Omega \Omega^y x = \Omega^{y+1} x$ Def.

According, then, to these symbolic rules we write the series

$x, \Omega x, \Omega \Omega x, \Omega \Omega \Omega x, \ldots$

Therefore I write in place of $\{x, \xi, \Omega \xi\}$:

$\{\Omega^0 x, \Omega^y x, \Omega^{y+1} x\}$

And I define:

$0 + 1 = 1$ Def.

$0 + 1 + 1 = 3$ Def.

(\text{u. s. f.)}

6.01

The concept of number is simply what is common to all numbers.

Die Zahl ist der Exponent einer Operation.

A number is the exponent of an operation.

6.02

Die allgemeine Form der Operation $\Omega(\eta)$ ist also: $[\xi, N(\xi)](\eta) = [\eta, \xi, N(\xi)]$.

Das ist die allgemeinste Form der Übergänge von einem Satz zum anderen.

And s o kommen wir zu den Zahlen:

Ich definiere

$x = \Omega^0 x$ Def. und

$\Omega \Omega^y x = \Omega^{y+1} x$ Def.

Nach diesen Zeichenregeln schreiben wir also die Reihe

$x, \Omega x, \Omega \Omega x, \Omega \Omega \Omega x, \ldots$

so:

$\Omega^0 x, \Omega^0+1 x, \Omega^{0+1+1} x, \Omega^{0+1+1+1} x, \ldots$

Also schreibe ich statt $\{x, \xi, \Omega \xi\}$:

$\{\Omega^0 x, \Omega^y x, \Omega^{y+1} x\}$

Und definiere:

$0 + 1 = 1$ Def.

$0 + 1 + 1 = 2$ Def.

$0 + 1 + 1 + 1 = 3$ Def.

(\text{u. s. f.)}

6.021

Die Zahl ist der Exponent einer Operation.

A number is the exponent of an operation.

6.022

Der Zahlbegriff ist nichts anderes als das Gemeinsame aller Zahlen, die allgemeinste Form der Übergänge von einem Satz zum anderen.

And this is how we arrive at numbers.

I give the following definitions

$x = \Omega^0 x$ Def.

$\Omega \Omega^y x = \Omega^{y+1} x$ Def.

So, in accordance with these rules, which deal with signs, we write the series

$x, \Omega x, \Omega \Omega x, \Omega \Omega \Omega x, \ldots$

in the following way

$\Omega^0 x, \Omega^{0+1} x, \Omega^{0+1+1} x, \Omega^{0+1+1+1} x, \ldots$

Therefore, instead of $\{x, \xi, \Omega \xi\}$, I write $\{\Omega^0 x, \Omega^y x, \Omega^{y+1} x\}$.

And I give the following definitions

$0 + 1 = 1$ Def.,

$0 + 1 + 1 = 2$ Def.,

$0 + 1 + 1 + 1 = 3$ Def.,

(\text{and so on).}

A number is the exponent of an operation.

The concept of number is simply what is common to all numbers, the general form
meine Form der Zahl.
Der Zahlbegriff ist die variable Zahl.

Und der Begriff der Zahlengleichheit ist die allgemeine Form aller speziellen Zahlengleichheiten.

6.03
Die allgemeine Form der ganzen Zahl ist: $[0, \xi, \xi+1]$.

6.031
Die Theorie der Klassen ist in der Mathematik ganz überflüssig.
Dies hängt damit zusammen, dass die Allgemeinheit, welche wir in der Mathematik brauchen, nicht die zufällige ist.

6.1
Die Sätze der Logik sind Tautologien.

6.11
Die Sätze der Logik sagen also Nichts.
(Sie sind die analytischen Sätze.)

6.111

6.112
Die richtige Erklärung der logischen Sätze ist, dass sie der formale Charakter des Satzes haben and the concept of numerical equality is the general form of all particular cases of numerical equality.

6.03
The concept number is the variable number.

6.031
And the concept of equality of numbers is the general form of all special equalities of numbers.

6.1
The general form of the cardinal number is: $[0, \xi, \xi+1]$.

6.11
The theory of classes is altogether superfluous in mathematics.
This is connected with the fact that the generality which we need in mathematics is not the accidental one.

6.111
The propositions of logic are tautologies.
Therefore the propositions of logic say nothing. (They are the analytical propositions.)

6.112
The correct explanation of logical propositions is, that they possess a character quite different from that of a proposition of natural science and this is a certain symptom of its being falsely understood.

6.112
The correct explanation of the proposi-
Sätze muss ihnen eine einzigartige Stellung unter allen Sätzen geben.

Es ist das besondere Merkmal der logischen Sätze, dass man am Symbol allein erkennen kann, dass sie wahr sind, und diese Tatsache schließt die ganze Philosophie der Logik in sich. Und so ist es auch eine der wichtigsten Tatsachen, dass sich die Wahrheit oder Falscheit der nichtlogischen Sätze nicht am Satz allein erkennen lässt.

Dass die Sätze der Logik Tautologien sind, das zeigt die formalen – logischen – Eigenschaften der Sprache, der Welt.

Dass ihre Bestandteile so verknüpft eine Tautologie ergeben, das charakterisiert die Logik ihrer Bestandteile.

Damit Sätze, auf bestimmte Art und Weise verknüpft, eine Tautologie ergeben, dazu müssen sie bestimmte Eigenschaften der Struktur haben. Dass sie so verbunden eine Tautologie ergeben, zeigt also, dass sie diese Eigenschaften der Struktur besitzen.


Es ist klar, dass man zu demselben

6.1201

positions must give them a peculiar position among all propositions.

It is the characteristic mark of logical propositions that one can perceive in the symbol alone that they are true; and this fact contains in itself the whole philosophy of logic. And so also it is one of the most important facts that the truth or falsehood of non-logical propositions can not be recognized from the propositions alone.

The fact that the propositions of logic are tautologies shows the formal—logical—properties of language, of the world.

That its constituent parts connected together in this way give a tautology characterizes the logic of its constituent parts.

In order that propositions connected together in a definite way may give a tautology they must have definite properties of structure. That they give a tautology when so connected shows therefore that they possess these properties of structure.

That e.g. the propositions “p” and “¬p” in the connexion “¬(p . ¬p)” give a tautology shows that they contradict one another. That the propositions “p ⊃ q”, “p” and “¬q” connected together in the form “(p ⊃ q) . (p) ≡ (q)” give a tautology shows that q follows from p and p ⊃ q. That “(x). fx ≡ fa” is a tautology shows that fa follows from (x). fx, etc. etc.

It is clear that we could have used for

positions of logic must assign to them a unique status among all propositions.

It is the peculiar mark of logical propositions that one can recognize that they are true from the symbol alone, and this fact contains in itself the whole philosophy of logic. And so too it is a very important fact that the truth or falsity of non-logical propositions cannot be recognized from the propositions alone.

The fact that the propositions of logic are tautologies shows the formal—logical—properties of language and the world.

The fact that a tautology is yielded by this particular way of connecting its constituents characterizes the logic of its constituents.

If propositions are to yield a tautology when they are connected in a certain way, they must have certain structural properties. So their yielding a tautology when combined in this way shows that they possess these structural properties.

For example, the fact that the propositions ‘p’ and ‘¬p’ in the combination ‘¬(p . ¬p)” yield a tautology shows that they contradict one another. The fact that the propositions ‘p ⊃ q’, ‘p’, and ‘¬q’, combined with one another in the form ‘(p ⊃ q) . (p) ≡ (¬q)’, yield a tautology shows that q follows from p and p ⊃ q. The fact that ‘(x). fx ≡ fa’ is a tautology shows that fa follows from (x). fx. Etc. etc.

It is clear that one could achieve the
Zweck statt der Tautologien auch die Kontradiktionen verwenden könnte.

Um eine Tautologie als solche zu erkennen, kann man sich, in den Fällen, in welchen in der Tautologie keine Allgemeinheitsbezeichnung vorkommt, folgender anschaulichen Methode bedienen: Ich schrei- be statt „p“, „q“, „r“ etc. „Wp F“, „Wq F“, „Wr F“ etc. Die Wahrheitskombinationen drücke ich durch Klammern aus, z. B.:

\[\begin{align*}
Wp F & \quad Wq F \\
Tp F & \quad Tq F
\end{align*}\]

und die Zuordnung der Wahr- oder Falschheit des ganzen Satzes und der Wahrheitskombinationen der Wahrheitsargumente durch Striche auf folgende Weise:

\[\begin{align*}
F & \quad F \\
Wp F & \quad Wq F \\
W & \quad W
\end{align*}\]

Dies Zeichen würde also z. B. den Satz \(p \supset q\) darstellen. Nun will ich z. B. den Satz \(\neg(p \cdot \neg p)\) (Gesetz des Widerspruchs) daraufhin untersuchen, ob er eine Tautologie ist. Die Form „\(\neg\xi\)“ wird in unserer Notation this purpose contradictions instead of tautologies.

In order to recognize a tautology as such, we can, in cases in which no sign of generality occurs in the tautology, make use of the following intuitive method: I write instead of “p”, “q”, “r”, etc., “Tp F”, “Tq F”, “Tr F”, etc. The truth-combinations I express by brackets, e.g.:

\[\begin{align*}
Tp F & \quad Tq F
\end{align*}\]

and the co-ordination of the truth or falsity of the whole proposition with the truth-combinations of the truth-arguments by lines in the following way:

\[\begin{align*}
F & \quad F \\
Wp F & \quad Wq F \\
W & \quad W
\end{align*}\]

This sign, for example, would therefore present the proposition \(p \supset q\). Now I will proceed to inquire whether such a proposition as \(\neg(p \cdot \neg p)\) (The Law of Contradiction) is a tautology. The form “\(\neg\xi\)” is written in our notation

\[\begin{align*}
T & \quad T
\end{align*}\]

and I use lines to express the correlation of the truth or falsity of the whole proposition with the truth-combinations of its truth-arguments, in the following way

\[\begin{align*}
F & \quad F \\
Wp F & \quad Wq F \\
W & \quad W
\end{align*}\]

So this sign, for instance, would represent the proposition \(p \supset q\). Now, by way of example, I wish to examine the proposition \(\neg(p \cdot \neg p)\) (the law of contradiction) in order to determine whether it is a tautology. In our notation the form “\(\neg\xi\)” is written as
geschrieben; die Form „ξ . η“ so:

Daher lautet der Satz \(\neg (p \land \neg q)\) so:

Setzen wir hier statt „\(q\)“ „\(p\)“ ein und untersuchen die Verbindung der äußersten \(W\) und \(F\) mit den innersten, so ergibt sich, dass die Wahrheit des ganzen Satzes allen Wahrheitskombinationen seines Argumentes, seine Falschheit keiner der

If here we put „\(p\)“ instead of „\(q\)“ and examine the combination of the outermost \(T\) and \(F\) with the innermost, it is seen that the truth of the whole proposition is coordinated with all the truth-combinations of its argument, its falsity with none of the

If we here substitute „\(p\)“ for „\(q\)“ and examine how the outermost \(T\) and \(F\) are connected with the innermost ones, the result will be that the truth of the whole proposition is correlated with all the truth-combinations of its argument, and its falsity with none
Wahrheitskombinationen zugeordnet ist.

Die Sätze der Logik demonstrieren die logischen Eigenschaften der Sätze, indem sie zu nichtssagenden Sätzen verbinden.

Diese Methode könnte man auch eine Nullmethode nennen. Im logischen Satz werden Sätze miteinander ins Gleichgewicht gebracht und der Zustand des Gleichgewichts zeigt dann an, wie diese Sätze logisch beschaffen sein müssen.

Daraus ergibt sich, dass wir auch ohne die logischen Sätze auskommen können, da wir ja in einer entsprechenden Notation die formalen Eigenschaften der Sätze durch das bloße Ansehen dieser Sätze erkennen können.


Dies wirft ein Licht auf die Frage, warum die logischen Sätze nicht durch die Erfahrung bestätigt werden können, ebenso wenig, wie sie durch die Erfahrung widerlegt werden können. Nicht nur muss ein Satz der Logik durch keine mögliche Erfahrung widerlegt werden können, sondern er darf auch nicht durch eine solche bestätigt werden können.

Now it becomes clear why we often of the truth-combinations.

The propositions of logic demonstrate the logical properties of propositions by combining them into propositions which say nothing.

This method could also be called a zero-method. In a logical proposition propositions are brought into equilibrium with one another, and the state of equilibrium then shows how these propositions must be logically constructed.

Whence it follows that we can get on without logical propositions, for we can recognize in an adequate notation the formal properties of the propositions by mere inspection.

If for example two propositions "p" and "q" give a tautology in the connexion "p ⊃ q", then it is clear that q follows from p.

E.g. that "q" follows from "p ⊃ q . p" we see from these two propositions themselves, but we can also show it by combining them to "p ⊃ q . p ⊴ q" and then showing that this is a tautology.

This throws light on the question why logical propositions can no more be empirically confirmed than they can be empirically refuted. Not only must a proposition of logic be incapable of being contradicted by any possible experience, but it must also be incapable of being confirmed by any such.

It now becomes clear why people have
Es wird jetzt auch klar, warum die Logik die Lehre von den Formen und vom Schließen genannt wurde. Es ist klar: Die logischen Gesetze dürfen nicht selbst wieder logischen Gesetzen unterstehen. 

(6.1224)


We can imagine a world in which the axiom of reducibility is not valid. But it is clear that logic has nothing to do with the question whether our world is really of this kind or not.

(6.123)

Die logischen Sätze beschreiben das Geist der Welt, oder vielmehr, sie stellen es dar. Sie „handeln“ von nichts. Sie setzen feel as though “logical truths” must be “postulated” by us. We can in fact postulate them in so far as we can postulate an adequate notation.

It also becomes clear why logic has been called the theory of forms and of inference.

It is clear that the laws of logic cannot themselves obey further logical laws.

(There is not, as Russell supposed, for every “type” a special law of contradiction; but one is sufficient, since it is not applied to itself.)

The mark of logical propositions is not their general validity.

To be general is only to be accidentally valid for all things. An ungeneralized proposition can be tautological just as well as a generalized one.

Logical general validity, we could call essential as opposed to accidental general validity, e.g. of the proposition “all men are mortal”. Propositions like Russell’s “axiom of reducibility” are not logical propositions, and this explains our feeling that, if true, they can only be true by a happy chance.

We can imagine a world in which the axiom of reducibility is not valid. But it is clear that logic has nothing to do with the question whether our world is really of this kind or not.

The logical propositions describe the scaffolding of the world, or rather they present it. They “treat” of nothing. They often felt as if it were for us to ‘postulate’ the ‘truths of logic’. The reason is that we can postulate them in so far as we can postulate an adequate notation.

Clearly the laws of logic cannot in their turn be subject to laws of logic.

(There is not, as Russell thought, a special law of contradiction for each ‘type’; one law is enough, since it is not applied to itself.)

The mark of a logical proposition is not general validity.

To be general means no more than to be accidentally valid for all things. An ungeneralized proposition can be tautological just as well as a generalized one.

The general validity of logic might be called essential, in contrast with the accidental general validity of such propositions as ‘All men are mortal’. Propositions like Russell’s ‘axiom of reducibility’ are not logical propositions, and this explains our feeling that, even if they were true, their truth could only be the result of a fortunate accident.

It is possible to imagine a world in which the axiom of reducibility is not valid. It is clear, however, that logic has nothing to do with the question whether our world really is like that or not.

The propositions of logic describe the scaffolding of the world, or rather they represent it. They have no ‘subject-matter’. 

Das Anzeichen des logischen Satzes ist nicht die Allgemeingültigkeit.


6.1231

6.1232

6.1233

6.124

6.125
Es ist möglich, und zwar auch nach der alten Auffassung der Logik, von vornherein eine Beschreibung aller „wahren“ logischen Sätze zu geben.

6.1251 Darum kann es in der Logik auch nie Überraschungen geben.

6.126 Ob ein Satz der Logik angehört, kann man berechnen, indem man die logischen Eigenschaften des Symbols berechnet.

Und dies tun wir, wenn wir einen logischen Satz „beweisen“. Denn, ohne uns um einen Sinn und eine Bedeutung zu kümmern, bilden wir den logischen Satz aus anderen nach bloßen Zeichenregeln.

Der Beweis der logischen Sätze besteht darin, dass wir sie aus anderen logischen Sätzen durch successive Anwendung gewisser Operationen entstehen lassen, die

6.1261 It is possible, also with the old conception of logic, to give at the outset a description of all “true” logical propositions.

6.1262 Hence there can never be surprises in logic.

Whether a proposition belongs to logic can be calculated by calculating the logical properties of the symbol.

And this we do when we prove a logical proposition. For without troubling ourselves about a sense and a meaning, we form the logical propositions out of others by mere symbolic rules.

We prove a logical proposition by creating it out of other logical propositions by applying in succession certain operations, which again generate tautologies out of the

They presuppose that names have meaning and elementary propositions have sense; and that is their connexion with the world. It is clear that something about the world must be indicated by the fact that certain combinations of symbols—whose essence involves the possession of a determinate character—are tautologies. This contains the decisive point. We have said that some things are arbitrary in the symbols that we use and that some things are not. In logic it is only the latter that express: but that means that logic is not a field in which we express what we wish with the help of signs, but rather one in which the nature of the natural and inevitable signs speaks for itself. If we know the logical syntax of any sign-language, then we have already been given all the propositions of logic.

It is possible—indeed possible even according to the old conception of logic—to give in advance a description of all ‘true’ logical propositions.

Hence there can never be surprises in logic.

One can calculate whether a proposition belongs to logic, by calculating the logical properties of the symbol.

And this is what we do when we ‘prove’ a logical proposition. For, without bothering about sense or meaning, we construct the logical proposition out of others using only rules that deal with signs.

The proof of logical propositions consists in the following process: we produce them out of other logical propositions by successively applying certain operations.
aus den ersten immer wieder Tautologien erzeugen. (Und zwar **folgen** aus einer Tautologie nur Tautologien.)


**6.1261**

In der Logik sind Prozess und Resultat äquivalent. (Darum keine Überraschung.)

**6.1262**

Der Beweis in der Logik ist nur ein mechanisches Hilfsmittel zum leichteren Erkennen der Tautologie, wo sie kompliziert ist.

**6.1263**

Es wäre ja auch zu merkwürdig, wenn man einen sinnvollen Satz **logisch** aus anderen beweisen könnte, und einen logischen Satz **auch**. Es ist vorzuherrschein klar, dass der logische Beweis eines sinnvollen Satzes und der Beweis in der Logik zwei ganz verschiedene Dinge sein müssen.

**6.1264**

Der sinnvolle Satz sagt etwas aus, und sein Beweis zeigt, dass es so ist; in der Logik ist jeder Satz die Form eines Beweises.

Jeder Satz der Logik ist ein in Zeichen dargestellter modus ponens. (Und den modus ponens kann man nicht durch einen Satz ausdrücken.)

**6.1265**

Immer kann man die Logik so auffassen, dass jeder Satz sein eigener Beweis ist.

**6.127**

Alle Sätze der Logik sind gleichberechtigt, es gibt unter ihnen nicht wesentlich Grundgesetze und abgeleitete Sätze.

first. (And from a tautology only tautologies **follow**.)

Naturally this way of showing that its propositions are tautologies is quite unessential to logic. Because the propositions, from which the proof starts, must show without proof that they are tautologies.

In logic process and result are equivalent. (Therefore no surprises.)

Proof in logic is only a mechanical expedient to facilitate the recognition of tautology, where it is complicated.

It would be too remarkable, if one could prove a significant proposition **logically** from another, and a logical proposition **also**. It is clear from the beginning that the logical proof of a significant proposition and the proof in logic must be two quite different things.

The significant proposition asserts something, and its proof shows that it is so; in logic every proposition is the form of a proof.

Every proposition of logic is a modus ponens presented in signs. (And the modus ponens can not be expressed by a proposition.)

Logic can always be conceived to be such that every proposition is its own proof.

All propositions of logic are of equal rank; there are not some which are essentially primitive and others deduced from that always generate further tautologies out of the initial ones. (And in fact only tautologies **follow** from a tautology.)

Of course this way of showing that the propositions of logic are tautologies is not at all essential to logic, if only because the propositions from which the proof starts must show without any proof that they are tautologies.

In logic process and result are equivalent. (Hence the absence of surprise.)

Proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases.

Indeed, it would be altogether too remarkable if a proposition that had sense could be proved **logically** from others, and so too could a logical proposition. It is clear from the start that a logical proof of a proposition that has sense and a proof in logic must be two entirely different things.

A proposition that has sense states something, which is shown by its proof to be so. In logic every proposition is the form of a proof.

Every proposition of logic is a *modus ponens* represented in signs. (And one cannot express the *modus ponens* by means of a proposition.)

It is always possible to construe logic in such a way that every proposition is its own proof.

All the propositions of logic are of equal status: it is not the case that some of them are essentially derived propositions.
Jede Tautologie zeigt selbst, dass sie eine Tautologie ist.

6.1271
Es ist klar, dass die Anzahl der „logischen Grundgesetze“ willkürlich ist, denn man könnte die Logik ja aus Einem Grundgesetz ableiten, indem man einfach z. B. aus Freges Grundgesetzen das logische Produkt bildet. (Freges wäre vielleicht sagen, dass dieses Grundgesetz nun nicht mehr unmittelbar einleuchte. Aber es ist merkwürdig, dass die Logik ja aus Einem Grundgesetz ableiten, indem man einfach z. B. aus Freges Grundgesetzen das logische Produkt bildet. (Freges würde vielleicht sagen, dass dies Grundgesetz nun nicht mehr unmittelbar einleuchte. Aber es ist merkwürdig, dass ein so exakter Denker wie Frege sich auf den Grad des Einleuchtens als Kriterium des logischen Satzes berufen hat.)

6.13
Die Logik ist keine Lehre, sondern ein Spiegelbild der Welt.
Die Logik ist transcendental.

6.2
Die Mathematik ist eine logische Methode.
Die Sätze der Mathematik sind Gleichungen, also Scheinsätze.

6.21
Der Satz der Mathematik drückt keinen Gedanken aus.

6.211
Im Leben ist es ja nie die mathematische Satz, den wir brauchen, sondern wir benützen den mathematischen Satz nur, um aus Sätzen, welche nicht der Mathematik gehöre, auf andere zu schließen, welche gleichfalls nicht der Mathematik gehöre.
(In der Philosophie führt die Frage: „wozu gebrauchen wir eigentlich jenes Wort, jenen Satz“ immer wieder zu wertvollen Einsichten.)

6.22
Die Logik der Welt, die die Sätze der there.

Every tautology itself shows that it is a tautology.

It is clear that the number of “primitive propositions of logic” is arbitrary, for we could deduce logic from one primitive proposition by simply forming, for example, the logical product of Frege’s primitive propositions. (Freges would perhaps say that this would no longer be immediately self-evident. But it is remarkable that so exact a thinker as Frege should have appealed to the degree of self-evidence as the criterion of a logical proposition.)

Logic is not a theory but a reflexion of the world.
Logic is transcendental.

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Die Logik der Welt, die die Sätze der there.

Every tautology itself shows that it is a tautology.

It is clear that the number of the ‘primitive propositions of logic’ is arbitrary, since one could derive logic from a single primitive proposition, e.g. by simply constructing the logical product of Frege’s primitive propositions. (Freges would perhaps say that we should then no longer have an immediately self-evident primitive proposition. But it is remarkable that a thinker as rigorous as Frege appealed to the degree of self-evidence as the criterion of a logical proposition.)

Logic is not a body of doctrine, but a mirror-image of the world.
Logic is transcendental.

Mathematics is a logical method.
The propositions of mathematics are equations, and therefore pseudo-propositions.
Mathematical propositions express no thoughts.

A proposition of mathematics does not express a thought.

In life it is never a mathematical proposition which we need, but we use mathematical propositions only in order to infer from propositions which do not belong to mathematics to others which equally do not belong to mathematics.

(In philosophy the question “Why do we really use that word, that proposition?” constantly leads to valuable results.)

The logic of the world which the propo-

(In philosophy the question, ‘What do we actually use this word or this proposition for?’ repeatedly leads to valuable insights.)

The logic of the world, which is shown
Logik in den Tautologien zeigen, zeigt die Mathematik in den Gleichungen.

Wenn zwei Ausdrücke durch das Gleichheitszeichen verbunden werden, so heißt das, sie sind durch einander ersetzbar. Ob dies aber der Fall ist, muss sich an den beiden Ausdrücken selbst zeigen.

Es charakterisiert die logische Form zweier Ausdrücke, dass sie durch einander ersetzbar sind.

Es ist eine Eigenschaft der Bejahung, dass man sie als doppelte Verneinung auffassen kann.

Es ist eine Eigenschaft von "1 + 1 + 1 + 1", dass man es als "(1 + 1) + (1 + 1)" auffassen kann.

Frege sagt, die beiden Ausdrücke haben dieselbe Bedeutung, aber verschiedenen Sinn.

Das Wesentliche an der Gleichung ist aber, dass sie nicht notwendig ist, um zu zeigen, dass die beiden Ausdrücke, die das Gleichheitszeichen verbindet, dieselbe Bedeutung haben, da sich dies aus den beiden Ausdrücken selbst ersehen lässt.

Und, dass die Sätze der Mathematik bewiesen werden können, heißt ja nichts anderes, als dass ihre Richtigkeit einzusehen ist, ohne dass das, was sie ausdrücken, selbst mit den Tatsachen auf seine Richtigkeit hin verglichen werden muss.

Die Identität der Bedeutung zweier Ausdrücke lässt sich nicht behaupten. Denn, um etwas von ihrer Bedeutung behaupten zu können, muss ich ihre Bedeutung kennen: und indem ich ihre Bedeutung kennen, sitions of logic show in tautologies, mathematics shows in equations.

If two expressions are connected by the sign of equality, this means that they can be substituted for one another. But whether this is the case must show itself in the two expressions themselves.

It characterizes the logical form of two expressions, that they can be substituted for one another.

It is a property of affirmation that it can be conceived as double denial.

It is a property of "1 + 1 + 1 + 1" that it can be conceived as "(1 + 1) + (1 + 1)".

Frege says that these expressions have the same meaning but different senses.

But what is essential about equation is that it is not necessary in order to show that both expressions, which are connected by the sign of equality, have the same meaning: for this can be perceived from the two expressions themselves.

And, that the propositions of mathematics can be proved means nothing else than that their correctness can be seen without our having to compare what they express with the facts as regards correctness.

The identity of the meaning of two expressions cannot be asserted. For in order to be able to assert anything about their meaning, I must know their meaning, and if I know their meaning, I know whether in tautologies by the propositions of logic, is shown in equations by mathematics.

If two expressions are combined by means of the sign of equality, that means that they can be substituted for one another. But it must be manifest in the two expressions themselves whether this is the case or not.

When two expressions can be substituted for one another, that characterizes their logical form.

It is a property of affirmation that it can be construed as double negation.

It is a property of ‘1 + 1 + 1 + 1’ that it can be construed as ‘(1 + 1) + (1 + 1)’.

Frege says that the two expressions have the same meaning but different senses.

But the essential point about an equation is that it is not necessary in order to show that the two expressions connected by the sign of equality have the same meaning, since this can be seen from the two expressions themselves.

And the possibility of proving the propositions of mathematics means simply that their correctness can be perceived without its being necessary that what they express should itself be compared with the facts in order to determine its correctness.

It is impossible to assert the identity of meaning of two expressions. For in order to be able to assert anything about their meaning, I must know their meaning, and I cannot know their meaning without know-
Die Gleichung kennzeichnet nur den Standpunkt, von welchem ich die beiden Ausdrücke betrachte, nämlich vom Standpunkte ihrer Bedeutungsgleichheit.

Die Frage, ob man zur Lösung der mathematischen Probleme die Anschauung brauche, muss dahin beantwortet werden, dass eben die Sprache hier die nötige Anschauung liefert.

Der Vorgang des Rechnens vermittelt eben diese Anschauung.

Die Rechnung ist kein Experiment.

Die Mathematik ist eine Methode der Logik.

Das Wesentliche der mathematischen Methode ist es, mit Gleichungen zu arbeiten. Auf dieser Methode beruht es nämlich, dass jeder Satz der Mathematik sich von selbst verstehen muss.

Die Methode der Mathematik, zu ihren Gleichungen zu kommen, ist die Substitutionsmethode.

Denn die Gleichungen drücken die Ersetzbarkeit zweier Ausdrücke aus und wir schreiten von einer Anzahl von Gleichungen zu neuen Gleichungen vor, indem wir, den Gleichungen entsprechend, Ausdrücke durch andere ersetzen.

So lautet der Beweis des Satzes $2 \times 2 = 4$:

$$(\Omega^\nu)^\mu x = \Omega^{\nu \times \mu} x \text{ Def.}$$

$$\Omega^{2 \times 2} x = (\Omega^2)^2 x = (\Omega^2)^{1+1} x = \Omega^2 \Omega^2 x$$

$$= \Omega^{1+1} \Omega^{1+1} x = (\Omega \Omega)(\Omega \Omega)x$$

$$= \Omega^1 \Omega \Omega^1 \Omega^1 x = \Omega^2 \Omega^2 \Omega x = \Omega^4 x.$$
Logical research means the investigation of *all regularity*. And outside logic all is accident.

The so-called law of induction cannot in any case be a logical law, for it is obviously a significant proposition.—And therefore it cannot be a law a priori either.

The law of causality is not a law but the form of a law.*

*"Law of Causality" is a class name. And just as in mechanics, for example, there are ‘minimum-principles’, such as the law of least action, so in physics there are causal laws, laws of the causality form.

Men had indeed an idea that there must be a "law of least action", before they knew exactly how it ran. (Here, as always, the a priori certain proves to be something purely logical.)

We do not believe a priori in a law of conservation, but we know a priori the possibility of a logical form.

All propositions, such as the law of causation, the law of continuity in nature, the law of least expenditure in nature, etc. etc., all these are a priori intuitions of possible forms of the propositions of science.

Newtonian mechanics, for example, brings the description of the universe to a unified form. Let us imagine a white surface.

The exploration of logic means the exploration of *everything that is subject to law*. And outside logic everything is accidental.

The so-called law of induction cannot possibly be a law of logic, since it is obviously a proposition with sense.—Nor, therefore, can it be an *a priori* law.

The law of causality is not a law but the form of a law.

‘Law of causality’—that is a general name. And just as in mechanics, for example, there are causal laws, laws of the causality form.

Indeed people even surmised that there must be a ‘law of least action’ before they knew exactly how it went. (Here, as always, what is certain *a priori* proves to be something purely logical.)

We do not have an *a priori belief* in a law of conservation, but rather a *a priori knowledge* of the possibility of a logical form.

All such propositions, including the principle of sufficient reason, the laws of continuity in nature and of least effort in nature, etc. etc.—all these are *a priori* insights about the forms in which the propositions of science can be cast.

Newtonian mechanics, for example, imposes a unified form on the description of the world. Let us imagine a white surface

(Wie man mit dem Zahlensystem jede beliebige Anzahl, so muss man mit dem System der Mechanik jeden beliebigen Satz der Physik hinschreiben können.)
Die Mechanik ist ein Versuch, alle wahren Sätze, die wir zur Weltbeschreibung brauchen, nach Einem Plane zu konstruieren.

Durch den ganzen logischen Apparat hindurch sprechen die physikalischen Gesetze doch von den Gegenständen der Welt.

Wir dürfen nicht vergessen, dass die Weltbeschreibung durch die Mechanik immer die ganz allgemeine ist. Es ist in ihr z. B. nie von bestimmten materiellen Punkten die Rede, sondern immer nur von irgendwelchen.

Obwohl die Flecke in unserem Bild geometrische Figuren sind, so kann die Mechanik nicht von ihren geometrischen Eigenschaften reden, sondern nur von den geometrischen Eigenschaften der Punkte.

Mechanics is an attempt to construct according to a single plan all true propositions which we need for the description of the world.

Through their whole logical apparatus the physical laws still speak of the objects of the world.

We must not forget that the description of the world by mechanics is always quite general. There is, for example, never any mention of particular material points in it, but always only of some points or other.

Although the spots in our picture are geometrical figures, geometry can obviously not speak of them in their geometrical properties, but only in their geometrical properties as points.

The laws of physics, with all their logical apparatus, still speak, however indirectly, about the objects of the world.

We ought not to forget that any description of the world by means of mechanics will be of the completely general kind. For example, it will never mention particular point-masses: it will only talk about any point-masses whatsoever.

Although the spots in our picture are geometrical figures, nevertheless geome-

6.342

Und nun sehen wir die gegenseitige Stellung von Logik und Mechanik. (Man könnte das Netz auch aus verschiedenartigen Figuren etwa aus Dreiecken und Sechsecken bestehen lassen.) Dass sich ein Bild, wie das vorhin erwähnte, durch ein Netz von gegebener Form beschreiben lässt, sagt über das Bild nichts aus. (Denn dies gilt für jedes Bild dieser Art.) Das aber charakterisiert das Bild, dass es sich durch ein bestimmtes Netz von bestimmt Feinheit vollständig beschreiben lässt.

So auch sagt es nichts über die Welt aus, dass sie sich durch die Newtonsche Mechanik beschreiben lässt; wohl aber, dass sie sich so durch jene beschreiben lässt, wie dies eben der Fall ist. Auch das sagt etwas über die Welt, dass sie sich durch die eine Mechanik einfacher beschreiben lässt als durch die andere.

Die Mechanik ist ein Versuch, alle wahren Sätze, die wir zur Weltbeschreibung brauchen, nach Einem Plane zu konstruieren.

6.343

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6.3431

Durch den ganzen logischen Apparat hindurch sprechen die physikalischen Gesetze doch von den Gegenständen der Welt.

6.3432

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The laws of physics, with all their logical apparatus, still speak, however indirectly, about the objects of the world.

We ought not to forget that any description of the world by means of mechanics will be of the completely general kind. For example, it will never mention particular point-masses: it will only talk about any point-masses whatsoever.

Although the spots in our picture are geometrical figures, nevertheless geome-
doch selbstverständlich die Geometrie gar nichts über ihre tatsächliche Form und Lage sagen. Das Netz aber ist rein geometrisch, alle seine Eigenschaften können a priori angegeben werden.

Gesetze wie der Satz vom Grunde, etc. handeln vom Netz, nicht von dem, was das Netz beschreibt.

Wenn es ein Kausalitätsgesetz gäbe, so könnte es lauten: „Es gibt Naturgesetze“.

Aber freilich kann man das nicht sagen: es zeigt sich.

In der Ausdrucksweise Hertz’s könnte man sagen: Nur gesetzliche Zusammenhänge sind denkbar.


Daher ist die Beschreibung des zeitlichen Verlaufs nur so möglich, dass wir uns auf einen anderen Vorgang stützen.

Ganz Analoges gilt für den Raum. Wo man z. B. sagt, es könne keines von zwei Ereignissen (die sich gegenseitig ausschließen) eintreten, weil keine Ursache vorhanden sei, warum das eine eher als das andere eintreten solle, da handelt es sich in Wirklichkeit darum, dass man gar nicht eines der beiden Ereignisse beschreiben kann, wenn nicht irgend eine Asymmetrie vorhanden ist. Und wenn eine solche Asymmetrie vorhanden ist, so können wir diese als Ursache des Eintreffens des einen und Nicht-Eintreffens

truly say nothing about their actual form and position. But the network is purely geometrical, and all its properties can be given 
apriori.

Laws, like the law of causation, etc., treat of the network and not what the network describes.

If there were a law of causality, it might run: “There are natural laws”.

But that can clearly not be said: it shows itself.

In the terminology of Hertz we might say: Only uniform connections are thinkable.

We cannot compare any process with the “passage of time”—there is no such thing—but only with another process (say, with the movement of the chronometer).

Hence the description of the temporal sequence of events is only possible if we support ourselves on another process.

It is exactly analogous for space. When, for example, we say that neither of two events (which mutually exclude one another) can occur, because there is no cause why the one should occur rather than the other, it is really a matter of our being unable to describe one of the two events unless there is some sort of asymmetry. And if there is such an asymmetry, we can regard this as the cause of the occurrence of the one and of the non-occurrence of the other.

try can obviously say nothing at all about their actual form and position. The network, however, is purely geometrical; all its properties can be given a priori.

Laws like the principle of sufficient reason, etc. are about the net and not about what the net describes.

If there were a law of causality, it might be put in the following way: There are laws of nature.

But of course that cannot be said: it makes itself manifest.

One might say, using Hertz’s terminology, that only connexions that are subject to law are thinkable.

We cannot compare a process with ‘the passage of time’—there is no such thing—but only with another process (such as the working of a chronometer).

Hence we can describe the lapse of time only by relying on some other process.

Something exactly analogous applies to space: e.g. when people say that neither of two events (which exclude one another) can occur, because there is nothing to cause the one to occur rather than the other, it is really a matter of our being unable to describe one of the two events unless there is some sort of asymmetry to be found. And if such an asymmetry is to be found, we can regard it as the cause of the occurrence of the one and the non-occurrence of the other.
Das Kant'sche Problem von der rechten und linken Hand, die man nicht zur Deckung bringen kann, besteht schon in der Ebene, ja im eindimensionalen Raum, wo die beiden kongruenten Figuren $a$ und $b$ auch nicht zur Deckung gebracht werden können, ohne aus diesem Raum herausbewegt zu werden. Rechte und linke Hand sind tatsächlich vollkommen kongruent. Und dass man sie nicht zur Deckung bringen kann, hat damit nichts zu tun.

Den rechten Handschuh könnte man an die linke Hand ziehen, wenn man ihn im vierdimensionalen Raum umdrehen könnte.

Was sich beschreiben lässt, das kann auch geschehen, und was das Kausalitätsgesetz ausschließen soll, das lässt sich auch nicht beschreiben.

Der Vorgang der Induktion besteht darin, dass wir das einfachste Gesetz annehmen, das mit unseren Erfahrungen in Einklang zu bringen ist.

Dieser Vorgang hat aber keine logische, sondern nur eine psychologische Begründung.

Es ist klar, dass kein Grund vorhanden ist, zu glauben, es werde nun auch wirklich der einfachste Fall eintreten.

Dass die Sonne morgen aufgehen wird, ist eine Hypothese; und das heißt: wir

The Kantian problem of the right and left hand which cannot be made to cover one another already exists in the plane, and even in one-dimensional space; where the two congruent figures $a$ and $b$ cannot be made to cover one another without moving them out of this space. The right and left hand are in fact completely congruent. And the fact that they cannot be made to cover one another has nothing to do with it.

A right-hand glove could be put on the left hand if it could be turned round in four-dimensional space.

What can be described can happen too, and what is excluded by the law of causality cannot be described.

The process of induction is the process of assuming the simplest law that can be made to harmonize with our experience.

This process, however, has no logical foundation but only a psychological one.

It is clear that there are no grounds for believing that the simplest course of events will really happen.

That the sun will rise to-morrow, is an hypothesis; and that means that we do not

It is an hypothesis that the sun will rise tomorrow: and this means that we do not
wissen nicht, ob sie aufgehen wird.  

Einen Zwang, nach dem Eines geschehen müsste, weil etwas anderes geschehen ist, gibt es nicht. Es gibt nur eine logische Notwendigkeit.

Der ganzen modernen Weltanschauung liegt die Täuschung zugrunde, dass die sogenannten Naturgesetze die Erklärungen der Naturscheinungen seien.

So bleiben sie bei den Naturgesetzen als bei etwas Unantastbarem stehen, wie die Älteren bei Gott und dem Schicksal. 

Und sie haben ja beide Recht, und Unrecht. Die Alten sind allerdings insofern klarer, als sie einen klaren Abschluss anerkennen, während es bei dem neuen System scheinen soll, als sei alles erklärt.

Die Welt ist unabhängig von meinem Willen.

Auch wenn alles, was wir wünschen, geschiehe, so wäre dies doch nur, sozusagen, eine Gnade des Schicksals, denn es ist kein logischer Zusammenhang zwischen Willen und Welt, der dies verbürgte, und den angenommenen physikalischen Zusammenhang könnten wir doch nicht selbst wieder wollen.

Wie es nur eine logische Notwendigkeit gibt, so gibt es auch nur eine logische Unmöglichkeit.

Dass z. B. zwei Farben zugleich an einem Ort des Gesichtsfeldes sind, ist unmöglich, und zwar logisch unmöglich, denn es ist durch die logische Struktur der Farbe ausgeschlossen.

know whether it will rise. 

A necessity for one thing to happen because another has happened does not exist. There is only logical necessity.

At the basis of the whole modern view of the world lies the illusion that the so-called laws of nature are the explanations of natural phenomena.

So people stop short at natural laws as something unassailable, as did the ancients at God and Fate.

And they are both right and wrong. But the ancients were clearer, in so far as they recognized one clear terminus, whereas the modern system makes it appear as though everything were explained.

The world is independent of my will.

Even if everything we wished were to happen, this would only be, so to speak, a favour of fate, for there is no logical connexion between will and world, which would guarantee this, and the assumed physical connexion itself we could not again will.

As there is only a logical necessity, so there is only a logical impossibility.

For two colours, e.g. to be at one place in the visual field, is impossible, logically impossible, for it is excluded by the logical structure of colour.

know whether it will rise. 

There is no compulsion making one thing happen because another has happened. The only necessity that exists is logical necessity.

The whole modern conception of the world is founded on the illusion that the so-called laws of nature are the explanations of natural phenomena.

Thus people today stop at the laws of nature, treating them as something inviolable, just as God and Fate were treated in past ages.

And in fact both are right and both wrong: though the view of the ancients is clearer in so far as they have a clear and acknowledged terminus, while the modern system tries to make it look as if everything were explained.

The world is independent of my will.

Even if all that we wish for were to happen, still this would only be a favour granted by fate, so to speak: for there is no logical connexion between the will and the world, which would guarantee it, and the supposed physical connexion itself is surely not something that we could will.

Just as the only necessity that exists is logical necessity, so too the only impossibility that exists is logical impossibility.

For example, the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour.
Denken wir daran, wie sich dieser Widerspruch in der Physik darstellt: Ungefähr so, dass ein Teilchen nicht zu gleicher Zeit zwei Geschwindigkeiten haben kann; das heißt, dass es nicht zu gleicher Zeit an zwei Orten sein kann; das heißt, dass Teilchen an verschiedenen Orten zu Einer Zeit nicht identisch sein können.

(Es ist klar, dass das logische Produkt zweier Elementarsätze weder eine Tautologie noch eine Kontradiktion sein kann. Die Aussage, dass ein Punkt des Gesichtsfeldes zu gleicher Zeit zwei verschiedene Farben hat, ist eine Kontradiktion.)

Alle Sätze sind gleichwertig.

Der Sinn der Welt muss außerhalb ihrer liegen. In der Welt ist alles, wie es ist, und geschieht alles, wie es geschieht; es gibt ihn keinen Wert – und wenn es ihn gäbe, so hätte er keinen Wert.

Wenn es einen Wert gibt, der Wert hat, so muss er außerhalb alles Geschehens und So-Seins liegen. Denn alles Geschehen und So-Sein ist zufällig.

Was es nichtzufällig macht, kann nicht in der Welt liegen, denn sonst wäre dies wieder zufällig.

Es muss außerhalb der Welt liegen. Darum kann es auch keine Sätze der Ethik geben. Sätze können nichts Höheres ausdrücken.

Es ist klar, dass sich die Ethik nicht aussprechen lässt. Die Ethik ist transcendental. (Ethik und Ästhetik sind Eins.)

Let us consider how this contradiction presents itself in physics. Somewhat as follows: That a particle cannot at the same time have two velocities, i.e. that at the same time it cannot be in two places, i.e. that particles in different places at the same time cannot be identical.

It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The assertion that a point in the visual field has two different colours at the same time, is a contradiction.

All propositions are of equal value.

The sense of the world must lie outside the world. In the world everything is as it is and happens as it does happen. In it there is no value—and if there were, it would be of no value.

If there is a value which is of value, it must lie outside all happening and being-so. For all happening and being-so is accidental.

What makes it non-accidental cannot lie in the world, for otherwise this would again be accidental.

It must lie outside the world.

Hence also there can be no ethical propositions.

Propositions cannot express anything higher.

It is clear that ethics cannot be expressed.

Ethics is transcendental. (Ethics and æsthetics are one.)

Let us think how this contradiction appears in physics: more or less as follows—a particle cannot have two velocities at the same time; that is to say, it cannot be in two places at the same time; that is to say, particles that are in different places at the same time cannot be identical.

It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colours at the same time is a contradiction. All propositions are of equal value.

The sense of the world must lie outside the world. In the world everything is as it is, and everything happens as it does happen: in it no value exists—and if it did exist, it would have no value.

If there is any value that does have value, it must lie outside the whole sphere of what happens and is the case. For all that happens and is the case is accidental.

What makes it non-accidental cannot lie within the world, since if it did it would itself be accidental.

It must lie outside the world. So too it is impossible for there to be propositions of ethics. Propositions can express nothing that is higher. It is clear that ethics cannot be put into words.

Ethics is transcendental. (Ethics and æsthetics are one and the same.)

(UND das ist auch klar, dass der Lohn etwas Angenehmes, die Strafe etwas Unangenehmes sein muss.)

Vom Willen als dem Träger des Ethischen kann nicht gesprochen werden.

(UND der Wille als Phänomen interessiert nur die Psychologie.)

Wenn das gute oder böse Wollen die Welt ändert, so kann es nur die Grenzen der Welt ändern, nicht die Tatsachen; nicht das, was durch die Sprache ausgedrückt werden kann.

Kurz, die Welt muss dann dadurch überhaupt eine andere werden. Sie muss sozusagen als Ganze abnehmen oder zunehmen.

Die Welt des Glücklichen ist eine andere als die des Unglücklichen.

Wie auch beim Tod die Welt sich nicht ändert, sondern aufhört.

Der Tod ist kein Ereignis des Lebens. Den Tod erlebt man nicht.

The first thought in setting up an ethical law of the form “thou shalt . . . ” is: And what if I do not do it? But it is clear that ethics has nothing to do with punishment and reward in the ordinary sense. This question as to the consequences of an action must therefore be irrelevant. At least these consequences will not be events. For there must be something right in that formulation of the question. There must be some sort of ethical reward and ethical punishment, but this must lie in the action itself.

(And this is clear also that the reward must be something acceptable, and the punishment something unacceptable.)

Of the will as the subject of the ethical we cannot speak.

And the will as a phenomenon is only of interest to psychology.

If good or bad willing changes the world, it can only change the limits of the world, not the facts; not the things that can be expressed in language.

In brief, the world must thereby become quite another, it must so to speak wax or wane as a whole.

The world of the happy is quite another than that of the unhappy.

As in death, too, the world does not change, but ceases.

Death is not an event of life. Death is not lived through.

When an ethical law of the form, ‘Thou shalt . . .’ is laid down, one’s first thought is, ‘And what if I do not do it?’ It is clear, however, that ethics has nothing to do with punishment and reward in the usual sense of the terms. So our question about the consequences of an action must be unimportant.—At least those consequences should not be events. For there must be something right about the question we posed. There must indeed be some kind of ethical reward and ethical punishment, but they must reside in the action itself.

(And it is also clear that the reward must be something pleasant and the punishment something unpleasant.)

It is impossible to speak about the will in so far as it is the subject of ethical attributes.

And the will as a phenomenon is of interest only to psychology.

If the good or bad exercise of the will does alter the world, it can alter only the limits of the world, not the facts—not what can be expressed by means of language.

In short the effect must be that it becomes an altogether different world. It must, so to speak, wax and wane as a whole.

The world of the happy man is a different one from that of the unhappy man.

So too at death the world does not alter, but comes to an end.

Death is not an event in life: we do not live to experience death.
Wenn man unter Ewigkeit nicht unendliche Zeitdauer, sondern Unzeitlichkeit versteht, dann lebt der ewig, der in der Gegenwart lebt.

Unser Leben ist ebenso endlos, wie unser Gesichtsfeld grenzenlos ist.

6.4312

Die zeitliche Unsterblichkeit der Seele des Menschen, das heißt also ihr ewiges Fortleben auch nach dem Tode, ist nicht nur auf keine Weise verbürgt, sondern vor allem leistet diese Annahme gar nicht das, was man immer mit ihr erreichen wollte. Wird denn dadurch ein Rätsel gelöst, dass ich ewig fortlebe? Ist denn dieses ewige Leben dann nicht ebenso rätselhaft wie das gegenwärtige? Die Lösung des Rätsels des Lebens in Raum und Zeit liegt außerhalb von Raum und Zeit.

(Nicht Probleme der Naturwissenschaft sind ja zu lösen.)

6.432

Wie die Welt ist, ist für das Höhere vollkommen gleichgültig. Gott offenbart sich nicht in der Welt.

6.4321

Die Tatsachen gehören alle nur zur Aufgabe, nicht zur Lösung.

6.44

Nicht wie die Welt ist, ist das Mystische, sondern dass sie ist.

6.45

Die Anschauung der Welt sub specie aeterni ist ihre Anschauung als – begrenztes – Ganzes.

Das Gefühl der Welt als begrenztes Ganzes ist das mystische.

6.5

Zu einer Antwort, die man nicht aus sprechen kann, kann man auch die Frage nicht aussprechen.

Das Rätsel gibt es nicht.

If by eternity is understood not endless temporal duration but timelessness, then he lives eternally who lives in the present.

Our life is endless in the way that our visual field is without limit.

The temporal immortality of the human soul, that is to say, its eternal survival after death, is not only in no way guaranteed, but this assumption in the first place will not do for us what we always tried to make it do. Is a riddle solved by the fact that I survive for ever? Is this eternal life not as enigmatic as our present one? The solution of the riddle of life in space and time lies outside space and time.

(It is not problems of natural science which have to be solved.)

How the world is, is completely indifferent for what is higher. God does not reveal himself in the world.

The facts all belong only to the task and not to its performance.

Not how the world is, is the mystical, but that it is.

The contemplation of the world sub specie aeterni is its contemplation as a limited whole.

The feeling that the world is a limited whole is the mystical feeling.

For an answer which cannot be expressed the question too cannot be expressed.

The riddle does not exist.

If we take eternity to mean not infinite temporal duration but timelessness, then eternal life belongs to those who live in the present.

Our life has no end in just the way in which our visual field has no limits.

Not only is there no guarantee of the temporal immortality of the human soul, that is to say of its eternal survival after death; but, in any case, this assumption completely fails to accomplish the purpose for which it has always been intended. Or is some riddle solved by my surviving for ever? Is not this eternal life itself as much of a riddle as our present life? The solution of the riddle of life in space and time lies outside space and time.

(It is certainly not the solution of any problems of natural science that is required.)

How things are in the world is a matter of complete indifference for what is higher. God does not reveal himself in the world.

The facts all contribute only to setting the problem, not to its solution.

It is not how things are in the world that is mystical, but that it exists.

To view the world sub specie aeterni is to view it as a whole—a limited whole.

Feeling the world as a limited whole—it is this that is mystical.

When the answer cannot be put into words, neither can the question be put into words.

The riddle does not exist.
Wenn sich eine Frage überhaupt stellen lässt, so kann sie auch beantwortet werden.

Skeptizismus ist nicht unwiderleglich, sondern offenbar unsinnig, wenn er bezweifeln will, wo nicht gefragt werden kann.

Denn Zweifel kann nur bestehen, wo eine Frage besteht; eine Frage nur, wo eine Antwort besteht, und diese nur, wo etwas gesagt werden kann.

Wir fühlen, daß selbst, wenn alle möglichen wissenschaftlichen Fragen beantwortet sind, unsere Lebensprobleme noch gar nicht berührt sind. Freilich bleibt dann eben keine Frage mehr; und eben dies ist die Antwort.

Die Lösung des Problems des Lebens merkt man am Verschwinden dieses Problems.

(Ist nicht dies der Grund, warum Menschen, denen der Sinn des Lebens nach langen Zweifeln klar wurde, warum diese dann nicht sagen konnten, worin dieser Sinn bestand.)

Es gibt allerdings Unaussprechliches. Dies zeigt sich, es ist das Mystische.

Die richtige Methode der Philosophie wäre eigentlich die: Nichts zu sagen, als was sich sagen lässt, also Sätze der Naturwissenschaft – also etwas, was mit Philosophie nichts zu tun hat –, und dann immer, wenn ein anderer etwas Metaphysisches sagen wollte, ihm nachzuweisen, dass er gewissen Zeichen in seinen Sätzen keine Bedeutung gegeben hat. Diese Methode

If a question can be put at all, then it can also be answered.

Scepticism is not irrefutable, but palpably senseless, if it would doubt where a question cannot be asked.

For doubt can only exist where there is a question; a question only where there is an answer, and this only where something can be said.

We feel that even if all possible scientific questions be answered, the problems of life have still not been touched at all. Of course there is then no question left, and just this is the answer.

The solution of the problem of life is seen in the vanishing of this problem.

(Is not this the reason why men to whom after long doubting the sense of life became clear, could not then say wherein this sense consisted?)

There is indeed the inexpressible. This shows itself; it is the mystical.

The right method of philosophy would be this: To say nothing except what can be said, i.e. the propositions of natural science, i.e. something that has nothing to do with philosophy: and then always, when someone else wished to say something metaphysical, to demonstrate to him that he had given no meaning to certain signs in his propositions. This method would be un-

If a question can be framed at all, it is also possible to answer it.

Scepticism is not irrefutable, but obviously nonsensical, when it tries to raise doubts where no questions can be asked.

For doubt can exist only where a question exists, a question only where an answer exists, and an answer only where something can be said.

We feel that even when all possible scientific questions have been answered, the problems of life remain completely untouched. Of course there are then no questions left, and this itself is the answer.

The solution of the problem of life is seen in the vanishing of the problem.

(Is not this the reason why those who have found after a long period of doubt that the sense of life became clear to them have then been unable to say what constituted that sense?)

There are, indeed, things that cannot be put into words. They make themselves manifest. They are what is mystical.

The correct method in philosophy would really be the following: to say nothing except what can be said, i.e. propositions of natural science—i.e. something that has nothing to do with philosophy—and then, whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give a meaning to certain signs in his
wäre für den anderen unbefriedigend – er hätte nicht das Gefühl, dass wir ihn Philosophie lehrten – aber sie wäre die einzige streng richtige.

Meine Sätze erläutern dadurch, dass sie der, welcher mich versteht, am Ende als unsinnig erkennt, wenn er durch sie – auf ihnen – über sie hinaustiegen ist. (Er muss sozusagen die Leiter wegwurf en, nachdem er auf ihr hinaufgestiegen ist.)

Er muss diese Sätze überwinden, dann sieht er die Welt richtig.
Wovon man nicht sprechen kann, darüber muss man schweigen.

satisfying to the other—he would not have the feeling that we were teaching him philosophy—but it would be the only strictly correct method.

My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.)

He must surmount these propositions; then he sees the world rightly.
Whereof one cannot speak, thereof one must be silent.

propositions. Although it would not be satisfying to the other person—he would not have the feeling that we were teaching him philosophy—this method would be the only strictly correct one.

My propositions serve as elucidations in the following way: anyone who understands me eventually recognizes them as nonsensical, when he has used them—as steps—to climb up beyond them. (He must, so to speak, throw away the ladder after he has climbed up it.)

He must transcend these propositions, and then he will see the world aright.
What we cannot speak about we must pass over in silence.
The translators’ aim has been to include all the more interesting words, and, in each case, either to give all the occurrences of a word, or else to omit only a few unimportant ones. Paragraphs in the preface are referred to as P1, P2, etc. Propositions are indicated by numbers without points [—the points have been restored for the side-by-side-by-side edition—]; more than two consecutive propositions, by two numbers joined by an en-rule, as 202–2021.

In the translation it has sometimes been necessary to use different English expressions for the same German expression or the same English expression for different German expressions. The index contains various devices designed to make it an informative guide to the German terminology and, in particular, to draw attention to some important connexions between ideas that are more difficult to bring out in English than in German.

First, when a German expression is of any interest in itself, it is given in brackets after the English expression that translates it, e.g. situation [Sachlage]; also, whenever an English expression is used to translate more than one German expression, each of the German expressions is given separately in numbered brackets, and is followed by the list of passages in which it is translated by the English expression, e.g. reality 1. [Realität], 55561, etc. 2. [Wirklichkeit], 206, etc.

Secondly, the German expressions given in this way sometimes have two or more English translations in the text; and when this is so, if the alternative English translations are of interest, they follow the German expression inside the brackets, e.g. proposition [Satz]: law; principle.

The alternative translations recorded by these two devices are sometimes given in an abbreviated way. For a German expression that is translated by the English expression, e.g. stand in a relation to one another; are related [sich verhalten: stand, how things; state of things].

Thirdly, cross-references have been used to draw attention to other important connexions between ideas, e.g. true, cf. correct; right: and a priori, cf. advance, in.

In subordinate entries and cross-references the catchword is indicated by ~, unless the catchword contains /, in which case the part preceding / is so indicated, e.g. accident; ~al [Zufall], 2.012, 2.0121, 3.34, 5.4733, 6.031, 6.1231, 6.1232, 6.3, 6.41

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