Putting Form Before Function: Logical Grammar in Frege, Russell, and Wittgenstein

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§1. *Introduction*

Years ago, Peter Geach suggested that Frege was "the most important single influence on Wittgenstein."[1] Such a remark can only be aimed to downplay the influence of Russell, the only plausible contender, at least with regard to Wittgenstein's early work. Lately many have suggested that certain doctrines of the *Tractatus* are best traced to Frege. Allegedly, Wittgenstein derived from Frege a holistic or "judgment-based" philosophy, which is in a much better position vis-à-vis Russell's "object-based" approach to explain phenomena such as the unity of propositions and facts, and why a (propositional) function cannot take itself as argument.[2] While I certainly do not deny that Frege influenced early Wittgenstein, the tendency to give Frege pride of place is due to a failure to appreciate the subtlety, complexity, and intricate development of Russell's thinking with regard to such matters. Underestimating Russell is, unfortunately, an all too common blunder.

As I shall argue, the relationship between these thinkers is greatly clarified when we examine their relative positions on the nature of logical forms and structures, on the relationship between structured complexes and their constituents, and on (propositional) functions as things derived from complexes. The evidence for Frege as the primary influence derives almost entirely from certain shared slogans to the effect that the sense of an entire sentence is more fundamental than that of the parts. On examination, the similarities be-

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[1] This quotation is from Coope et al., 1970, p. 5. I follow Reck 2002a (p. 34n) in attributing it to Geach.

[2] See, e.g., Gerrard 2002, Reck 1997, Diamond 1991 and Ricketts 1985; a less severe version of this tendency is found in Conant 2002. Recently, Ricketts (2002) seems to have weakened his view somewhat. The labels "judgment based" and "object based" come from Hylton 1991, though I would not place him in the same company. For a contrasting view, see Stevens 2003.

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tween them quickly evaporate. In Russell, however, we find a complex and subtle position in which the insights of Wittgenstein are clearly prefigured. It is to Russell and not Frege we must look if we are to understand the Tractarian doctrines that complete propositions are more fundamental than functions, and the impossibility of a function's taking itself as argument. This has not been seen because of misunderstandings regarding Russell's type theory, the nature of Russellian propositional functions, and their relationship to the propositions that are their values.

In this paper, I firstly discuss Frege's views on how functions are to be understood as derived from thoughts and how functions are to be distinguished from objects. I suggest that his views on these topics are problematic, and that it is unlikely that they influenced the core of Wittgenstein's position. Next comes a more extended discussion of the development of Russell's position from 1903 until his first interactions with Wittgenstein in 1911. I then discuss Wittgenstein's views, portraying them as, if anything, more Russellian than Russell's own. I conclude with a discussion and evaluation of the changes to Russell's philosophy instigated by Wittgenstein's, work as well as the relevance of these matters to the debate over "inexpressible truths" in the *Tractatus*.

§2. *Frege on the alleged priority of thoughts over concepts*

According to Frege's mature theory, a proposition (*Satz*) has for its referent (*Bedeutung*) a truth-value, and has for its sense (*Sinn*) a thought (*Gedanke*). Propositions consist of two kinds of expressions: proper names and function expressions. In "Socrates is wise," "Socrates" is a proper name, "… is wise" is a function expression. The proper name refers to an object, here the person Socrates, and the function expression refers to a function, in this case, the function whose value is

the True for all wise things as arguments, and the False for all others. Concepts (*Begriffe*), the referents of predicates, are functions of one argument whose values are always truth-values. The referent of a complex expression is determined functionally by the referents of the parts. However, the referent of a complex expression is not a whole *containing* the referents of the parts. The phrase "the square root of …" refers to a function, and "nine" refers to a number, but the referent of "the square root of nine"—three—is not a whole consisting of the number nine and the function. To use Frege's own example from 1919, Sweden is not a constituent of Stockholm, despite the fact that the name "Sweden" is a constituent of the phrase "the capital of Sweden" (*NLD* p. 255).

Frege contrasts this with the realm of sense. The thought expressed by the whole, he repeatedly tells us, is a complex formed of the senses of the parts (e.g., *GG* §32; *CT* pp. 390-391; *NLD* pp. 254-55; *PMC* pp. 79-80, 98, 149; *IL* p. 191; *LM* pp. 225, 243; *DV* p. 378). The senses of the function expressions have a sort of "unsaturatedness" that calls for completion by other senses, and thereby provide the logical glue holding the thought together (*BuG* pp. 193-194; *CT* pp. 390-391). This suggests that the constituent senses making up a thought must in some sense be prior to it, and that it is built up from them. Moreover, it suggests that the *structure* of a thought is to be explained in virtue of the nature of the parts themselves, and in what way or ways these parts are capable of being completed. The sense of "… is wise" can be competed by the sense of "Socrates", but the reverse is impossible. These views are implemented in the syntax of Frege's logical language. No complex expression can be well-formed without containing at least one function expression. Frege argues explicitly that the same is true of natural language (*BuG* pp. 193-94). The reason is that without a func-

tion sign, there is nothing to unite or bind together the senses of the other signs into a unified, judgeable content.

However, this general account seems to conflict with certain of Frege's other statements. He explicitly claims that a complete thought is prior to its parts. In 1919, he writes, "… I do not begin with concepts and put them together to form a thought; I come by the parts of a thought by analyzing the thought" (*NLD* p. 253). This echoes statements made earlier in his career to the effect that concepts do not have independent existence but instead are gotten at by "decomposing" or "splitting up" a judgeable content (see, e.g., *BLC* p. 17; *PMC* p. 101). In the context of Frege's earlier philosophy, the view that concepts are gotten by breaking apart content is more straightforward, because "judgeable content" had not yet been split up into "thoughts" and "truthvalues" per the theory of sense and reference, and arguably concepts and judgeable contents can be found on the same plane. After the sense/reference distinction is in place, however, concepts are the *referents* of function expressions. Concepts cannot literally be gotten at by splitting up thoughts, because concepts are not senses and only senses can be parts of thoughts (see, e.g., *NLD* p. 225; *PMC* p. 163). Indeed, concepts cannot be understood as gotten from decomposing anything. As we have seen, concepts are functions, and Frege does not hold the view that functions form wholes with their arguments.[3]

The only semi-plausible reading of these remarks, then, would be to attribute to Frege a view according to which thoughts are to be understood as prior to the *senses* whose referents are concepts, and those *senses* are gotten by decomposing or splitting apart thoughts. Concepts are *indi-*

rectly derived from thoughts because the senses of function expressions are derived from breaking apart thoughts, and concepts are understood only through such senses.

But this reading too is problematic. Frege has given us no *independent* explanation of how to understand the constitution and unity of thoughts without understanding them as put together from incomplete and complete senses. If incomplete senses are derived from, or secondary to, thoughts, then it must be possible to explain such things without making reference to incomplete senses. In general it does not seem possible to break apart, analyze, or split apart something that does not have a definite structure to begin with. If thoughts were seamless wholes, it is hard to see how the very notion of decomposition could make sense. Some have attributed to Frege the view that a thought does not on its own have a determinate inner structure, and instead it is only through *our* acts of analysis that a thought can be said to have a structure at all.[4] But making the structure of a thought dependent on *our* acts of analysis seems contrary to Frege's robust anti-psychologism and leaves his position almost completely obscure. Even if Frege did hold such a view, it cannot have influenced Wittgenstein's position in the *Tractatus*, as Wittgenstein does think that propositions have unique determinate analyses (*TLP* 3.25).

I shall argue below that both Russell and Wittgenstein gave an account of how we can understand the form and make-up of propositions without presupposing the notion of a (propositional) function. When their views are understood, it is obvious that Wittgenstein's position is an elaboration on Russell's. Frege pays lip service to a view accord-

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[3] Frege does sometimes talk as if the whole/part relationship can be found at the realm of reference as well. See, e.g., *SuB* p. 165. However, I believe this is a slip, which Frege clearly redresses with his discussion of "the capital of Sweden", as I have argued elsewhere (see Klement 2002, pp. 68-9).

[4] See, e.g., Hodes 1982; Currie 1985; Garavaso 1991; Ruffino 1991. Such readings are motivated also to explain why Frege thinks that thoughts can be analyzed in multiple ways. However, I have argued elsewhere that Frege's claims to this effect are not inconsistent with the view that thoughts have determine structures (Klement 2002, pp. 76-88).

ing to which complete logical forms (thoughts) are to be understood as prior to functions, but missing from his account is any explanation of the form or structure of a thought that does not presuppose functions or their senses as more fundamental. There have been attempts within the secondary literature in the past few decades to clear or patch up Frege's position (e.g., Dummett 1981, pp. 261-91, etc.), but most involve going beyond anything Frege explicitly said, or involve highlighting certain aspects of his writings that are far from evident in the surface of those he had published during his lifetime. So it is unlikely these aspects of Frege's work were much of an influence on Wittgenstein, especially as Wittgenstein's grasp of the more subtle aspects of Frege's philosophy can certainly be questioned (as in, e.g., Goldfarb 2002).

§3. *Frege on the distinction between functions and objects*
These issues are pertinent to assessing both the viability of certain of Frege's other positions as well as how much influence on Wittgenstein we can plausibly take them to have had. Consider next the distinction Frege makes between objects and functions of different levels. In his essay "Funktion und Begriff," Frege takes issue with the prevalent view in mathematics according to which an expression containing a variable, e.g.,

$$2x^3 + x$$

represents a function. Frege points out that because the variable letter "x" indicates a number indefinitely, the expression "$2x^3 + x$" likewise indicates a number indefinitely. Instead, Frege claims that the function itself would be better represented as "$2 \cdot (\ )^3 + (\ )$" (see, e.g., *FuB* 140-141). Frege contrasts functions with objects and introduces objects in

this way:

> … the question arises what it is that we are here calling an object. I regard a regular definition as impossible, since we have here something too simple to admit of logical analysis. It is only possible to indicate what is meant. Here I can only say briefly: An object is anything that is not a function, so that an expression for it does not contain an empty place. [*FuB* 147]

The distinction is central to Frege's philosophy and forms the basis for a hierarchy of levels of functions: first, there are *first-level* functions, i.e., functions that take objects as arguments; then there are *second-level* functions, i.e., functions that take first-level functions as arguments; and so on. Frege suggests that these distinctions are founded "deep in the nature of things" (*FuB* p. 156), and claims that confusing functions and objects or confusing functions of different levels is "just about the grossest possible" confusion (*GG* p. 24). Frege invokes the distinction to reply to a certain formulation of Russell's paradox (*PMC* pp. 132-133). Because a function of the $n+1^{th}$ level takes entities of the nth level as argument, no concept has itself among its own arguments, and so one does not face a problem regarding concepts not true of themselves.

Despite their centrality in his philosophy, Frege's philosophical argumentation and explanation for these distinctions are not entirely satisfactory. While he strongly intimates that the notions of *object* and *function* are incapable of definition,[5] his usual explanation is that the hallmark of a

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[5] This comes out in the passage from *FuB* quoted earlier, and also in an extended passage in *BuG* (pp. 182-183) where he claims that his explanation of concepts was "not meant as a proper definition". This latter passage, I think, can be a little misleading. That discussion centers around first-level concepts. Among first-level functions, it is possible to define what makes concepts distinct from other first-level functions, and indeed, the second-level concept of being a *concept* as opposed to a non-concept function (i.e., the second-level function mapping all first-level concepts to the True, and all other first-level functions to the False) can be defined outright in Frege's logical language. What is indefinable is the notion of being a concept or function *as opposed to* an object.

function is that the expression for it exhibit an empty spot or spot for completion, whereas the hallmark of an object is that the expression for it not exhibit any such empty spot (see, e.g., *FuB* 140-41, *GG* §§1-2, *WIF* 290-91). If the distinction is founded in the nature of things, and not in language, this would seem to characterize the difference by inessential features. Since this is not meant as a strict definition, this problem can be put aside. However, even if this way of characterizing the difference is meant only to get things right extensionally, some clarification is in order. Some objects and functions are referred to by more than one expression, and some likely are not referred to by any expression at all. It is imprecise to characterize a function as something "*the expression for which*" contains an empty place, as if there were always a unique expression. We might instead characterize a function either as (1) an entity that *can* be the referent of a expression that *does* contain an empty spot, or as (2) an entity that *cannot* be the referent of any expression that does *not* contain an empty spot. Similarly, we might characterize an object either as (3) an entity that *can* be the referent of an expression *not* containing an empty spot, or as (4) an entity that *cannot* be the referent of an expression that does contain an empty spot. The differences between these characterizations will be trivial only if we accept the following bridge principle: *that no entity can be both the referent of an expression containing an empty spot and the referent of an expression not containing an empty spot.* Without accepting this principle, the distinction may either not be exhaustive or not be exclusive. In an earlier work, *Die Grundlagen der Arithmetik*, Frege characterized a concept as "that which can be the predicate of a singular judgment," and an object as "that which can be the subject of the same" (*GL* p. 77n). This suggests readings (1) and (3) above, and these characterizations certainly don't, on their own, logically rule out that something could be both a concept and an object. However, Frege seems implicitly to accept the bridge principle, and hence

without giving the matter the attention it deserves, he seems to take (1) and (2), and (3) and (4), as interchangeable characterizations.

Frege's inattentiveness to this issue is pertinent to the position taken in his apologetic article, "Über Begriff und Gegenstand," where he discusses the phrase "the concept horse", used as grammatical subject. Benno Kerry had called into question the exclusivity of the distinction, pointing out that the phrase "the concept horse" as used, e.g., in "the concept horse is a concept easily attained" seems to refer to a concept but also, as it is not itself incomplete in any way, must refer to an object. In this response, Frege accuses Kerry of deviating from what he meant by "concept". Frege explains the distinction this way:

> A concept (as I understand the word) is predicative. [*Footnote:* It is, in fact, the referent of a grammatical predicate.] On the other hand, a name of an object, a proper name, is quite incapable of being used as a grammatical predicate. [*BuG* p. 183]

Here again, Frege characterizes concepts as those entities that can be referred to by predicates, and objects as those entities that can be referred to by proper names (broadly construed). He then goes on to argue that a proper name is "incapable of being used as a grammatical predicate". His argument that proper names and grammatical predicates are distinct is eminently plausible, and we need not rehearse it here. However, Frege seems to move very quickly to a strong conclusion:

> We have here a word 'Venus' that can never be a proper predicate, although it can form part of a predicate. The referent of this word is thus something that can never occur as a concept, but only as an object. [*BuG* p. 184]

However, even if the word "Venus" never occurs as a predicate, this does not mean there cannot be a *distinct* word or phrase that *can* occur as a predicate and has the same referent as "Venus". Frege seems to be taking the bridge principle for granted, because only then could he conclude that *in virtue* of being the referent of a proper name, Venus cannot *also* be the referent of a predicate. Frege does warn against equating the referent of "Venus" with the concept referred to by "is Venus" or "is none other than Venus", and his warning does seem to be apt. But this does not rule out there being some *other* predicate that refers to it. Of course, on independent grounds, it is not plausible to equate the planet Venus with any function. In this particular case, it may be that the referent of "Venus" can be referred to only by proper names. However, this imports knowledge about what sorts of objects planets are. We do not yet have any grounds for the conclusion that a referent of a proper name can never also be the referent of a predicate.

Frege responds to Kerry's example of "the concept *horse* is a concept easily attained" by admitting that "the concept horse" refers to an object. Frege's understanding of "proper name" is broad enough to include phrases following the definite article as typically used to refer to an individual. While admitting that the words "the concept horse" designate an object, Frege concludes that "on that very account they do not designate a concept" (*BuG* p. 184). Assuming that the essential criterion for something's being a concept is that it be capable of serving as the referent of a predicate expression, Frege's argument goes too fast. Without the bridge principle, establishing of something that it is the referent of a proper name does not by itself show anything about whether or not it could be a concept. Again, Frege seems to be assuming the principle implicitly.

One argument that Frege *might* have given for the bridge principle would run like this:

Premise 1: If two expressions refer to the same entity, then it is always possible to replace an occurrence of one in a significant sentence with an occurrence of the other, and the result will also be a significant sentence with the same truth-value as the original sentence.

Premise 2: An occurrence of a complete expression (e.g., a proper name) can never replace an occurrence of an incomplete expression (e.g., a predicate), or vice-versa, *salva congruitate* much less *salva veritate*.

Conclusion: A complete expression can never refer to the same entity as an incomplete one.

For future reference, let us call this the *replacement argument*. Frege did explicitly endorse something at least *similar* to Premise 1. Elsewhere he wrote, "if we replace one word of the sentence by another having the same referent … this can have no effect on the referent of the sentence" (*SuB* p. 162)—recall that the referent of a sentence is its truth value—but it is not entirely clear whether or not he had in mind cases in which the words were of different grammatical types, or cases in which the result is not itself a sentence. In any case, no explicit appeal to such a principle is made in motivating the function/object distinction, and as far as I can tell, Frege never (explicitly, at least) advances an argument such as the above.[6]

Does Frege then give a different argument for the principle? Later in the discussion of "the concept horse", he writes:

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[6] Nor would he have been convincing were he to advance such an argument. See Wright 1998 for discussion.

> In logical discussions one quite often needs to say something about a concept, and to express this in the form usual for such predicates—viz. to make what is said about the concept into the content of the grammatical predicate. Consequently, one would expect that what is referred to by the grammatical subject would be the concept, but the concept as such cannot play this part, in view of its predicative nature: it must first be converted into an object, or, more precisely, an object must go proxy for it. [*BuG* p. 186]

Although it is difficult to extract an argument here, this is the closest he comes to providing support for the bridge principle. Here Frege speaks of the "predicative nature" of concepts. He intimates that it is in virtue of this nature that concepts are *suitable* referents of predicates, and *unsuitable* for being referents of names. It is not yet clear what it means to have a predicative nature, but if the referent of a predicate expression *must* have such a nature, and the referent of a proper name must *lack* such a nature, then the referents of predicates and the referents of proper names could not overlap. However, without further discussion of what it is to have a predicative nature, and what it is to have a nameable nature, and why the two are exclusive, this phraseology seems to do nothing more than label the problem, not solve it.

What are we to make of the notion of having a "predicative nature"? In a footnote, Frege explains the predicative nature of concepts as "a special case of the need of supplementation, the 'unsaturatedness', that I gave as the essential feature of a function" (*BuG* p. 187). Frege's position thus stands or falls with his making good on an argument to the effect that functions, as referents of incomplete expressions, must be unsaturated, while the referents of proper names must not be unsaturated. However, the divisions of "unsaturated" and "saturated" seem most applicable to the realm of *sense*, not the realm of reference. Frege writes in *BuG* (p. 193):

> … not all the parts of a thought can be complete; at least one must be 'unsaturated', or predicative; otherwise they would not hold together. For example, the sense of the phrase "the number 2" does not hold together with that of the expression "the concept prime number" without a link. We apply such a link in the sentence "the number 2 falls under the concept *prime number*"; it is contained in the word "falls under" … and only because their sense is thus 'unsaturated' are they capable of serving as a link.

This line of reasoning may have a hope of establishing that the *sense* of "the concept *horse*" is complete whereas the *sense* of a predicate expression must be incomplete. However, just as the same entity can be the referent of multiple expressions, the same entity can be presented by more than one sense. Hence it does not improve Frege's position to characterize an object as an entity that can be presented by a complete sense, and a function as an entity that can be presented by an incomplete sense. Even assuming that complete and incomplete senses are mutually exclusive, without an analogue of the bridge principle for senses, we have not yet got what is needed. To put the same point another way, even if "the concept horse" and "… is a horse" cannot have the same sense, it by no means follows that they cannot have the same referent. If they can, then the category of objects and the category of concepts are not mutually exclusive.

It is true, he does *transfer* the complete/incomplete distinction from the realm of sense to the realm of reference, writing about a concept expression that "the words 'unsaturated' and 'predicative' seem more suited to the sense than the referent; still there must be something on the part of the referent which corresponds to this" (*CSM* p. 119n), and that

"we can, metaphorically speaking, call the concept unsaturated too" (*IL* p. 193). However, the metaphor is not a good one, and the word "unsaturated" does not seem *at all* suited to the concept. As discussed earlier, in the realm of reference, concepts do not form *wholes* with their arguments, and so there is nothing analogous to the literal *completion* of an incomplete sense by its component sense that is found in the realm of sense. All that could be meant by "unsaturatedness" in the realm of reference is that functions must be capable of taking arguments and yielding values. But Frege gives us no argument for thinking that objects must *lack* this property. Without further elaboration, Frege is left without an explanation for why a concept cannot take itself as argument, which casts doubt on the philosophical motivation of his entire hierarchy of levels.

Moreover, as Russell explicitly argued, Frege's view seems self-defeating, since asserting the view that the concept referred to by "… is a horse" cannot be the referent of any proper name requires doing precisely what it claims to be impossible (*PMC* p. 134; *PoM* pp. 45-46, 510). One would need to assert something like "The referent of '… is a horse' is not the referent of any expression with a complete sense," and for this proposition to say what we want, it will be necessary that the phrase "the referent of '… is horse'" refer to the referent of "… is a horse". However, if it *does*, then the referent of "… is a horse" can be referred to by an expression with a complete sense, since "the referent of '… is horse'" is such an expression.

To some extent, Frege faces this problem precisely because he characterizes both functions and objects as the *things* or *entities* referred to by different types of expressions. If he instead had a different kind of view, according to which function expressions and object expressions were not meaningful in the same way—say, one according to which only proper names are taken to *refer to things*—then perhaps he would be on better footing.[7] But at least on the surface, the way in which Frege thinks that both types of expressions are meaningful is that they refer to things by expressing senses. The question then remains as to whether the *things* can ever be the same. Frege's thinking on these matters is problematic at best. We shall see, however, that these are issues to which Russell had given quite a lot of thought in his logical prime, and that his mature position is far closer to Wittgenstein's than Frege's is.

§4. *Common misreadings of Russell's early logic*

I shall argue that Russell, unlike Frege, had a clear conception of exactly how complete propositions can be considered logically more primitive and propositional functions more derivative, and that this explanation provides us with a clear answer to why it is impossible to assert the value of a propositional function with itself as argument. First, however, I must clear away certain common misunderstandings of Russell's work that have their origins in the notation and vocabulary used by later logicians.

In contemporary first-order predicate logic—the stuff we teach our undergraduate students—an atomic statement that predicates a property to an individual, e.g., "Alison is

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[7] See Diamond 1991 and Ricketts 1986 for attempts to defend Frege in this way. Their responses involve recasting Frege's apparent Platonic realism about functions and giving a strong reading of Frege's content principle "never to ask for the meaning of a word in isolation, but only in the context of a proposition" (*GL* p. x), according to which it is a mistake to think, as Diamond puts it, that one can refer to a concept by using a phrase predicatively, "turn on yourself", and then refer to the same thing with a proper name, since what *it is* to refer to concept is to use a phrase predicatively. Besides any questions we may have about whether it does justice to Frege's Platonism, this response presupposes that Frege could provide an intelligible notion of using a phrase predicatively that it is not circularly explained in terms of which expressions have incomplete senses or refer to concepts, i.e., a notion of logical form characterizable independently of such notions, which is highly questionable.

friendly," is written "*Fa*", and is read aloud "*a is F*". In natural language, usually, the subject phrase precedes the predicate phrase. So why the reverse in predicate logic? Historically, the answer is that contemporary predicate logic represents a simplification of Frege's logic of functions and Russell's logic of propositional functions, and so "*F*" precedes the "*a*" to coincide with the older mathematical usage of the notation "*f(x)*" as it occurs in equations such as

$$f(x) = x^2 + 5x + 6$$
$$f(x, y) = xy - 3x + 4y - 12$$

etc. For Frege, predicate expressions *just are* function expressions, and there is no difference between predicate expressions and the function expressions of mathematics in terms of syntax or even in terms of the general nature of their referents.

It is well known that, for Russell, ordinary mathematical functions are thought to be less fundamental than "propositional functions", and that an expression involving a mathematical function is to be regarded as a sort of definite description (e.g., *PoM* p. 264; *OD* p. 426; *SRML* pp. 524-526; *ML* pp. 92-93). So expressions for the values of an ordinary mathematical function are eliminated by means of the quantificational analysis he gives for descriptive phrases. However, it seems widely regarded that the notation of contemporary predicate logic matches Russell's understanding of propositional functions, even early on. In reading "*Fa*", Russell would have regarded the "*a*" as standing for an individual, and the "*F*" for a propositional function of one argument. In reading "*Rab*", Russell would have regarded the "*R*" as standing for a two-place propositional function, and so on. Indeed, such contemporary notation is widely used in reconstructing or discussing Russell's work.

However, anachronism is a danger here, especially with regard to earlier phases of Russell's thought. To be sure,

Russell's logical notation is historically connected with the notation of contemporary predicate logic, and one can easily find in Russell's writings notations such as "$\phi(a)$" and "$F(a)$", where this is to be understood as the value of a propositional function for a given argument. However, before the 1920s, when Russell uses such notation, the "ϕ" or "F" (or other letter) is *always* used either as a *variable* or *schematically*. Russell did not "translate" ordinary-language simple statements such as "Anne is friendly" into expressions of the form "*Fa*" where the "*F*" is used for a *constant* propositional function. Early Russell did not use, and would not have recognized, the notation "*Fa*" used for a *variable-free* formula.

Russell sometimes spoke of the logical notation of *Principia Mathematica* as a language that contained a "syntax" but lacked a vocabulary (e.g., *PLA* p. 58). The goal of *PM* was to reduce mathematics to logic and did not require the use of any constants besides logical constants. Some might think the lack of non-logical constants is sufficient to explain why the form "*Fa*" was never used with "*F*" as a constant. However, Russell gives plenty of examples of variable-free atomic statements in the introduction to *Principia*, and there he leaves the English as is, writing "this is red" and "Socrates is human" (*PM* pp. 39, 50). Moreover, when he gives examples of propositional functions whose values are atomic, he writes them in the form "$\hat{x}$ is hurt" (*PM* p. 15), or "$\hat{x}$ is human" (*PM* p. 39), and never in a form such as "$F\hat{x}$". Even if Russell *had* introduced non-logical vocabulary into the official notation of *PM*, the notation of contemporary predicate logic would not have appeared.[8]

Why is this important? Primarily, it helps clear away certain possible misunderstandings of Russell's position.

[8] Here and elsewhere I ignore the fact that *PM* was a collaboration. Most citations are to chapter II of the introduction to the first edition, which is certainly Russell's work, for it is nearly identical to the paper "The Theory of Logical Types," published first in French under Russell's name alone. See *CPBR* v. 6, pp. 3-31.

Representing "Anne is friendly" as "Fa" makes it seem as if the complex corresponding to this statement (i.e., the proposition early on, or the fact later on) consists of two entities, one represented by "F" and one by "a". One constituent is Anne, represented by "a". Writing the "F" first makes it seem as if the entity it represents is a function, and hence as if the proposition or fact consists of these two entities. Propositional functions and individuals are of different logical types, so here we have a complex composed of two different types of entities, and it would be impossible to have a complex of the form FF, where the entity F replaces the entity a in the complex.

On this (mis)reading, the propositional function represented by "F" would appear to be the same as the universal *Friendliness*. Consider then the hierarchy of Russell's theory of types:

Type 1: Propositional functions whose arguments are individuals

Type 2: Propositional functions whose arguments are functions of type 1

Type 3: Propositional functions whose arguments are functions of type 2

(etc.)

This hierarchy is then often taken to be the same as the following hierarchy:

Type 1: Universals that apply (or not) to particulars (things that aren't universals)

Type 2: Universals that apply (or not) to universals of type 1

Type 3: Universals that apply (or not) to universals of type 2

(etc.)

When applied to Russell's views prior to his having been influenced by Wittgenstein, this is a misapprehension. As I shall substantiate later in the paper, the truth is that Russell does not regard propositional functions as constituents of atomic propositions or facts. He *does* regard universals as constituents of such complexes, and hence, universals cannot be equated with propositional functions. The hierarchy of types of propositional functions is not a hierarchy of universals. In order to understand Russell properly it is crucial to put out of one's mind the conception of what an atomic statement "looks like" that comes from contemporary predicate logic. Bernard Linsky (1999, chap. 2) is to be credited for pointing out that Russell is committed to a distinction between universals and propositional functions, but concludes that his understanding of the relationship between them is left obscure. While certainly Russell is not as clear about it as he could have been, it is possible to get a sense of his views by carefully considering their development.

§5. *Propositions and concepts in* The Principles of Mathematics

The first major statement of Russell's views on logical grammar occurs in his 1903 classic *The Principles of Mathematics*. The focus is on the notion of *propositions* understood as mind-independent complexes. There he claims that the grammar of the sentences used to express propositions is a mostly reliable guide to understanding their make-up,[9] and even that "[t]he correctness of our philosophical analysis of a proposition may … be usefully checked by the exercise of assigning the meaning of each word in the sentence" (*PoM* p. 42). The proposition expressed by "Plato loves Socrates" consists of Plato, *Love,* and Socrates. He even suggests that "Socrates is human" expresses a proposition with three con-

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[9] However, for discussion of certain exceptions to Russell's taking grammar as a guide to analysis during this period, see Makin 2000 (pp. 74-75) and Levine 2001. However, I do not agree with these authors on all points.

stituents, and that the copula has for its meaning a special sort of relation (*PoM* p. 49). However, a proposition is not an aggregate of its constituents; it is a kind of unity, and the relationship of the constituents to that unity is fundamentally different from the usual relation of whole and part (*PoM* pp. 139ff). Moreover, there are different ways in which an entity can occur in a proposition. An entity can occur *as term*, i.e., as *logical subject*, or it can occur *as concept*, i.e., predicatively. All entities are capable of occurring as term, but only some are capable of occurring predicatively. Those that can occur both kinds of ways are called *concepts*; those that cannot occur as concept are called *things*. Socrates is a thing, whereas *Humanity*, *Love*, and *Wisdom* are concepts. While *Wisdom* occurs *as concept* in *Socrates is wise*, it occurs as term in *Wisdom is a virtue*.

Although Russell does not think of concepts as functions, in many ways his position is exactly of the sort one would imagine Frege adopting had he not accepted the bridge principle. An entity occurs *as concept* in a proposition when it is the contribution a predicative expression makes to the proposition expressed by a sentence; an entity occurs *as term* when it is the contribution a name makes to the proposition expressed by a sentence. Concepts are those entities that *can* occur as concept; terms are those that *can* occur as term. While Russell is quite aware that no *expression* can play both predicative and referring roles at once, he does not assume that because an entity occurs in one way in one proposition, the same entity cannot occur the other way in another proposition. While admitting that some terms, *things*, are not suitable to occur as concept, he concludes that all concepts can occur as term. This is because, as we've seen, he believes it to be "self-contradictory" to suggest of any given entity that it is incapable of occurring as logical subject, be-

cause any statement to that effect would require doing precisely what it claims impossible (*PoM* p. 46).[10] The structure of the proposition expressed by "*Humanity* can only occur in a proposition as concept" reveals its own absurdity.

It is worth asking, briefly, then, how Russell would have responded to a version of the *replacement argument* considered in §3.[11] Since Russell thinks that the word "Wisdom" as it occurs in "Wisdom is a virtue" represents the same entity as does "wise" in "Socrates is wise," one might claim that he is then committed to holding that we could replace "Wisdom" with "wise" in "Wisdom is a virtue" while preserving the proposition expressed and thereby also its truth-value. However, "wise is a virtue" does not seem grammatically well-formed, much less to express a true proposition. Russell, however, would remind us that he marks a distinction between *occurring as term* and *occurring as concept*. The grammatical form a word takes, while not contributing an(other) entity to the proposition expressed, nevertheless indicates something about the proposition, i.e., what modes of occurring its constituents do have. Because, by grammatical custom, the adjective "wise" is used to indicate the concept Wisdom when it occurs *as concept*, and the noun "Wisdom" is used when it occurs *as term*, "wise is a virtue" is awkward and contravenes the grammatical rule. Of course, this series of words *might* nevertheless be used successfully to express a proposition, e.g., by a child or foreigner who has not mastered the noun/adjective distinction in English. If so, then the proposition in question is the same as that ex-

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[10] It has been pointed out to me (by a referee) that Russell's label of "self-contradictory" may be too harsh. It might be more accurate to say that the view is pragmatically self-defeating. The theory does not contradict itself in the straightforward sense of both committing itself to accepting, and committing itself to rejecting, the very same claim; instead, the theory makes claims that can be seen to be false by considering the way the theory itself must be stated.

[11] Indeed it is possible that Russell encountered similar arguments in his debates with Bradley. See, e.g., *CPBR* v. 2, pp. 136-146, and v. 6, pp. 349-53.

pressed by "Wisdom is a virtue" and has the same truth-value. Indeed, Russell explicitly says that if the verb form "kills" is used in a place usually occupied by a grammatical subject, it means the same thing as would the verbal nouns "Killing" or "to kill" (*PoM* p. 48).

While Russell did not at the time consider language to be an important subject for philosophical inquiry and therefore did not explicitly develop many views about it, the above considerations show that it would be uncharitable to attribute to him a simplistic theory of linguistic meaning whereupon all there is to the meaning of a word is the entity it contributes to the proposition. For Russell as early as 1903, a full understanding of the meaning of a word requires not only an understanding of what entity is designated but also an understanding of *how* that entity occurs within the proposition (whether as term or as concept, etc.). Typically, this additional element of the meaning of the word is indicated by the grammatical form the word takes. However, the possibility of making sense of "wise is a virtue" shows that this element can also be appreciated by considering the position of the word relative to the other words in the sentence. Although he does not turn it into a slogan the way Frege (and later Wittgenstein) did, Russell too is committed to the principle that the meaning of a word cannot be fully appreciated except in the context of a complete sentence.[12]

Russell does not yet use the word "universals" for concepts in *PoM*, but the label is appropriate, and later he assimilates his early distinction between things and concepts to the distinction between particulars and universals (see, e.g., *RUP* p. 169). However, there is a broader logical category that includes both: *terms*. All entities are terms; indeed,

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[12] Russell's theory of descriptions is widely known to result in some contextuality in his theory of meaning, with its "incomplete symbols" having no meaning of their own but contributing to the meaning of the sentences wherein they appear. However, we here see that a kind of context principle held for Russell even *before* 1905.

Russell claims that "entity", "term", "logical subject", "individual", and "unit" are all synonymous (*PoM* p. 43). Concepts, i.e., universals, are not things, but they are terms and hence *individuals*. The distinction between things and concepts is, to be sure, a *logical distinction*, a difference in logical kind. Because Socrates is a thing, not a concept, the name "Socrates" cannot replace the word "loves" in "Plato loves Socrates" without creating nonsense. But the distinction between things and concepts is not a theory of types in the usual sense. Words representing things can *sometimes* meaningfully replace words representing concepts. Moreover, the distinction does not generate an infinite hierarchy. The concepts that occur predicatively in propositions in which the logical subject is a concept need not be distinct from those that occur in propositions in which the logical subject is a thing.

Indeed, Russell's views seem antithetical to more radical divisions of logical type. An argument parallel to the above argument that concepts must also be terms would be available to establish the termhood of any entity, so Russell is committed to a logical category—terms or individuals—to which all entities belong. It is true that Russell provisionally puts forth a theory of types in an appendix to the *PoM* which identifies classes and "ranges" as *objects* distinct from individuals or terms. However, he does not seem at all pleased with the theory, claiming about it that "the fact that a word can be framed with a wider meaning than *term* raises grave logical problems", and citing his argument that concepts must be terms (*PoM* p. 55n). Indeed, in the appendix itself, because identity statements are meaningful regardless of type, Russell is tempted to think there must be a universal type subsuming all others (*PoM* p. 525)—a concession that could easily be used to re-introduce the paradoxes. It is therefore unsurprising that by May 1903, the same month

PoM appeared in print, Russell had already rejected this early theory of types, announcing in a letter to Frege that "I believe I have discovered that classes are entirely superfluous" (*PMC* p. 158). Apparently, he found it more coherent to deny reality to classes altogether than to admit them but deny that they were terms or individuals. (More on this below.)

It is widely held, I think, that later on when he adopted the mature theory of types of *Principia Mathematica*, Russell (again) abandoned his insistence that every entity must be capable of occurring as logical subject in a proposition, and therefore is logically an individual, and came to hold that there are different types of things making up the world. However, this is not so. The theory of types in *PM*, unlike that in the appendix to *PoM*, focuses not on types of classes but types of propositional functions. However, as we shall see, by then Russell no longer regarded propositional functions as extra-linguistic entities. This has not been seen, in part, because too many mistakenly read Russell as having equated concepts/universals with propositional functions and (since he was clearly a realist about the former) attribute to him a realism about propositional functions. The first step in showing this reading to be mistaken comes in demonstrating that even when Russell *did*, early on, have an ontology of propositional functions as extra-linguistic entities, he did not equate them with universals. We begin by considering his views on propositional functions in *PoM*.

§6. Propositional Functions in **The Principles of Mathematics**

Russell derived the notion of a propositional function indirectly from Peano, who explains certain of his notations as follows:

> If propositions *a* and *b* contain the indeterminate objects *x, y*, …, … then $a \supset_{x, y...} b$ means: whatever *x, y* … may be,

from the proposition *a* one deduces *b*. [Peano 1889, p. 87]

> Let *a* be a proposition containing the indeterminate *x*; then the expression [*x* ε] *a*, which is read *the x for which a,* …, means the class composed of the individuals that satisfy condition *a*. [Peano 1889, p. 90]

Unfortunately, Peano was the progenitor of Russell's use/mention sloppiness. What he means is that if α and β are well-formed formulae containing the letters "*x*" and "*y*", used as variables, then $\ulcorner \alpha \supset_{x, y} \beta \urcorner$ is a well-formed formula to the effect that $\ulcorner \alpha \supset \beta \urcorner$ holds for all values of the variables, and $\ulcorner [x \, \varepsilon] \, \alpha \urcorner$ is a well-formed term naming the class of those possible values of "*x*" for which α is true. (The notation $\ulcorner [x \, \varepsilon] \, \alpha \urcorner$ later became $\ulcorner x \ni \alpha \urcorner$, with the 'ᴈ' read "such that".) Then it seems that by "proposition", Peano means *well-formed formula*, and by "variables" he means *letters not given a constant meaning*, and so his "propositions containing variables" are just open formulae. But Peano, unlike Russell, did not belabor the philosophical or metaphysical commitments of his logic, and sometimes he is simply confused about use and mention. In his writings, we find axioms and theorems such as the following (Peano 1889, p. 90):

$$a \, \varepsilon \, \mathrm{P} \, . \supset : [x \, \varepsilon] \, a \, . \varepsilon \, \mathrm{K}$$

Here, apparently, P is the class of all propositions, K the class of all classes, and it is asserted that if *a* is a proposition, the class of *x* such that *a* is a class. This is a use/mention mess: if the antecedent is false, would the consequent even be well-formed?

The symbolic logic Russell advocates in *PoM* is based mostly on Peano's logic, with a few changes. Russell chides both McColl and Peano for not distinguishing propositions

and propositional functions, noting that "*x* is a man" is not itself true or false, and thus not a proposition (*PoM* pp. 12-13, 19-20). He introduces propositional functions as follows:

> In any proposition … which contains no real [i.e., free] variables, we may imagine one of the terms, not a verb or an adjective, to be replaced by other terms: instead of "Socrates is a man" we may put "Plato is a man," "the number 2 is a man," and so on. Thus we get successive propositions all agreeing except as to the one variable term. Putting *x* for the variable term, "*x* is a man" expresses the type of all such propositions. A propositional function in general will be true for some values of the variable, and false for others. [*PoM* pp. 19-20]

Russell is tempted to define a propositional function as a class of propositions all differing from one another by the variation of a constituent, but he notes that the attempted definition, utilizing the notion of *class of all entities such that…*, requires a propositional function and therefore cannot be used non-circularly. Russell also considers defining them as relations-in-extension holding between certain definite entities and propositions in which those entities are combined with others in certain specifiable ways. Here the propositional function represented by "*x* is human" would be understood as the relation that holds between Socrates and *Socrates is human*, Boston and *Boston is human*, etc. But this definition suffers a similar defect, as definitions via relations-in-extension also presuppose propositional functions (*PoM* p. 509; cf. *FO* p. 53).

Speaking loosely, Russell equates propositional functions with "propositions containing a variable" (*PoM* p. 263). It would have been better to say that a propositional function is a complex unity just like a proposition except containing a variable rather than a definite term as one of its constituents (cf. *PoM* p. 356). Although he often speaks loosely, initially

defining a propositional function as an "expression" of a certain kind (*PoM* p. 13), it is clear that this is a slip: neither propositions, nor terms, nor variables are linguistic entities in Russell's early parlance. Russell explains variability in terms of his early (still somewhat crude) theory of denoting. According to this theory, the denoting concept *the Queen* is an entity such that, when it occurs in a proposition, the proposition is not about the concept, but about some entity (e.g., Elizabeth herself) to which the concept bears a special relation (*PoM* pp. 53, 62-63). Denoting concepts of the *the*-variety are unique in that, when they denote, they denote unambiguously, i.e., they denote one individual in particular. Other varieties of denoting concepts do not share this feature. Denoting concepts of the *any*-variety denote *ambiguously*. The denoting concept *any number*, Russell tells us, denotes only one number but no number in particular—admitting that there are difficulties with this view that he does not know how to solve (*PoM* p. 77).

I discuss the nature of these sorts of denoting concepts and the difficulties they engender elsewhere (Klement forthcoming). The important thing in this context is that Russell explains propositional functions in terms of his theory of ambiguously denoting denoting concepts. Although admitting certain complications, in general Russell likens the functioning of the variable letter "*x*" in the expression for a propositional function to the denoting concept *any term* (*PoM* pp. 89, 94). The denoting concept indicated by "*x*" denotes one term, though no term in particular, and hence "*x* is human", as a whole, designates one proposition, though none in particular. The propositional function *x is human*, in effect, is explained as a sort of "dependent variable", having an ambiguous denotation in virtue of its constituent variable's ambiguous denotation (*PoM* pp. 263-64). Russell, however, admits that his views on the relationship between

variables and propositional functions are far from fully settled in *PoM* (pp. 80, 89).

Russell's explanation of a propositional function as a proposition-like unity containing a variable is in some ways the polar opposite of the view expressed by Frege that the "*x*" that occurs in "$2x^3 + x$" does not belong to the function, and so the function would be better symbolized with "2.( )^3 + ( )". Indeed, Russell was a harsh critic of Frege's contention that one can, in general, get at a function or any other kind of single entity by simply "pulling out" a definite entity from a complete proposition (*PoM* pp. 505-508). This goes along with his rejection (as separable entities) of what he calls *assertions*, i.e., those purported entities gotten at by analyzing a proposition into logical subject and remainder (representing what is asserted about the subject) (*PoM* pp. 84-88). Russell's rejection of this view of functions is bound up with his views regarding the unity of propositions. First, consider such propositions as *Socrates loves Plato, Xantippe loves Plato, the number 2 loves Plato, Plato loves Plato*. These are the values of the function *x loves Plato*. They all have a sort of unity provided by the relation-in-intension *Love* (which Russell then called a *verb*); *Love* occurs in these propositions *as concept*, that is, it actually relates the two entities that occur there as term. (This can be seen in part by the fact that "loves" in the above examples is given a verbal declension.) These propositions share a common *form* in which one entity is related by *Love* to Plato. This form cannot be preserved by simply *removing* the logical subject, because in that case, in what remains, *Love* no longer relates anything to Plato. What they share is not simply the entities *Love* and Plato, nor even such an entity (if there is one) as *Loving Plato*. (Here, note the natural switch to the noun form!) The proposition *Loving Plato is easy to do* is not a value of the function *x loves Plato*.

The presence of the variable in the propositional function is thus necessary both to preserve the unity of the function as an entity and properly to encode the logical form of its values.

Russell writes, "the ϕ in ϕx is not a separate and distinguishable entity: it lives in the propositions of the form ϕx and cannot survive analysis", i.e., "the functional part of a propositional function is not an independent entity" (*PoM* p. 88). Russell's terminology and notation on these points can often cause confusion;[13] he admits that the variable that appears in the notation for a propositional function is not part of its "functional part", i.e., not part of that which remains constant between the various values of the function, but he denies that there *is* any such separable entity as *that-which-all-the-values-have-in-common*. The only proper way to represent their commonality is to retain the *variable* as part of the function to indicate how the entities are arranged in the values. As Russell puts it later on in *PoM* when summarizing his views:

> … in general it is impossible to define or isolate the constant element in a propositional function, since what remains, when a certain term, wherever it occurs, is left out of a proposition, is in general no discoverable kind of entity. Thus the term in question must not simply be omitted, but replaced by a *variable*. [*PoM* p. 107]

So we should not write "is human", or as Frege does, "( ) is human", for the function, but "*x* is human" so as to have a "schematic representation of *any* proposition of a certain type" (*PoM* p. 29).

[13] Indeed, Russell is often read wrongly here as denying that *propositional functions* are entities separable from their values. But it is fairly clear if one reads these passages in context that he distinguishes the function from its "functional part", and that when he speaks of the "ϕ in ϕx", by "ϕ" he means the functional part, or the *assertion*, not the function as a unity that contains the variable. I argue this in greater length in Klement (forthcoming).

It is fairly clear then that early Russell, unlike Frege, regards forms as prior to functions. A proposition is a unity; its unity is provided by certain of its constituents' occurring in it *as concept*, i.e., in a special way that holds together the other entities in that proposition. A propositional function is *itself* a kind of unity built up just like a proposition, except that rather than containing a definite entity at some position, it contains instead an *ambiguously denoting denoting concept*. On pain of circularity, the unity of a propositional function cannot be explained by the application of function to argument. Indeed, Russell rejects the Fregean notion of an "incomplete" function, and with it, any possibility of explaining the unity of propositions in virtue of such incompleteness. Russell opts instead for explaining propositional unity in terms of the occurrence of relations-in-intension or universals occurring *as concept*. The very approach demands a distinction between universals and functions. The universal *Love* is capable of different modes of occurrence. When it occurs as concept, the propositions in which it occurs are instances of the function "*x* loves *y*", but it is not identical with this function. The function contains variables; the relation does not. The *values* of the function (e.g., *Socrates loves Plato*) contain *Love*, but not the function. Moreover, our grasp of such propositions as *Socrates loves Plato*, while surely requiring an understanding of ("acquaintance with") the concept of *Love*, do not require a prior understanding of the function "*x* loves *y*", and indeed, an understanding of the function comes by first considering the variable-free proposition and imagining a term in it replaced by others.

It then seems that early Russell, unlike Frege, is entitled to say that propositional functions are always derived from complete logical forms (propositions) rather than vice-versa, because Russell, unlike Frege, provides an account of how it is that a complete logical form can be understood and represented *independently of* understanding it as built up from function and argument. In the sentences "Socrates is human" and "Plato loves Socrates", none of the individual words stands for a function: they all stand for individuals, some occurring as term, some occurring as concept. The function becomes involved only when variables are introduced. The story, however, does not end in 1903.

§7. *The maturation of Russell's views on functions and complexes: 1903-1905*

Russell read Frege carefully only while finishing the *Principles*. Realizing the great overlap between their views, Russell added an appendix on Frege and made minor changes to the body (see Grattan-Guinness 1996). While he criticizes many aspects of Frege's philosophy, including his take on functions and concepts, Russell was obviously intrigued. His deference for Frege led even to a last minute footnote endorsing Frege's ill-fated "solution" to Russell's paradox from the appendix to the second volume of *Grundgesetze*. That endorsement did not last long, but over the next year and a half, Russell moved in the direction of Fregean views and interests. His posthumous works from 1903-1904 bear Fregean titles such as "Functions and Objects" and "On Meaning and Denotation". I like to call this Russell's "Fregean period".

One strikingly Fregean aspect of the period is his brief consideration of a view according to which functions and concepts are identified, and functions are taken as that which provides "the logical genesis of all complexes" (*FO* p. 50; cf. *OMDP* p. 287). He continues:

> In common language, verbs, prepositions and in a sense adjectives, express functions; the words that do not express functions may all be called, by a slight extension,

proper names. Thus functions … are simpler than their values: their values are complexes formed of themselves together with a term.

Here Russell takes the relation *Love* to be a function, which, when adhering with an argument, forms a proposition. Other sorts of complexes result when the function is what he then called "denoting" rather than "undenoting" or "propositional". An example of the former is the function expressed by "the positive square root of …". If it takes an argument, say, the number four, the result is again a complex, consisting of the function and the argument. This is what Russell calls a "denoting complex" and can be thought of as a complex meaning that *denotes* the number two according to Russell's quasi-Fregean theory of meaning and denotation.[14] When such a complex occurs in a proposition, the proposition is not about it but instead about the object denoted. So if we consider the complete proposition expressed by the sentence "The square root of four is even", one constituent is a denoting complex, and it is in virtue of it that the proposition is about the number two. The remainder of the proposition, the function *is even*, also forms complexes, but it forms propositional complexes rather than denoting complexes. Here Russell says the function is simpler than its values, which shows that he no longer regards the variable as a constituent of the function (for otherwise, the function and its values would be equally complex).

Russell's views here are not fully Fregean, yet there certainly is inspiration from Frege. In some ways, he does a better job than Frege himself making good on some of Frege's slogans. Here Russell's functions really *are* incomplete and really can be understood as combining with their arguments to form wholes. Granted, for what Russell then called "denoting functions", the wholes so formed are not themselves the values, but they are their representatives in propositions (just as complex Fregean senses are the representatives of objects in thoughts). Moreover, it was during this period, inspired by Frege's treatment of value-ranges (*Werthverläufe*) of functions, that Russell (*PMC* p. 158, cf. *TLP* 6.031) reached the conclusion that classes were "entirely superfluous", and that functions could be made to do all the work of classes (see Klement 2003).

However, these quasi-Fregean views did not last. One reason is that they put function before form. If universals (qualities and relations) are understood as functions, functions are some of the basic building blocks of propositions. They cannot, on pain of circularity, be understood as things gotten at only by analysis or by decomposing complete propositions. Propositions presuppose functions, and their forms cannot be understood independently from their genesis by means of functions. This required thinking of functions as independent entities, and true to his old self, Russell continues to believe it impossible for there to be any fully real entities that cannot occur as logical subjects. Hence he could not find a philosophically adequate way of eliminating the assumption that there must be a type subsuming all entities, and therefore that some functions must take all entities, including themselves, as argument (see, e.g., *FO* p. 52). However, this gave him no way to escape from the functions version of Russell's paradox. He described the failing of his views of this period in a 1906 letter to Jourdain:

> Then, in May 1903, I thought I had solved the whole thing by denying classes altogether; I still kept propositional functions, and made ϕ do duty for $\acute{z}(\phi z)$. I treated

[14] The vocabulary of "denoting complexes"—new to these 1903 manuscripts—even makes its way into Russell's famous attack on Fregean theories of meaning in *OD* rather than the vocabulary of "denoting concepts" found in *PoM*. Whether this has any bearing on how to interpret *OD* is a topic for another occasion.

ϕ as an entity. All went well till I came to consider the function W, where $W(\phi) . \equiv_\phi . \sim\phi(\phi)$. This brought back the contradiction, and showed that I had gained nothing by rejecting classes. [Quoted in Grattan-Guinness 1977, p. 78]

Notice how simple and straightforward the reasoning is here. He describes the problem with the view as being that it treats functions as entities, suggesting that any view that does so must admit that they could then constitute their own arguments, and thus that a proper solution must come from denying that they are entities. However, so long as functions are presupposed as the logical basis of all complexes, they cannot easily be scrapped.

Throughout the next two years we find Russell again and again returning to the question as to whether functions or complexes are more fundamental (see, e.g., *CF* pp. 69ff; *OFns* pp. 98f; *FN* pp. 251f; *NF* pp. 265f; *OMD* pp. 338f). In late 1903 or early 1904, he returns to a view according to which functions and concepts are distinguished, and it is through concepts' occurring predicatively that complexes are formed. He also returns to a view of propositional functions as denoting entities involving variables. In particular, a propositional function is described as a "dependent variable", or a denoting complex that ambiguously denotes propositions, with which proposition it denotes depending on the object ambiguously denoted by the "independent variable", which it contains as a constituent (see, e.g., *OMD* pp. 342-55). In April 1904, prompted by a letter from Whitehead (see *CPBR* v. 4, p. xxiv), Russell begins using the circumflex notation familiar from his later writings as a method of talking about a propositional function itself as opposed to an arbitrary value. He describes it explicitly as a way of speaking about a denoting concept or meaning as opposed to its denotation:

The circumflex has the same sort of effect as inverted commas have. E.g., we say

Any man is a biped;
"Any man" is a denoting concept.

The difference between $p \supset q .\supset. q$ and $\hat{p} \supset \hat{q} .\supset. \hat{q}$ corresponds to the difference between any man and "any man". [*FN* pp. 128-129]

The relation between a propositional function and one of its values is here described as the relation of denoting concept and entity denoted (see also *OMD* p. 355). This relation is not one of whole and part, as Russell says explicitly during this period. To see this, one must understand the connection Russell draws between propositional functions and modes of combination. Take the proposition expressed by "A is greater than B"; Russell tells us that its constituents are A, *Greater than*, and B (*OFns* p. 98). Here, *Greater than* is not a function but a relation. The proposition is more than the sum of its constituents; the constituents are combined in a certain way. In this case, the relation *Greater than*, which might occur as logical subject in a different proposition, occurs in *this* proposition *as relating* A to B. The mode of combination involved here is shared by all propositions in which one thing is related to another via a binary relation (in intension). This mode of combination, Russell tells us, is not a constituent of those propositions having this form, reasoning as follows:

For if it were, it would be combined with the other constituents to form the complex; hence we should need to specify the mode of combination of the constituents with their mode of combination; thus what we supposed to be the mode of combination of the constituents would only be a mode of combination of *some* of the constituents. [*OFns* p. 98]

Russell uses the notation "$\hat{x}\hat{R}\hat{y}$" for this mode of combination, suggesting that a mode of combination is in essence an ambiguously denoting denoting complex consisting *entirely* of variables, so that no constituent is shared by the values. Typically, a *function* is a mixture of form (mode of combination) and content, or a particular way of combining entities with certain other, possibly constant, entities. So "$\hat{x}$ is greater than A" and "A is greater than $\hat{y}$" represent functions; the former represents one mode of combining *Greater than*, A, and something else to form a proposition, the latter representing a different mode. However, the only constituents of the values of the functions are *Greater than*, A, and whatever other entity enters in. The functions are not constituents, any more than the mode of combination is a constituent. Indeed, it is possible to conceive of the value without conceiving of it *as* the value of any function. As Russell puts it, "there is no $\phi'\hat{x}$ *presupposed* by $\phi'x$" (*FN* p. 118); while the proposition "A is greater than B" presupposes or contains A, *Greater than*, and B, it does not presuppose any functions. It of course *does presuppose* the relation *Greater than*, and so the relation cannot be equated with a function (see, e.g., *FN* p. 130).

Russell comes quickly to the conclusion that the fact that a function is not a constituent of its values is one of great importance. He writes, "I think it is fairly certain that the root of the Contradiction is the fact that $\phi'\hat{x}$ is not a constituent of $\phi'x$" (*FN* p. 154). He takes this insight in different directions. At times, he is tempted to think that even though functions are not constituents of their values, they *are* always entities, and hence must occur as constituents (logical subjects) in some propositions, albeit not, in general, those propositions that are their values (*OFns* pp. 98-99). However, coupled with the assumption that for every proposition containing some term *A*, the function or mode of combination of *A* with the other constituents is a separate entity, this view falls prey to the paradox. Other times, he concluded that *sometimes* there is a separable entity, sometimes there is not (e.g., *OSL* pp. 78-84). However, he was never able to delimit under what conditions there was such an entity adequately.

The situation was resolved only in 1905 when Russell decided to reject the assumption that there is *ever* such a separable entity, having come upon a theory that allowed him to do the work of functions without having them in his ontology.

§8. The substitutional theory and the birth of ramification

In 1905, the new theory of descriptions allowed Russell to do without what he had previously called denoting functions. Doing without propositional functions was relatively more tricky. In late 1905, Russell came to the view that the notion of a propositional function could be replaced by the notion of the substitution of one term for another within a proposition. The resulting "substitutional theory" (see, e.g., *OS*, *STCR*) made use of a four-place relation written "$p/a;b!q$", which means that *q* results from *p* by substituting *b* for *a* wherever *a* occurs as term in *p*. For example, this relation would hold when *p* is the proposition *Socrates is wise*, *a* is Socrates, *b* is Plato, and *q* is the proposition *Plato is wise*. Rather than representing what is shared by *Socrates is wise* and *Plato is wise* by portraying them as two values of the same propositional function, one portrays them as the first and last relata of this relation.

In this way apparent discourse about the propositional function $\hat{x}$ *is wise* can be proxied by talk about the pair of *p* and *a*; its value for any argument *x* can be replaced with the description $(\imath q)(p/a;x!q)$, i.e., the proposition that results by replacing Socrates with *x* in *Socrates is wise*, treated using the

mechanics for the theory of descriptions. All propositions of this form could then be represented in this way (including p itself, which is trivially the same as $(\imath q)(p/a;a!q)$). Claims he previously would have understood as about the function $\hat{x}$ *is wise*, he now analyzes as saying something more complicated about p and a. In this way, the substitutional theory could make do with only one style of variable, including in its range all genuine entities, including propositions and their constituents (*STCR* p. 171; *LPL* p. 206).

It should be noted that in abandoning propositional functions as he had understood them, Russell was not abandoning universals. *Wisdom* is still very much a constituent of the proposition that *Socrates is wise*, and there is no way of understanding its make-up otherwise. He is instead abandoning the notion that there is a *single entity* corresponding to the commonality of *form and content* shared by *Plato is wise*, *Socrates is wise*, etc. (but not by *Wisdom is a virtue*). Nevertheless, this commonality is not lost. He does not proxy $\hat{x}$ *is wise* simply using the pair Socrates and *Wisdom*, but using Socrates and the complete proposition *Socrates is wise*. We use a complete proposition as "prototype" rather that just mentioning the other constituents over and above Socrates, because they alone would not provide us with an understanding of the proposition's form or the ways in which the entities are to be combined.

This substitutional "proxy" for propositional functions leads to certain formal results very similar to those of simple type theory. Those non-paradoxical assertions seemingly about "propositional functions" that can be formulated in simple type theory can also be formulated in proxy form in the substitutional theory, and yet those would-be assertions ruled out in simple type theory similarly cannot be formulated in the substitutional theory. In particular, the substitutional theory provides a ready-made solution to the functions version of the paradox. A "value" of the function $\hat{x}$ *is wise* has been replaced by the notion of something that results when an entity replaces Socrates in *Socrates is wise*. However, there is no *single* entity corresponding to the function to swap for Socrates, and it is meaningless to speak of simultaneously swapping Socrates with a *pair* of entities, since a pair of entities is not itself an entity. So would-be discourse about a function taking "itself" as argument cannot be translated into the theory. Russell uses the proxy for functions to provide a proxy for classes, and the classes version of the paradox is similarly solved. However, metaphysically, the substitutional theory is not committed to any hierarchy of entities. Indeed, exactly what Russell most likes about the theory is that it avoids the paradox without postulating any *entities* not regarded as individuals:

> … it affords what at least seems to be a complete solution of all the hoary difficulties about the one and the many; for, while allowing that there are many entities, it adheres with drastic pedantry to the old maxim that, 'whatever is, is one.' [*STCR* p. 189]

This view gives a thorough-going priority to complete propositions over functions, and this priority moreover *explains why a function cannot take itself as argument*. In 1906, the diagnosis of the paradox is that functions are not really entities at all but are merely *façons de parler* used to speak about the results of substitutions with complete propositions. Given that he had previously understood functions as denoting entities, eschewing commitment to them as entities was *demanded* by his own argumentation in "On Denoting" that any theory that requires disambiguating truths about denoting entities themselves from those about their denota-

tions leads to "inextricable tangles" (*OD* p. 422).[15]

Many Frege sympathizers hold that Frege's views on functions as entities gotten by decomposing thoughts, and on functions as incomplete entities, provide him with a sound philosophical rationale to suppose it incoherent to imagine a function taking itself as argument. But as we have seen, Frege's views on these matters are not quite so clear cut when carefully scrutinized. Here we find in Russell a very sophisticated position in which an account of functions "as derivative entities" (or, as not really entities at all) is explained in detail, and a full explanation is given as to why a "function" cannot take itself as argument.

Unfortunately, while the substitutional theory solved Russell's paradox in this form, not all logical paradoxes involve something as simple as a function taking itself as argument. He soon found Cantorian paradoxes resulting from his continuing assumption that propositions are individuals.[16] Because the assumption of propositions as entities was necessary for his proxy for propositional functions, and that proxy was in turn necessary for his "no classes" form of logicism, Russell was in need of a fix. The fix was his *ramified theory of types*, first presented in the paper "Mathematical Logic as Based on the Theory of Types." There he divides propositions into various orders depending on what entities they quantify over. Those that do not quantify over propositions are put in the lowest order, those that quantify over lowest order propositions are put in the next higher order.[17] While Russell does also return in that paper to a vocabulary of "propositional functions" of different types, he describes this as a convenience, and briefly sketches the substitutional-theory method of eliminating commitment to them (*ML* p. 77).

Substitution provides a philosophical explanation for types; this leaves only orders to be explained. Russell defines an *individual* as "something destitute of complexity" (*ML* p. 76). Propositions, by contrast, are "essentially complex" (*ML* p. 96; cf. *NT* p. 503). If everything destitute of complexity is an individual, the simplest complexes—among which Russell would surely include elementary propositions—must only have individuals as constituents. This may seem to conflict with what Russell writes:

> In an elementary proposition we can distinguish one or more *terms* from one or more *concepts*; the *terms* are whatever can be regarded as the *subject* of the proposition, while the concepts are the predicates or relations asserted of these terms. The terms of elementary propositions we will call *individuals*; these form the first or lowest type. [*ML* p. 76]

Here it may seem that he thinks that only *some* entities making up an elementary proposition are individuals, i.e., those occurring *as term*. But where in the type theory do the other entities fall? Are they propositional functions? No; these are to be eliminated! They are certainly not propositions. The solution is to remember that Russell believes that concepts have a two-fold nature; they *can* predicatively oc-

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[15] It is striking that in Russell's manuscript "On Fundamentals", in which the theory of descriptions is first introduced, almost the bulk of the manuscript *after* the introduction of the theory of descriptions sees Russell attempting to find a way to do logic without treating variables as akin to *any*-style denoting concepts as he had previously (see *OF* pp. 385-413). Clearly Russell immediately saw the tension between his old views on functions and the new theory of denoting he wanted to adopt.

[16] In his published writings, Russell's usual example was the *Liar paradox*, taken in the form "All the propositions I am currently expressing are false" (see, e.g., *PL passim*, *LPL* pp. 204-213, *ONT* pp. 46-47). However, there were other, non-semantical paradoxes involving quantification over propositions that do not, like the Liar paradox, involve any "contingent assumptions". For excellent further discussion of the paradoxes plaguing substitution, see Landini 1998, chap. 8.

[17] Russell himself did not stick to the contemporary vocabulary between "order" and "type" used in contemporary discussions of ramification, but it is now too common and too helpful to forgo.

cur in propositions, but they can also occur as subject. There is no evidence that Russell has changed his mind that it is self-contradictory to deny of a given concept that it can occur as logical subject. So concepts *are* subjects in some elementary propositions, and therefore concepts are individuals, even in the ramified type theory of *ML*.

But wait: doesn't this re-introduce the paradox in the concepts-not-applicable-to-themselves form? If *Blueness* is an individual, then we can ask whether or not *Blueness is blue*, and it isn't. If being a concept (i.e., *Concepthood*) is a quality some individuals have and some individuals lack, then we can ask whether *Concepthood* is a concept, and it is. Isn't there then a concept *Non-self-applicability*, of which we can ask whether or not it is non-self-applicable?

No, Russell is safe. There is nothing in his philosophy preventing him from having a *sparse* view of universals. Granted, he seems to want an abundant theory of *propositional functions*, but he is not committed to a concept or universal for every propositional function.[18] Consequently, he is not committed to such a concept as non-self-applicability. It is true that for genuine concepts, those contained in atomic propositions, one can ask whether or not they apply to themselves. Suppose *Blueness* is one. Then we have a proposition *Blueness is blue*. Presumably, this is false, and so the proposition ~(*Blueness is blue*) is true. Remember that on the substitutional theory, propositional functions are proxied through the notion of substitution, which is defined so as to treat of logical subjects only. The matrix consisting of ~(*Blueness is blue*) and *Blueness* proxies the propositional function ~($\hat{x}$ *is blue*) only. To get something more relevant,

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[18] Russell's terminology on some of these points can cause confusion. When he means universals, Russell usually uses either of the words "predicate" or "quality", but sometimes he also uses the word "property", though when he says "property" he *usually* means *propositional function*. For an explicit statement of the difference, see *MPD* p. 124, and for further discussion, see Linsky 1999, chap. 2.

we would have to suppose that there is such a relation (in intension) as *instantiating* or *exemplifying* or simply *having*. Russell early on admitted such a relation (see *PoM* pp. 45, 49), but even if he still does, no contradiction looms. From the matrix pair of ~(*Blueness has Blueness*) and *Blueness*, we can proxy the propositional function ~($\hat{x}$ *has* $\hat{x}$) and thereby speak of any given proposition obtained by substituting some individual (including some other universal) for Blueness in ~(*Blueness has Blueness*). However, Russell is not committed to a single entity corresponding to this matrix, and so does not have to face this version of the paradox. The overarching view is this: *individuals* are those *simple* entities out of which propositions are formed, including both universals and particulars. "Logical fictions" constructed by means of propositions, including classes and propositional functions, are not *really anything at all*. Individuals are metaphysically prior to propositions, but propositions are conceptually prior to propositional functions and classes, the latter notions being mere *façons de parler*. But 1908 is not the end of the story, either.

§9. *Russell's mature position in* Principia Mathematica

The type theory presented in the 1910 first edition of *PM* differs from that presented in 1908. Nowhere in *PM* does Russell invoke the substitutional theory. Why the change? As we've seen, to avoid antinomy, Russell needed to divide propositions into different orders. As I have attempted to make plain, Russell was *never* philosophically satisfied with any theory that divided genuine entities into logical types. All entities must be capable of occurring as logical subject, for if not, the proposition that such entities are not logical subjects becomes impossible and the theory is self-undermining. This was Russell's reason for thinking that concepts were individuals in *PoM* and the reason he soon abandoned his first theory of types in 1903. It also played a

pivotal role in the birth of the substitutional theory, as he rejected his early ontology of propositional functions explicitly because he could not see how, if they were entities, he could avoid the assumption that they could be their own arguments. Without a philosophical *explanation* for orders similar to the philosophical explanation of *types* that comes from the substitutional theory, ramification was philosophically suspect. His response was predictable. Just as he had abandoned classes, denoting complexes and propositional functions, when the assumption that they were entities (and hence logical subjects on par with concrete individuals) led to paradox, Russell excised propositions from his ontology. He restructured his ontology around *facts*, and adopted a new theory of judgment (the multiple-relations theory) not requiring propositions as relata in propositional attitudes. Of course, this meant throwing out the substitutional theory's method of proxying propositional functions, which required treating a proposition as a singular, albeit complex, entity. Russell needed a new approach.

In *PM*, Russell reintroduces the vocabulary of "propositional functions" and sometimes even returns to the terminology of "ambiguous denotation" (e.g., *PM* p. 39). Propositional functions are divided into types, and are distinguished from individuals. Rather than invoking the substitutional theory, Russell elaborates on his type theory by invoking what he calls the "vicious-circle principle", or the principle that "whatever involves *all* of a collection must not be one of the collection" (*PM* p. 37). However, there is widespread disagreement as to what propositional functions now amount to. Some, e.g., Peter Hylton (1990) and James Levine (2001), allege that Russell returned to an ontology of intensional entities containing variables that ambiguously denote their values. Others give a nominalistic understanding of the "propositional functions" spoken of in *PM* (e.g., Landini 1998). The opposing parties of this debate typically have a very different understanding of the vicious-circle principle, and whether it is a semantic or an ontological principle. While Russell's sloppiness with regard to use and mention hinders a definitive resolution of the dispute, I think we have very good reason to think the realist reading is mistaken. Let us first look at the reasons against the realist reading and then attempt to sketch what Russell's views probably were.

Russell later described his philosophical development as a "retreat from Pythagoras" (*MPD* p. 154): beginning with a robust metaphysics of abstracta, he slashed more and more away with Occam's razor. Re-adopting propositional functions would be a large step back towards Pythagoras. One of the chief advantages he saw for the theory of descriptions was that it had made it possible to excise denoting entities from his metaphysics. Through most of the period prior to 1905, Russell equated propositional functions with dependent variables, which in turn he identified with a kind of ambiguously denoting denoting complex. Those who attribute to Russell a realism about propositional functions must read him either as having backslid to his older view—highly unlikely given what is at stake—or as having adopted some wholly different conception of propositional functions. However, nowhere does Russell present any new or revised theory about what sort of non-linguistic entity a propositional function could be. As the 1906 letter to Jourdain (quoted earlier) testifies, Russell had thought he had found convincing reason against thinking of propositional functions as entities. Attributing a robust realism about propositional functions to Russell in 1910 leaves an explanatory gap as to why Russell would ever have abandoned his first (1903) "no classes" theory. It would also leave unanswer-

able the question as to what the "values" of these functions would be, since Russell has dropped propositions as extra-linguistic entities from his metaphysics, a problem noted by many commentators on *PM* (see, e.g., Church 1974). There is also later textual evidence, such as his claim in *My Philosophical Development* that "Whitehead and I thought of a propositional function as an expression" (*MPD* p. 92) and indeed as "only an expression" (*MPD* p. 62). But most of all, the view simply reads Russell as *abandoning* his scruples that there must be a type of occurrence in facts—occurring as subject or term—of which every genuine entity is capable.

In order to get a better view of Russell's position, we must understand how he uses the word "proposition" in *PM*. Unfortunately, his usage fluctuates. Sometimes he says simply that propositions are "incomplete symbols" (akin to definite descriptions or class terms), which of course means that propositions are *symbols*, i.e., parts of language. Other times he expresses himself by saying that propositions are non-entities: things which he had imagined to be real, but as fictional as Meinong's golden mountain. There is yet another usage. Since he now believes that someone who sincerely says "Socrates is wise" is one constituent of a fact in which she stands in a "multiple relation" to Socrates and *Wisdom*, Russell sometimes hints that talk about a "proposition" *Socrates is wise* is just a roundabout way of talking about all those facts in which Socrates and *Wisdom* occur in certain positions as relata in judgments.[19]

We cannot afford to be similarly inconsistent. When not explicitly discussing their status as entities, Russell usually means by "propositions" nothing other than sentences that

[19] It is this last usage that eventually becomes the account of propositions Russell suggests in the 1913 *Theory of Knowledge* manuscript (p. 115), on which a proposition is identified with the existence fact of there being a subject bearing some propositional attitude relation to the entities making up the content of the attitude. Russell is not quite to this position in 1910, and indeed *PM* does not yet countenance general or existence facts. More on this below.

express judgments, true or false. Propositional functions are introduced in *PM* as follows:

> By a "propositional function" we mean something which contains a variable x, and expresses a proposition as soon as a value is assigned to x. That is to say, it differs from a proposition solely by the fact that it is ambiguous: it contains a variable of which the value is unassigned (*PM* p. 40).

On the reading given here, variables are now not abstract ambiguously denoting entities but just *letters* used in a particular way within formal languages, and a propositional function is just *an open formula*: it is a sentence that contains a free variable. So far this is just terminological. Making good on a nominalist reading of Russell's use of propositional-function variables and quantifiers requires giving to Russell a reading that does not commit him to a special kind of entity corresponding to open formulae or quantified over by higher-order variables. In other words, we need to provide a *semantics* for the statements of the language of *PM* without postulating special complex intensions corresponding to propositional functions. Although Russell does not present a semantic theory up to the standards of rigor practiced by contemporary formal semanticists, I believe at least the core of such a theory can be extracted from a close reading of *PM*'s introduction. When this is done, it can readily be seen that the theory of types is not an ontological theory of metaphysical divisions but a theory of types of meaningful expressions, only *one type of which* is directly related to entities. It can further be seen that the vicious-circle principle is not a principle about ontological dependence but about the conditions under which sentences have adequate truth conditions.

The key to these points lies in understanding how Section III of Chapter II of the introduction to *PM*, entitled

"Definition and Systematic Ambiguity in Truth and Falsity", fits in the context of those sections that precede and follow it. These sections deal with the vicious-circle principle (Section I), the nature of propositional functions (Section II), why a propositional function is restricted to arguments of a certain type (Section IV), and the hierarchy of functions and propositions (Section V). Realist readings of *PM* typically pay little or no attention to Section III nor attempt to give an explanation for why it is placed where it is. On my own reading, this section is placed where it is because it contains the seeds for the semantic theory Russell intends for *PM*, and it is this semantic theory that ultimately provides the philosophical justification for the vicious-circle principle and the type theory as a whole. The section largely consists in Russell's attempts (admittedly crude by modern standards) to characterize the different notions of truth applicable to different kinds of formulae.[20] When these characterizations are examined, it is seen that at *no* stage does Russell posit complex intensional entities as the ontological correlates of propositional functions.[21]

He begins with a sketch of the truth conditions for what he calls "elementary propositions". Unfortunately, Russell's terminology in the 1910 *PM* is not as rich as the terminology he employs in later works, and he does not yet utilize the distinction between "atomic" and "molecular" (cf. *PLA* chap. III). Officially, Russell includes under the rubric of "elementary propositions" both (what would later be called) atomic propositions and *quantifier-free* molecular propositions (*PM* pp. 54, 91-92). However, sometimes, as in Section III of Chapter II of the introduction to *PM*, when Russell discusses "elementary" propositions, his attention seems narrowly focused on atomic propositions. Here, when Russell describes elementary propositions and judgments, he describes them as taking forms such as "*a* has the relation *R* to *b*" and "*a* has the quality *q*" (*PM* p. 44). Notice that Russell writes "quality *q*" and not "quality *ϕ*" or anything suggesting a function. (Again, we must remember that he does *not* utilize the notation of contemporary predicate logic, where the form "*Fa*" makes the "*F*" look functional.) Russell does not say what individuals, qualities, and relations (here, in intension) there are or have names in his language, but this is precisely because *PM* lacks an extra-logical vocabulary. I take it that the words "the quality" and "has the relation" in these examples are extraneous. Really, what Russell means is that if "*q*" stands for a simple quality (i.e., a universal to which one thing may or may not correspond), and "*a*" names an individual, then "*a* is *q*" is an elementary proposition. Similarly, if "*r*" stands for a relation-in-intension of the sort that relates two individuals, then "*a r*'s *b*" is an elementary proposition, etc.[22]

Notice that it is still Russell's view that names of universals can occur in subject position, so that "*q* is *q*" and "*r r*'s *r*" would also be well-formed. This is explicit in other works of the same period, such as the 1911 paper "Le Réalisme analytique", where he writes:

> … in every complex there are two kinds of constituents: there are terms and the relation which relates them: or there might be (perhaps) a term qualified by a predicate. Note that the terms of a complex can themselves be re-

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[20] Ever since van Heijenoort 1967, there has been a tendency to think that Russell's (and Frege's) views are antithetical to the very project of giving a semantic theory. However, I do not find van Heijenoort's arguments convincing. This issue cannot be fully addressed here, but for persuasive criticism of this tendency, see Landini 1996; 1998 (chaps. 1 and 10).

[21] I follow Landini (1998, chap. 10) in taking this general line of interpretation; Landini and I differ over the some the details, however. See, e.g., note 25 below.

[22] Here I use lowercase "*r*" as opposed to uppercase "*R*" because we are not dealing with relations-in-extension but relations-in-intension, and uppercase letters "*P*", "*Q*" and "*R*" are officially used for relations-in-extension in the symbolism of *PM*. One should not confuse a proposition "*aRb*", where "*R*" is used for a relation-in-extension, with the sort of notation found at the elementary level, as the former sort of notation is actually an abbreviation for a complex formula involving higher-order quantification (see *PM* pp. 81-82).

lations, as, for example, in the statement that priority implies diversity. But there are some terms which appear only as terms and can never appear as predicates or relations. These terms are what I call *particulars*. The other terms found in a complex, those which can appear as predicates or relations, I call *universals*. [*RA* p. 135]

Russell gives no indication that he believes that the proposition "priority implies diversity" needs rewording or restructuring into a form in which these universals appear predicatively (e.g., as "$(x)(y)(x$ is prior to $y \supset x \neq y)$"), and indeed, his point would be lost if he did. A year later, in the paper "On the Relations of Universals and Particulars", Russell again defines particulars as those entities that can "only" occur as subject in a fact, while universals are those that "can" occur as predicates or relations in facts. While he does not explicitly say that universals *do* appear as subject, his phrasing seems designed to leave that possibility open, and indeed, there is no indication that he thinks that the theory of types is at all relevant to the distinction between universals and particulars; *both*, as the basic building blocks of facts, are individuals. As he puts it in another work of the period, "an individual is a being in the actual world, as opposed to the beings of the logical world" (*SAIT* p. 44). As with the case of *ML*, the assumption that both universals and particulars are individuals does not lead to paradox, as Russell's ontology of genuine universals is sparse.

All the signs making up a variable-free atomic proposition stand for individuals. However, a proposition is not a mere concatenation of names: one of the signs must stand for a universal occurring predicatively. We can assume that the copula "is" occurring in "q is q", while not standing for an entity, helps indicate the *form* of the proposition, i.e., that the second occurrence of "q" stands for a quality occurring *as a*

concept.[23] An atomic proposition corresponds to what Russell calls an "elementary judgment", which is a fact in which a certain subject stands in a multiple relation to those entities. In that fact, the relation or quality occurs as just one relatum in the multiple relation, but the judgment is *true* if and only if there is a fact in which that universal holds of the other entity or entities in the judgment. So for example, "*a is q*" is true if and only if the entity *a* has quality *q*, and "*a r's b*" is true if and only if the entity *a* stands in relation *r* to *b*, as one would expect (cf. *ONTF*).

While it is possible for an atomic proposition to be a value of a propositional function, Russell makes it clear that atomic propositions are not constructed from signs for propositional functions: indeed, this is part of his *definition* of an elementary proposition (*PM* p. 54). He writes:

> It should be remembered that a function is not a constituent of its values: thus for example the function "$\hat{x}$ is human" is not a constituent of the proposition "Socrates is human." [*PM* pp. 54-55]

Indeed, the values of a propositional function are said to be prior to the function (*PM* pp. 39-40). While Russell pushes his theory of propositional comprehension or epistemology into the background in *PM*, in other works of the period he makes it clear that to understand a proposition one must be acquainted with the entities represented by all its independently significant parts, including any universals (e.g., *KAKD* pp. 149-150; *RA* pp. 133-135). However, understanding an atomic proposition does not require acquaintance with any functions. As Russell writes in *PM* (p. 39), "the proposition 'Socrates is human' can be perfectly apprehended without regarding it as a value of the function 'x is human.'" While

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[23] He seems to have abandoned the view of *PoM* (p. 49) that the copula indicates a special kind of entity or relation. Russell is explicit about this in works written not long after *PM* (see, e.g., *TK* p. 98; *OKEW* p. 40).

the word "human" that occurs in "Socrates is human" must be thought to represent a universal, this universal is certainly not equated with (the ontological correlate of) a function.

The facts that make true atomic propositions true only consist of individuals. Obviously, then, their semantics does not require positing any intensional entities corresponding to propositional functions. But that's the easy case. When we turn to molecular and quantified propositions, Russell is less clear about what his semantics presupposes metaphysically. While he clearly posits different kinds of "judgment" corresponding to different kinds of propositions (*PM* pp. 44-46), he is not as clear about what the constituents are of those kinds of judgment-facts, nor what the constituents are of the facts that would make the judgments true or false.[24] It is certainly possible that his mind was not entirely made up on the issue when he wrote *PM*, and indeed, one of his chief projects to follow *PM* was to make efforts to fill in the philosophical details of his logical views. The topic of molecular and quantified judgments was due to be covered in the *Theory of Knowledge* work of 1913, but unfortunately he abandoned it before reaching those topics. In *PM*, Russell does not discuss moleculars separately, perhaps because he had no separate label for them as opposed to atomic elementary propositions. However, we can imagine his having one of two views. Either a true molecular proposition corresponds to a single molecular fact, or it is simply made true indirectly through the truth or falsity of its constituent propositions. Russell's 1910 view was most likely the latter of the two (and that is certainly his view a few years down the road—see *PLA* p. 74). On that view there is simply more than one fact involved, each of which itself consists of nothing but individuals. On either reading it is implausible to suppose that Russell would regard the ontological correlates of propositional functions—were he to admit any such things—as involved in the truth conditions of molecular propositions. Indeed, even when he clearly does have in mind both atomic and molecular quantifier-free propositions when speaking of "elementary propositions", he is explicit that elementary propositions "contain no functions" (*PM* p. 54).

For general propositions, the explicit doctrine of *PM* is that they don't correspond to a new kind of complex. Russell makes it clear that once a notion of truth, so-called "first truth", has been given for quantifier-free propositions, it is necessary then to move to a new notion of truth for propositions containing (first-order) quantifiers. His exposition (*PM* p. 42) of this unfortunately is once again sloppy about use and mention:

> Consider now the proposition $(x).\phi x$. If this has truth of the sort appropriate to it, that will mean that every value ϕx has "first truth." Thus if we call the sort of truth that is appropriate to $(x).\phi x$ "second truth", we may define "$\{(x).\phi x\}$ has second truth" as meaning "every value of $\phi \hat{x}$ has first truth," *i.e.* "$(x).(\phi x$ has first truth)".

This discussion is clearly meant to be carried out in the metalanguage, and we must forgive his use of an object-language quantifier where he should be using the quantificational devices of the metalanguage, and his use of "ϕ", which appears to be a variable of the object-language, when he seems to be using "ϕx" schematically for any wff with no

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[24] To be sure, later commentators have tried to foist a theory upon him. Cocchiarella (1987, pp. 196-206) gives him a theory that would require the supposition of an ontology of complex intensional entities corresponding to propositional functions as constituents of general judgments. Indeed, Cocchiarella's interpretation, mistakenly equating universals with propositional functions in this period of Russell's thought, requires propositional functions as intensional entities even to make sense of atomic judgments. However, everything we have said up till now mitigates against this interpretation. Hochberg (2000) gives a similar misreading.

bound variables and "*x*" as its only free variable. What he means is that a quantified proposition is true if and only if every proposition got by replacing its variable with an individual constant is true, or more precisely:

> Consider ⌜(*x*). *A*⌝ where *A* contains the variable "*x*" free but no additional bound or free variables. The proposition ⌜(*x*). *A*⌝ has second truth iff for every individual constant **c**, the proposition obtained by replacing "*x*" with **c** throughout *A* has first truth.

Here, Russell makes the truth or falsity of a quantified proposition depend entirely on the truth or falsity of its instances. He even goes so far as to suggest that when a quantified judgment is true, the only facts that correspond to it are the facts pointed to by its instances, writing:

> If ϕx is an elementary judgment, it is true when it *points* to a corresponding complex. But (*x*).ϕx does not point to a corresponding complex: the complexes are as numerous as the possible values of *x*. [*PM* p. 46]

Suppose that every individual has quality *q*. Then "(*x*)(*x* is *q*)" is true, but it is made true entirely by the complexes consisting of *q* and the other individuals. Nothing else is involved; certainly Russell suggests nothing to the effect that there is any additional *general fact* involving ontological counterparts to quantifiers, variables, or propositional functions.

This might not seem adequate. Notice that Russell's statement of the truth conditions of quantified statements is given in terms of another quantified statement. As he puts it, "{(*x*).ϕx} has second truth" means "(*x*).(ϕx has first truth)", which appears to be a circular way of explaining the semantics of quantification. Even if the second quantifier is metalinguistic, as Russell probably intended it, one might

see metaphysical difficulties. The quantified statement does not seem to be made true by the atomic complexes alone; it seems to require additionally that these be *all the possible complexes* of that form, which would appear to be something over and above the atomic complexes. Russell realized this problem later on, and at least by 1918 his metaphysics clearly involves *general facts* (*PLA* p. 101), but in *PM* there is no hint of this.

The real issue, however, involves propositions with *higher-order* quantification. Clearly, if Russell held strictly that ontological commitment comes with the use of bound variables, there could be no denying that he held an ontology of propositional functions as complex intensional entities. Russell does give a quantificational analysis of existence statements. But we must not take things too far; he is happy to speak of such things as "the existence of classes", even on his "no classes" theory. However, one would need to say more about Russell's intended semantics for quantification if one were to argue that Russell's views on existence are tantamount to a theory about genuine ontological commitment. In fact, Russell seems to endorse a substitutional (or Kripkean), as opposed to an ontological (or Tarskian), semantics for quantification—and this coheres with the account given above for the first-order quantified propositions. This interpretation was suggested years ago by Grattan-Guinness (1977, p. 75), and explored in detail by Sainsbury (1979, pp. 274ff) and more recently by Landini (1998, pp. 278f).[25]

The core idea is that a formula involving a *free* higher-order variable, e.g., "$\phi!a$", has as its substitution instances

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[25] However, Landini does not give a substitutional reading for the individual variables. On my own reading, Russell gives a substitutional reading for all variables; however, the individual variables are unique in that their substitution instances—"logically proper names"—are always direct representatives of individuals. For all other types of variables, there is no direct connection between their substitution instances and any entities: this explains the special metaphysical significance of these variables.

first-order formulae containing the constant "*a*" occurring in logical subject position (i.e., a position accessible to a name of a particular) one or more times. This includes "*a* is *q*", "*a* is *q* ⊃ *a r*'s *b*", "(*y*)(*y* is *q* ⊃ *a r*'s *y*)", and so forth. Just as a proposition involving an individual quantifier is true just in case all its substitution instances are, similarly a proposition involving a higher-order quantifier is as well. So "(*ϕ*)(*ϕ*!*a*)" has "third truth" if and only if all closed substitution instances of "*ϕ*!*a*" have "second truth". Russell's different notions of truth (*PM* pp. 41-47) are derived from the recursive truth definition for his language; ramification is *required* in order for the truth conditions not to be circular. Russell uses the notation "*ϕ*!*a*" for a function whose arguments are predicative functions of *a*, i.e., those not involving quantification over functions; if we allowed impredicative expressions to be substitution instances of "*ϕ*!*a*", then the conditions under which "(*ϕ*)(*ϕ*!*a*)" has third truth could not be defined in terms of second truth alone. This, and not any ontological doctrine, is the heart of the "vicious-circle principle". Russell seems to have anticipated Kripke's suggestion (Kripke 1976) that higher-order quantification can be given a substitutional semantics only if the allowable instances of the variables cannot contain expressions involving bound variables of the same order.

The upshot of this for our present discussion is that all it means to say that "*x̂* is *q*" and "(*y*)(*y* is *q* ⊃ *x̂ r*'s *y*)" are values the variable "*ϕ*" involved in "(*ϕ*)(*ϕ*!*a*)" is that the truth-conditions of "(*ϕ*)(*ϕ*!*a*)" require the truth of both "*a* is *q*" and "(*y*)(*y* is *q* ⊃ *a r*'s *y*)". There is no hint that Russell is ontologically committed to a special kind of entity denoted by the expression "(*y*)(*y* is *q* ⊃ *x̂ r*'s *y*)". Circumflexion is only used in his informal discussions, and indeed, there is good evidence that Russell did not take it to be a term-forming

operator in *PM*.[26] Just as Russell thought that, metaphysically, "(*x*)(*x* is *q*)", if true, would not correspond to a single fact but simply be made true by those facts making its instances true, the "third truth" of a proposition quantifying over predicative propositional functions consists entirely in the "second truth" of all its instances. None of this requires an ontology of complex attributes. Now of course, for those propositional functions (open sentences) of the form "*x* is *q*" and "*x* is *p*", Russell is committed to an entity—a universal—which is expressed by the "*q*" or the "*p*", but in general one cannot isolate a single entity or universal that correlates with the function. In the case of "(*y*)(*y* is *q* ⊃ *x r*'s *y*)", Russell certainly isn't committed to some universal of *Bearing-relation-r-to-all-things-that-are-q*. If "(*y*)(*y* is *q* ⊃ *a r*'s *y*)" is true, this is because all propositions of the form "*y* is *q* ⊃ *a r*'s *y*" are true, and these are made true by many distinct facts, some of which involve *a*'s bearing the *r*-relation to some other individual that is *q*, etc. To be sure, "(*y*)(*y* is *q* ⊃ *a r*'s *y*)" says something about *a*, and indeed, in a loose sense of "property", it ascribes a property to *a*, but it does not simply attribute a universal to *a* (see *MPD* p. 124).

If this is the right interpretation of the semantics of *PM*, then it becomes evident that Russell's ontology could not have included ontological correlates of propositional func-

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[26] See Klement 2003, pp. 27, 35; Landini 1998, pp. 264-267. One question that arises is why the device is needed at all in *PM*. If propositional functions and propositions are linguistic items, as I have suggested, why does Russell not just use quotation marks to talk about a function? Really, Russell *shouldn't* need circumflexion in order to talk about a propositional function as opposed to its values. The reason he does is that, in his informal discussion, he has the sloppy habit of using object-language variables where he ought to be using metalinguistic schematic letters (see p. 27 above). Hence he'll talk about "the proposition '*x* is mortal'" for different "values of *x*" where he should be talking about "the proposition ⌜**n** is mortal⌝", where "**n**" is a metalinguistic schematic letter for object-language *constants*. However, given his own sloppy practice, he needs a way of making it clear when he is *not* using object-language variables schematically in his informal discussion, whence the circumflex.

tions. The facts that make elementary propositions true consist entirely of individuals, and Russell is fairly explicit that the truth or falsity of quantified propositions of a given order in the ramified hierarchy is to be defined recursively in terms of the truth or falsity of those of lower order. Ultimately, then, the only *facts* involved are those relevant to the truth or falsity of elementary propositions, and these facts consist entirely of individuals. The theory of types, therefore, is not a hierarchy of types of entity but only of types of meaningful expression. The "vicious-circle principle" is, in effect, a consequence *of* the recursive nature of the truth conditions for quantified propositions. As Russell himself wrote a few years before *PM*, "it is important to observe that the vicious-circle principle is not itself the solution of vicious-circle paradoxes, but merely the result which a theory must yield if it is to afford a solution of them" (*LPL* p. 205). The *solution* to the paradoxes comes in the recognition that classes, propositions, propositional functions, etc. are not genuine entities but merely constructs of logic; once the semantics of language seemingly about such "entities" is understood, the would-be assertions giving rise to the paradoxes are revealed as lacking proper truth conditions.

Why is it impossible for a propositional function to take itself as argument? In the new metaphysics of *PM*, Russell is no longer committed to propositions or propositional functions at the ontological level, but even at the linguistic level propositions and their forms are prior to functions. One begins with definite propositions such as "*a* is *q*", "*b* is *q*", etc. The propositional function "$\hat{x}$ is *q*" is simply gotten at by thinking of the name "*a*" in "*a* is *q*" as replaceable by any other name of an individual; this is what all these propositions have in common. However, "$\hat{x}$ is *q*" is not the name of any individual, or indeed of any entity at all. So a would-be proposition of the form "($\hat{x}$ is *q*) is *q*" does not count as an

instance. Higher *type* functions are gotten at by beginning with propositions such as "$(x)(x$ is $q)$" and "$(x)(x$ is $p)$", noting their commonality, and thinking of the "*x* is *q*" part of "$(x)(x$ is $q)$" as replaceable by any other expression in which "*x*" occurs free. This gets us to the function "$(x).\phi x$" and then to higher-order quantification. The function "$\hat{x}$ is *q*" grammatically fits the argument spot of "$(x).\phi x$" but "$(x).\phi x$" does not fit its own argument spot. Generally, according to the hierarchy of truth and falsehood endorsed as part of *PM*'s intended semantics, the truth of a proposition involving a propositional function is defined recursively in terms of the truth of its values. This is why Russell concludes that, in order to be a "well-defined function", a function cannot "have among its values anything which presupposes the function" and that "a function cannot be definite until its values are definite", which he describes as "perhaps the most fundamental case" of the vicious-circle principle (*PM* p. 39). But again, the *explanation* for what makes the circle vicious makes reference to the truth-hierarchy invoked by the semantics of the language.

Russell's 1910 type-hierarchy is not a hierarchy of particulars, universals that apply to universals, universals that apply to universals, etc. This reading misunderstands it completely. It is a hierarchy of different types of expressions that are meaningful in different ways. This was true even in *PM*, before Russell met Wittgenstein.

§10. *Wittgenstein and Russell*
We noted earlier the tendency to think that the "judgment based" and fact/proposition–centered aspects of Wittgenstein's views in the *Tractatus* are something derived from Frege and certainly not from the "object based" Russell. We have now sketched enough of their views to see that this is an error. To be sure, Wittgenstein espouses Fregean-

sounding slogans such as "only in the context of a proposition has a name meaning" (*TLP* 3.3). He also makes it clear that facts are in some ways metaphysically prior to objects (1.1, 2.0121) and that a fact is not exhausted by its constituents alone but has a definite form or structure (2.031, 2.032). But Russell too thinks that a complex has a *mode of combination* that is not itself another constituent, and that, understanding how the individual represented by a given sign occurs in the complex represented requires placing it in the context of an entire proposition. Consider, e.g., that in "*a* is *q*" the sign "*q*", while representing an individual, represents it *occurring as concept*, whereas in "*q* is *p*" the sign "*q*" indicates the same entity *occurring as logical subject*. While Russell does ontologically place individuals first, it is not as if he thinks it *possible* for there to be individuals that do not occur in any complexes or have a nature separable from their occurrence therein.

Indeed, when we get down to the specific topics of the priority of complete propositions over functions and the explanation for why a function cannot take itself as argument, Wittgenstein's views are almost *thoroughly* Russellian. A proposition (*Satz*), in the *Tractatus*, is a logically articulated picture of a fact (*Tatsache*) expressed in a perceptible form (3.1, 4.01). Elementary propositions depict atomic facts or simple states of affairs (*Sachverhalte*). A simple state of affairs consists of a number of objects combined with a definite structure (2.01, 2.031-2.032). Elementary propositions are *themselves* facts, and they consist of *names* similarly combined together with a definite structure (2.13-2.15; 3.21-3.22). The configuration of names in the proposition corresponds to the configuration of objects in the pictured fact (2.16-2.2). It is immediately clear from this basic sketch that Wittgenstein thinks that an elementary proposition consists of names and names only, and a state of affairs consists of ob-

jects and objects only. So unlike Frege, Wittgenstein does not think that an elementary proposition is built up from function and argument, with the function representing a different kind of entity from its argument. Wittgenstein's view is *much* closer to Russell's view that all the signs in an atomic proposition represent individuals, and that such propositions can be grasped without understanding them as values of any functions.

What is surprising about Wittgenstein's view is that it does not seem as if a proposition could consist entirely of *names*. "Plato Xantippe Socrates" is not a proposition. This sort of worry lay behind Frege's contention that every proposition must contain a sign representing a function (e.g., *BuG* p. 193). Before understanding how Wittgenstein solves this problem we must recall how Russell solved it. A Russellian atomic proposition is not a mere list of names like "*a q*". As we have seen, it is also not, as it is often taken to be, something of the form "*Fa*" or "*Rab*". Russellian atomic propositions instead take forms such as "*a* is *q*" and "*a r's b*". While all the signs that stand for the entities making up the corresponding complex represent individuals, words such as "is" and the declension of "*r's*" as a verb (i.e., in English, "loves" rather than "Love" or "hates" rather than "Hatred"), while themselves not indicating a different entity, instead indicate something about how those individuals in the complex are combined. So a proposition is not a mere list of names; the grammatical structure of the sentence indicates the form of the corresponding complex.

Wittgenstein's solution is along the same continuum, and if anything more radical. In early 1913, he wrote to Russell with the following news:

> I have changed my views on "atomic" complexes. I now think that qualities, relations (live love) etc. are all

copulae! … The reason for this is a very fundamental one: I think that there cannot be different Types of things! … For instance if I analyse the proposition Socrates is mortal into Socrates, mortality and $(\exists x, y)\epsilon_1(x, y)$ I want a theory of types to tell to tell me that "mortality is Socrates" is nonsensical, because if I treat "mortality" as a proper name (as I did) there is nothing to prevent me to make the substitution the wrong way around. … What I am *most* certain of is… the fact that all theory of types must be done away with by a theory of symbolism showing that what seem to be different *kinds of things* are symbolized by different kinds of symbols … . [*NB* pp. 121-122]

To understand both what Wittgenstein's view is here, and how he differs from Russell, one must place the above in the context of questions Russell had been working on after the appearance of *Principia*. In the 1912 manuscript "What is Logic?", Russell reopened the question as to what a form is. Having swapped an ontology of facts for an ontology of propositions, he now believes that it is facts, not propositions, that exhibit forms, though he maintains that a "*form* is a something, though not a constituent of the complexes" having it (*WIL* p. 56). He notes that two facts or complexes have the same form "if one can be obtained from the other by mere substitution of new terms in other places" (*WIL* p. 55); this, however, does not tell us what a form, in and of itself, *is*. We saw earlier that in 1904, Russell thought of "$\hat{x}\hat{R}\hat{y}$" as representing a *mode of combination*, then understanding it as a denoting complex consisting entirely of variables, with variables themselves understood as extra-linguistic entities. Eight years later, having adopted, in the meantime, a purely linguistic view of variables, Russell is now uncertain about the nature of forms, noting that a "form is not a mere symbol: a symbol composed entirely of variables symbolizes a form, but it is not a form" (*WIL* p. 56). Since Russell understands logic as the *a priori* science of form

or structure, he begins working on a new conception.[27]

In the 1913 *Theory of Knowledge* manuscript, Russell suggests the following account of what a form might be taken to be:

> We require of the form that there shall be one form, and only one, for every group of complexes which "have the same form"; also, if possible, it would be convenient to take as the form something which is not a mere incomplete symbol. We may secure these *desiderata* by taking as the form the fact that there are entities that make up complexes having the form in question. This sounds circular, but what is intended is not circular. For example, the form of all subject-predicate complexes will be the fact "something has some predicate"; … . This logical nature of this fact is very peculiar … since "something" is nothing … our new fact … contains no constituent at all … . In a sense, it is simple, since it cannot be analyzed. At first sight, it seems to have a structure, and therefore to be not simple; but it is more correct to say that it *is a* structure. [*TK* p. 114]

During this period, Russell uses the sign "ϵ" in place of the copula in subject-predicate statements, writing that the "ϵ" "merely serves, like a bracket, to indicate relative position"—i.e., it contributes something about form, not content (*TK* p. 113; cf. *WIL* p. 56). Of course, the set-theoretic use of "ϵ" is not meant here.[28] So the form of the fact that makes

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[27] For more on the nature and importance of forms for Russell during this period, see Griffin 1980. For more on Russell's views on logic as the *a priori* science of form, see also Landini 2002.

[28] The set-theoretic usage of "ϵ" in Russell's work derives from Peano. As early as 1903, however, Russell complained that Peano did not clearly distinguish the usage of "ϵ" for the relation of class-membership from its usage for the relation of a term to a concept it instantiates, both representing possible translations of the ordinary language words "is a" (*PoM* pp. 68, 77-78). Russell himself seems to think the sign is appropriate for either purpose, provided that one is clear about which is meant. Non–set-theoretic usages of "ϵ" are found periodically in Russell's writings, and indeed, Russell uses it in the marginalia of his copy of Frege's *Grundgesetze* where Frege speaks of the relation of an object "falling under" a concept (see Linsky, forthcoming).

the atomic proposition "Socrates ϵ mortal" true is equated with the fact that makes the existentially quantified statement "$(\exists x, \alpha).x \epsilon \alpha$" true (where the "$\alpha$" is a special variable ranging only over those universals that can occur as concept in subject-predicate statements). Curiously, Russell describes this fact as being *without constituents* and therefore without complexity. Clearly, this is a difficult conception to wrap one's head around. As mentioned above, Russell abandoned the manuscript before delving much into the nature of quantified judgments or facts; it is possible that he had intended to flesh out his view in later chapters.

Wittgenstein's letter from early 1913, quoted above, shows that he is also attempting to solve this problem in this period. There, Wittgenstein is almost certainly following Russell in using "ϵ_1" as a sign for the copula. The view he begins with is Russell's own view; that in "Socrates is mortal", the words "Socrates" and "mortal" both represent individuals, and "is" helps to indicate the form in which they are arranged in the corresponding fact, with Wittgenstein representing this form as "$(\exists x, y)\epsilon_1(x, y)$". Wittgenstein is critical of this view, but we need to be clear about *what* Wittgenstein is criticizing. It is not that Russell believes in different types of things and Wittgenstein denies this. Socrates and *Mortality* and *Love* are all individuals for Russell. However, Russell does have a distinction in logical *kind* between those entities that can occur as only logical subject (particulars) and those that have a two-fold nature (universals). For Russell, "a is q" (i.e., "$\epsilon_1(a, q)$") and even "q is q" are well-formed but only if "q" names a universal. Wittgenstein points out that Russell's type theory does *not*—precisely *because* it treats universals as individuals—explain why "a is q" is meaningful but "q is a" is not, when "a" names a particular. His solution is to push a step further and treat words standing for qualities and relations the way Russell treats the copula, i.e., not as names of

constituents of the corresponding facts but merely indicating something about their forms.

In the *Tractatus* itself, Wittgenstein claims that in a proposition such as "aRb", the "R" is not a name and does not correspond to an object in the state of affairs. Instead, *flanking* the sign "R" is one possible way for the names "a" and "b" to relate to each other in the fact that *is* the proposition, and this form of names' relating to each other signifies one possible way the objects *named by* "a" and "b" relate to each other in reality. Wittgenstein puts it this way:

> Instead of, 'The complex sign "aRb" says that a stands to b in the relation R', we ought to put, '*That* "a" stands to "b" in a certain relation says *that aRb*." [3.1432]

Unfortunately, since Wittgenstein gives us no example of an *interpreted* elementary proposition, we have to make do with something that is demonstrably *not* elementary and, for the moment, treat it as though it were. Consider "Plato loves Socrates". Here, "loves" is not a name and does not signify any entity in the fact that Plato loves Socrates. Rather, this particular way in which the two names combine into a written sign, i.e., "Plato" and "Socrates" flanking "love" with "Plato" on the left and "Socrates" on the right—this *fact*—signifies a certain kind of way Plato and Socrates themselves can enter into a state of affairs. Russell's view is that in addition to Plato and Socrates there is the universal Love, which, in this case, occurs *as relating* Plato and Socrates. The form is not a constituent but is indicated by the word "loves" occurring grammatically as a verb. Wittgenstein, however, treats the "R" in "aRb" the way Russell treats those features of a proposition that indicate form; "a" and "b" flanking "R" mirror some purely structural or formal aspect of the state of affairs, in which *the only constituents or entities* involved are a and b. A Tractarian elementary proposi-

tion might use the *notation* of contemporary predicate logic, with "*Fa*" and "*aRb*" among the possibilities, but this should absolutely *not* be understood as a move away from Russell towards Frege. In effect, Wittgenstein is taking Russell's response to the Fregean worry about all signs in a proposition indicating the same type of entity and *pushing it a step further*; Wittgenstein is out-Russelling Russell, eliminating not only *types* of entities (with Russell) but even the difference in *logical kind* represented by Russell's thing/concept distinction (cf. Landini 2004, p. 104).

Wittgenstein's 1913 letter to Russell, in suggesting that even relations like *Love* are "copulae" puts the point more strongly than it needs to be put. The view Wittgenstein advanced in *TLP* does require that "*R*" in "*aRb*", where "*aRb*" is a truly elementary proposition, not be a name or a sign for an object in the fact depicted and instead only indicate something about how the objects in the fact are arranged. However, he does not necessarily need to accept the conclusion that the word "loves" in "Socrates loves Plato" indicates something purely structural or formal about the fact in question. This proposition needs further analysis, and it is not at all clear that in its proper analysis, "loves" will appear in any similar fashion. Moreover, his mature position is at least *compatible* with a position on which what are ordinarily thought of as universals are not regarded as "relating relations" but are instead treated simply as objects. On this view, "*a* loves *b*" would be interpreted as involving three names; "*a*", "love", and "*b*". Here the logical syntax of the proposition would be interpreted not as two names flanking "love" but as one name preceding another name, with the second name preceding the letter "s" (used to conjugate a verb), followed by a third name. Here the object corresponding to the word "love" plays no special role in providing the structure of the fact. The structure of the fact is

distinct from any constituent and is not indicated by any of the names in the proposition but instead by the proposition's syntactic form. This sort of refined view is suggested by remarks of Wittgenstein made after 1913, such as his 1915 remark in the *Notebooks* that "[r]elations and properties, etc., are objects too" (*NB* p. 61), and his 1930 or 1931 comment to Desmond Lee that the "objects" of the *Tractatus* also include relations (see *WLC* p. 120). It is only marginally different from views at times held by Russell, but notice how deeply un-Fregean it is.

Wittgenstein's views on functions are also more Russellian than Fregean. First, as others have noted, like Russell and unlike Frege, Wittgenstein seems only to have in mind *propositional functions* when he speaks of functions (see Hylton 1997, pp. 94f; Goldfarb 2002, p. 195). Secondly, Wittgenstein introduces functions only *after* propositions had already been fully explained. Unfortunately, Wittgenstein is not as explicit about what he means by a "function" (*Funktion*) as he is about what he means by such things as "names", "objects", "propositions", and even "operations" and "truth-functions", which he contrasts with functions proper. However, his notion of a function is clearly tied up with his understanding of what he calls "expressions" (*Ausdrücke*) and "symbols" (*Symbole*).[29] These are introduced as follows:

> [3.31] I call any part of a proposition that characterizes its sense an expression (or a symbol) … .
>
> Everything essential to their sense that propositions can have in common with one another is an expression.

[29] The first occurrence of the word "function" in *TLP* comes at 3.318, where Wittgenstein writes "I conceive the proposition—like Frege and Russell—as a function of the expressions contained in it." According to the numbering system of *TLP*, this is meant to elaborate further upon 3.31, where expressions are first introduced.

An expression is the mark of a form and a content.

[3.311] An expression presupposes the form of all the propositions in which it can occur. It is the common characteristic mark of a class of propositions.

Wittgenstein suggests that one can begin with the realization that "*aRb*", "*cRd*", and "*bRc*" have something in common: an expression. By itself, however, "*R*" is not a good depiction of these propositions' "common characteristic mark", which involves, in addition to the letter "*R*", a certain constant form. The letter "*R*" itself does not signify any *thing* in "*aRb*", so the important commonality in these propositions is not "*R*" but *two names flanking "R"*. A better depiction makes use of variables in place of the names and hence appears: "*xRy*". Wittgenstein calls "*xRy*" a "propositional variable" (*Satzvariable*). The expression that "*aRb*", "*cRd*", and "*bRc*" have in common is that which is shared by all *values* of this propositional variable; the expression is "represented" or "depicted" (*dargestellt*) by means of the propositional variable (3.313). Although he is not explicit about this, I believe that Wittgenstein equates *functions* with propositional variables.

Wittgenstein's suggestions about propositional variables echo suggestions made by Russell about propositional functions as early as the *Principles of Mathematics*. First, one begins with a certain class of propositions sharing not only a certain amount of common *content* but also a common *form*. Their commonality cannot be represented simply by *removing* the part that varies between them; it must be replaced with a variable in order to preserve the common form that the members of the class share and that would otherwise be lost. The difference of course is that in 1903 Russell was talking about propositions understood as objective complexes, and Wittgenstein is talking about propositions un-

derstood as interpreted symbols. However, as we have seen, Russell himself had in the meantime abandoned his ontology of propositions and propositional functions as entities containing variables and had reinterpreted both as being purely linguistic items.

Wittgenstein also follows Russell in holding the *values* of a function to be prior to the function itself. "*xRy*" is not a constituent of "*aRb*", and the latter can be grasped without explicitly thinking of it as a value of the former. Indeed, the way a propositional variable is introduced is by stipulating those sentences whose commonality is to be captured by it:

The determination [*Festsetzung*] of values of the propositional variable is done by *indicating the propositions* whose common mark the variable is.

The determination is a description of these propositions.

The determination will therefore deal only with symbols not with their meaning.
And *only* this is essential to the determination, that it is only a description of symbols and asserts nothing about what is symbolized. [*TLP* 3.317]

Recall that for the mature Russell, propositional functions are gotten from propositions by identifying some grammatically significant piece that one imagines being replaced by other expressions; with the substitutional semantics for variables intended by Russell for *PM*, the values of a propositional function are merely its substitution instances. Wittgenstein is explicit that the "values" of a propositional variable are to be identified not by anything having to do with the *meanings* of the signs involved but merely that they (the values), *as symbols*, match the description of the features a *symbol* must have in order for them to be included in the class of propositions whose common form and content the

propositional variable is stipulated to represent. The propositional variable *presupposes* the forms of the propositions bearing the common mark in question (*TLP* 3.311). I interpret *TLP* 3.317 as Wittgenstein's unique way of recommending a substitutional interpretation of the variables.

If I am right in thinking that Wittgenstein thinks of functions as "propositional variables", then functions are purely linguistic items; they are just like propositions except containing a variable, i.e., they are open sentences. Functions are not "out there in the world", except in the sense that propositions (sentences) are "out there in the world", and they are not constituents of facts, except insofar as propositions themselves are facts. Functions are linguistic devices used when we wish to speak *at once* about all propositions that have common characteristics. E.g., the sentence "~$(\exists x).Fx$" negates all propositions whose common form is represented by "Fx".[30]

We can also obtain functions by beginning with a class of *non*-elementary propositions, e.g., "$(x).Fx :\supset Fa$", "$(x).Gx :\supset Ga$", and so on. These can be stipulated as the values of the propositional variable (function) "$(x).\phi x :\supset \phi a$". Because a function encodes the structure of its values, it must always be as structurally complex as those values; the constituent variable letter is used in place of any constituent part of any of the values that is not found in all the values.

Why can't a function take itself as argument? The answer is that functions are abstracted from the propositions that are their values and therefore cannot be contained in them. The function "Fx" is the common mark of those propositions in which a name stands to the right of "F", so all its values look like: "Fa", "Fb", "Fc". There could not be a

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[30] Wittgenstein would construe this proposition as an application of the N operator, which is a truth-functional operator that can be applied to any number of propositions. At 5.52, he writes, "If ξ has its values all the values of a function *fx* for all values of *x*, then N($\overline{\overline{\xi}}$) = ~$(\exists x).fx$."

value of this function that looked like "$F(Fx)$", because it is not an instance of a name standing to the right of the letter "F"; it has the wrong syntactic form. That is not to say that "$F(Fx)$" is illegitimate; it only shows that the outer "F" cannot be part of the same symbol or expression as the inner "F". The same sign can occur as parts of two different symbols; but when this happens this is purely accidental and indicates nothing in common between the two symbols or what they represent (3.321-3.323).

A form such as "$F(Fx)$" might be used significantly, since, after all, we *can* construct propositions dealing with functions and their values. Indeed, "~$(\exists x).Fx$" is such a proposition. However, Russell's paradox, in its "propositional function" variety, seems to involve an function abstracted from sentences of like-form that speak about functions:

> The function Gx [x is green] does not apply to itself.

> The function Bx [x is blue] does not apply to itself.

and so on. The notion of "not applying to itself" is not really well-considered, but suppose for the moment that it were. Suppose furthermore that we use capital "F", *when its argument is a itself function of the form "Gx" or "Bx"*, as shorthand for what the above propositions have in common, and lowercase "fx" as a second-order variable for what varies between them. The propositional variable abstracted from the above list would then be written "$F(fx)$". We can then make sense of what Wittgenstein goes on to say about Russell's paradox at 3.333:

> Let us suppose that that the function $F(fx)$ could be its own argument: in that case there would be a sentence '$F(F(fx))$' in which the outer function F and the inner

function *F* must have different meanings, since the inner
one has the form φ(*fx*) and the outer one has the form
ψ(φ(*fx*)). Only the letter '*F*' is common to the two func-
tions, but the letter by itself signifies nothing.
… That disposes of Russell's paradox.

If we were to attempt to take the propositional variable
dealing with functions "*F*(*fx*)" and make it its own argu-
ment, we would end up with "*F*(*F*(*fx*))". But this is not a
value of the function that the internal "*F*" is a part of, since
all its values have a form such as "*F*(*Gx*)". As Wittgenstein
says, the "*fx*" within "*F*(*fx*)" provides a *prototype* for the ar-
gument to "*F*(*fx*)", and this very function does not match this
prototype. "*F*(*Fx*)" and "*F*(*F*(*fx*))", as we have seen, are well-
formed, but here the internal "*F*"s must be part of a different
symbol than the external "*F*"s.

How Russellian is this solution to the paradox? Very.
Both of their solutions begin by rejecting the Fregean idea
that functions are involved in the simplest of propositions,
and that the unity or form of a proposition is to be under-
stood in terms of the application of a function to arguments.
Both arrive at functions by beginning with classes of propo-
sitions sharing a certain commonality of form that is found
when some one member of the class can be gotten from oth-
ers by replacing some significant part of the proposition by
other expressions of the same grammatical type. A proposi-
tional function is then understood as what one gets by turn-
ing what differs between these propositions of like form into
a variable. Unlike Frege, they both believe that a function
presupposes its values, not vice-versa. Really, for both, the
reason a function cannot be its own argument is that it does
not have the right grammatical structure to "fit" into its own
argument spot. E.g., for Russell, if we begin with the propo-
sitions (sentences) "*a* is red", "*b* is red", etc., what differs
between them is that a different sign representing an indi-

vidual occurs as subject. The function derived is "*x̂* is red",
and its values are propositions in which a sign representing
an individual precedes "is red". There is no such value as
"(*x̂* is red) is red" because that is not a sentence in which a
name precedes "is red". Wittgenstein's explanation is al-
most exactly the same. A function is derived from a class of
propositions (sentences); one replaces what differs between
them with variables. The variable in a function represents a
"prototype" of its argument, and hence the function cannot
replace the variable because it does not match the prototype
(cf. 3.332).

Of course, the *Tractatus* does contain some critical re-
marks aimed at Russell, e.g.:

> 3.33 In logical syntax the meaning of a sign should never
> play a rôle. It must be possible to establish logical syn-
> tax without meaning the *meaning* of a sign: *only* the de-
> scription of expressions may be presupposed.

> 3.331 From this observation we turn to Russell's 'theory
> of types'. It can be seen that Russell must be wrong, be-
> cause he had to mention the meaning of signs when es-
> tablishing the rules for them.

These sections, and 3.331, especially, are the source of the
misreading that while Russell's theory of types is a hierarchy
of entities, Wittgenstein's is merely one of different kinds of
signs. This is oversimplified. As we have seen, neither of
them are committed to a hierarchy of entities. The only con-
stituents of facts for Russell (circa 1910-1912) are individuals,
and for Wittgenstein, the only constituents are objects.

Their views differ over what makes two expressions
grammatically similar or dissimilar. For Russell, one gram-
matical kind of expression is that of proper names, which are
signs indicating individuals *as appearing as logical subject*.
(Notice that for Russell, not all signs indicating individuals

are proper names.) What makes a sign have *this* kind of syntactic role is that it *means* an entity occurring in a certain kind of way. It is from this definition that Russell's definition of all the other grammatical types begins. Type 1 propositional functions are open sentences whose variable has replaced a proper name, and type 2 propositional functions are open sentences whose variable has replaced a type 1 propositional function. While this hierarchy is not one of different types of entities making up facts, Russell still violates Wittgenstein's dictum that logical syntax should be able to be established without recourse to semantics.

Wittgenstein would apply type-distinctions directly by examining what possible senses the constituent expressions *could* have, given that, according to the picture theory, the proposition must share its logico-pictorial form with the reality in represents. Hence the study of logical syntax *generally* need only concern itself with "the description of expressions". Now of course, syntactic facts have semantic ramifications: elementary propositions can represent states of affairs only if they have the same structures. Different elementary propositions that can be conceived of as values of the same function must differ only with regard to which names appear at which positions, and *therefore* correspond to states of affairs differing from each other only in regard to which entities occur at which positions.

This subtle difference between their views leads to differences as to how they react to *would-be* propositions in which a function *seems* to take itself as argument. As we saw in the previous section, Russell believes that the truth of a proposition involving a propositional function is defined recursively in terms of the truth of its instances. Hence Russell concludes that a would-be proposition in which a function seems to take itself as argument cannot, given the semantics he intends for *PM*, be given truth conditions non-circularly

and is therefore "not significant" or "meaningless" (*PM* pp. 40-41). In the *Tractatus* 3.331 Wittgenstein, however, says not that "$F(F(fx))$" is meaningless but rather that the two capital "F"s are parts of different symbols. He sees no difficulty in reading this as a function taking a function as argument; only we must realize that the two functions are not the same, because their values would be propositions having different syntactic structures and hence different logico-pictorial forms. This connects with Wittgenstein's claims at 5.473:

> Logic must look after itself.
>
> If a sign is possible, then it also capable of signifying. Whatever is possible in logic is also permitted. (The reason why 'Socrates is identical' means nothing is that there is no property called 'identical'. The proposition is nonsensical because we have failed to make an arbitrary determination, and not because the symbol, in itself, would be illegitimate.)
>
> In a certain sense, we cannot make mistakes in logic.

We can, if we wish, give a sense to "$F(F(fx))$", but we need not worry about this leading to paradox, because we simply *cannot* turn it into a function taking itself as argument. Doing that would require giving the internal and external "F"s the same "logico-syntactical employment", which cannot be done. As Wittgenstein says, "we cannot give a sign the wrong sense" (5.4732).

However, we can operate under illusions about what senses symbols have, and even under the illusion that we have given a sign a sense when we have not (5.4733). We might think mistakenly that by giving "F" a sense when it appears in a construction with the form "$F(fx)$", we have also given it a sense when it first appears in "$F(F(fx))$". Wittgenstein claims that confusing the same sign which occurs as

parts of different symbols leads to "the most fundamental confusions" in philosophy, and that this is the motive behind artificially constructed logical languages (3.324-3.325). His criticism of Russell is that Russell seems to think that the paradox is only solved for those languages that *prevent* the construction of signs that *in virtue* of how they are constructed *must be meaningless*. Wittgenstein, however, thinks that no signs we can create are necessarily meaningless; the rules adopted in artificial languages are just pragmatic ones to help prevent psychological confusion. The difference comes out nicely in their 1919 exchange of letters after Russell had first received a draft of the *Tractatus*:

> [Russell wrote] "The theory of types, in my view, is a theory of correct symbolism: a simple symbol must not be used to express anything complex: more generally, a symbol must have the same structure as its meaning." [Wittgenstein responds:] That's exactly what one can't say. You cannot prescribe to a symbol what it *may* be used to express. All that a symbol *can* express, it *may* express. This is a short answer, but it is true! [*NB* pp. 130-131]

The proposition "$F(F(fx))$" is not ruled out because it is "incorrect"; excluding such forms from a logical language only minimizes the temptation to become betwitched into thinking that it *might* express something it *cannot*.

Of course, it would be somewhat uncharitable to say that Russell thought that something like "$F(F(f\,\hat{x}))$" *must* be ill-formed; if different and appropriate type-subscripts were added to the "F"s, it would become *obvious* that they are parts of different symbols. Russell is taking for granted that it is advantageous to have a form of symbolism that avoids using the same sign in different ways; in suggesting that such constructions are meaningless, he is merely pointing out the impossibility of giving it a semantic interpretation

that treats the two "F"s similarly. Russell thinks that it is both possible and worthwhile to *say* this. Wittgenstein thinks this impossibility is *shown* by the sentence's structure and cannot properly be *said*. As Wittgenstein put it in his 1914 notes dictated to Moore, "a THEORY of types is impossible" (*NB* p. 109).

§11. Conclusion

Recently in the *Tractatus* literature there has been great controversy about whether or not early Wittgenstein's final position committed him to ineffable truths, those that can only be *shown*, not *said*. Those on both sides of the debate relate it to Frege's struggle with the function/concept divide. Frege admits that his own explanations of that theory, *in virtue of* the facets of meaningfulness the theory intends to capture, unavoidably require him to misuse language (*PMC* p. 136; *BuG* p. 194). The debate over Wittgenstein's views has spilled over to Frege; was Frege committed to deep truths that somehow escape expression (Geach 1976), or was the purpose of his explanations to get us to see beyond thinking that there is anything there to express (Diamond 1991), or are his views left in a state of tension (Conant 2002)? Wittgenstein is then presented as responding to these concerns. The debate then becomes: is he a philosopher-mystic attempting to say, or to use Ramsey's phrase "whistle", what cannot be said (Geach 1976, Hacker 2000), or alternatively, is his work aimed at getting us to see such nonsense as the plain and simple nonsense it is (Diamond, Conant, Ostrow 2002 and others)?

I do not intend to solve this debate over the *Tractatus*, but merely register my belief that it cannot profitably continue without a better understanding of Russell's influence. Unfortunately, when Russell *is* mentioned in these contexts, it is only briefly, and usually his theory of types is simply

thrown in as another theory that requires truths that cannot be put into words (Ricketts 1996, pp. 61-62; Proops 2001, pp. 174-175; Ostrow 2002, p. 64). Authors who do this, naturally, take the theory of types to be a theory of different kinds of genuine entity. But Russell's *pre-Wittgenstein* views do not commit him to any such thing. Russell was one of the first to criticize views such as Frege's function/object distinction on the grounds that they lead to their own unspeakability. All too often Russell's theory of types is taken as an abandonment of these scruples. It was not. Russell's contention that every *genuine* entity must be capable of being a logical subject was the *primary principle* underlying the development of his views on logical grammar. The crudest readings of the theory of types take it as a commitment to different types of classes or sets of which the same things cannot meaningfully be said, and of course, naturally these readings attribute to Russell views about logical syntax that cannot be stated without violating themselves (see, e.g., Soames 2003, pp. 152-154). Fortunately, one needs only minimal exposure to Russell's writing to know that he eschewed commitment to classes, and that the type theory applicable to classes is just a *byproduct of* the type theory for propositional functions in which talk of classes is reconstructed. If classes were entities then the rules governing discourse about them could not be so restrictive, as Russell explained in 1910:

> I have … discovered that it is possible to give an interpretation to all propositions which verbally employ classes, without assuming that there really are such things as classes at all. … That it is meaningless … to regard a class as being or not being a member of itself, must be assumed for the avoidance of a more mathematical contradiction; but *I cannot see that this could be meaningless if there were such things as classes.* [*SERB* p. 357; emphasis added]

It is precisely because classes are logical constructions that such restrictions on meaningfulness exist; for *genuine* entities, there are no such restrictions. Indeed, his entire notion of a logical construction is bound up with the realization that "none of the raw material of the world has smooth logical properties, but that whatever appears to have such properties is constructed artificially in order to have them" (*IPoM* p. xi). Classes are logical constructions; type restrictions apply to them in virtue of their construction out of propositional functions.

If Russell's type theory is committed to inexpressible truths, they must be truths about propositional functions. However, we have seen that the 1910 Russell did not have a realism about propositional functions either; propositional functions of different types are just open sentences with grammatically different kinds of variables. It is not clear that there is there is any "truth" that the theory leaves unexpressible. The value of a propositional function is what one gets by substituting an appropriate kind of expression in the argument place, but the substituted expression is *used* there, not mentioned. The value of one propositional function of type $n + 1$ for another propositional function of type n as argument is not a proposition that asserts something *about* the argument. If we *do* ascend into the metalanguage in order to mention propositional functions, the theory of types does not require that their names be put in different types (*PM* p. 48n). So when we're actually speaking *about the expressions*, there is nothing that can be said about one that cannot be said about others, and indeed, it is perfect possible to say things such as that one is of a certain type, and another is of a different type. This was appreciated years ago by Gödel (1944, pp. 148-149), who wrote that a "realistic interpretation" of the theory of types entails that "the fact that an object x is (or is not) of a given type … cannot be expressed by

a meaningful proposition", noting in footnote that "this objection does not apply to the symbolic interpretation of the theory of types … because there one does not have objects but only symbols of different types." What Gödel has in mind by "the symbolic interpretation" is a reading according to which propositions and propositional functions are merely linguistic entities, and the values of a propositional function are what one obtains by replacing one "fragment of a proposition" with others that "fit together" with remainder. This is precisely the sort of view we argued that Russell had in 1910, and it leaves Russell with nothing to "whistle".

If Russell's views had not changed after 1910, his claim in the introduction to the *Tractatus* (*TLP* p. 23) that the saying/showing distinction is rendered superfluous by the hierarchy of languages would have been entirely appropriate. Unfortunately, however, his views had changed between 1910 and 1921, largely due to Wittgenstein's influence. However, it is doubtful that the changes were the right ones. He seems to have been most sensitive to criticisms Wittgenstein made early on, such as in the 1913 letter quoted above, that he had no explanation for why "*a* is *q*" is meaningful but "*q* is *a*" is not when "*a*" names a particular and not a universal. The influence is apparent long before the *Tractatus* appeared. In *The Theory of Knowledge*, Russell adopted a view on which, while it is admitted that universals can occur as logical subject in facts and propositions, their mode of occurrence is always different from that of particulars (*TK* pp. 92, 123). By 1918, Russell is more radical. In his logical atomism lectures, almost immediately after confessing that a "very great deal of what I am saying in this course of lectures consists of ideas I derived from my friend Wittgenstein", Russell goes on to say:

> Understanding a predicate is quite a different thing from understanding a name. By a predicate, as you know, I mean the word that is used to designate a quality such as red, white, square, round. … To understand a name you must be acquainted with the particular of which it is a name, and you must know that it is the name of that particular. You do not, that is to say, have any suggestion of the form of a proposition, whereas in understanding a predicate you do. … The importance of that is in connection with the theory of types, which I shall come to later on. It is in the fact that a predicate can never occur except as a predicate … .
>
> The different sort of words, in fact, have different sorts of uses and must be kept always to the right use and not to the wrong use, and it is fallacies arising from putting symbols to wrong uses that lead to the contradictions concerned with types. [*PLA* pp. 67-68]

This and similar passages occurring in Russell's writing over the next decade (e.g., in the second edition of *PM*, p. xix) are almost certainly the cause of the common misreading of Russell's earlier type theory according to which the hierarchy of individuals, propositional functions, etc., is equated with the hierarchy of particulars, universals true of particulars, and so on. Something like this became Russell's view only *after* having been influenced by Wittgenstein. Indeed, Ramsey later complained to Wittgenstein that "of all your work, he [Russell] seems now to accept only this: that it is nonsense to put an adjective when a substantive ought to be, which helps in his theory of types" (see Monk 1990, p. 219). Ramsey did not think this change did justice to Wittgenstein's insights and took Russell to task for it (Ramsey 1925).

Ramsey's criticisms notwithstanding, the change represents a betrayal against Russell's years of attempting to find a solution to the paradoxes without presupposing different types of entities. While Russell still does not *equate* propositional functions with universals, he intimates that universals are only indicated by expressions occurring predicatively, suggesting a form. Because predicative expressions, by

definition, cannot occur as subject, no proposition can represent a universal occurring as subject. Now particulars and individuals *are* equated, and universals are left as a distinct type of entity. Even given the greater connection he now sees between functions and universals, he cannot transfer his nominalism about the former into a nominalism about the latter. The reason is that without entities occurring in that peculiar unifying way of which only relations are capable, Russell has no way of explaining the unity of facts. In 1919, he is explicit that he still thinks of relations and qualities as constituents of complexes, indeed, as the constituents doing the relating (*OP* pp. 287-288). But this view, far from "helping with his theory of types", as apparently Russell at the time believed, really *destroys* its philosophical rationale, because Russell now can no longer hold that grasping a complete logical form is possible without grasping the ontological counterpart of a function. This puts him, in essence, back to his views in 1903-1904 and without a explanation for why these genuine entities, parts of the real word, should not be individuals nor occur as logical subject.

Russell was better off before meeting Wittgenstein. To be sure, Russell's views of the era of 1910s *PM* are not without their problems; we noted earlier that Russell's contentions about the truth conditions of quantified propositions are somewhat suspect. But at the very least, then, Russell was not wedded to a metaphysics of different types of genuinely real entities. As Russell himself was one of the first to point out, such theories tend to be self-undermining because they cannot be stated without violating their own strictures (see e.g., *PoM* p. 46; *PMC* p. 134). It *was precisely Wittgenstein's influence* that lead him to a view that entails that there are truths inexpressible in any language. It's certainly true that the post-Wittgenstein Russell is explicit, perhaps even more

so than before, that the theory of types is really a theory of correct symbolism, writing in *PLA* (p. 139) that "the theory of types is really a theory of symbols, not of things. … [I]n that sense in which there are particulars, you cannot say either truly or falsely that there is anything else." But Russell is still a realist about universals and hence committed to holding that there are things, and *genuine things*, that are not particulars. But *that* truth, that *there are not only* particulars but other genuine things, corresponds to no meaningful sense of "there are", in any language respecting type distinctions as he now understand them.

The issues regarding the relationship between universals and predicates, the possibility of making what is "meant" by a predicate into what is meant by the subject of some sentence, the nature of logical form and its relation to grammatical form touched on by Frege, Russell, and Wittgenstein a century ago are still very much in the air. Any viable theory of "logical grammar" must explain how the rules of logical grammar arise without themselves falling into metaphysical or logical incoherence. Frege's attempt to motivate the rules of logical grammar stemming from his function/object distinction as being grounded "deep in the nature" of the entities involved was, I think, not successful. Russell's post-Wittgenstein position, which is really a backslide to the views of his 1903-1904 Fregean period, is also flawed. The views of pre-Wittgenstein Russell still seem worthy of consideration, precisely because *they do not try* to explain the rules governing logical grammar in virtue of the nature of the entities involved; all *genuine* entities are capable of occurring as logical subject. Those "entities" that cannot are not really entities and only appear as such due to a failure to understand the true logical form of those propositions in which they allegedly occur. Insofar as Wittgen-

stein's views are a descendant of the views of early Russell, they have similar merits. What makes early Russell's views so attractive is that *type-distinctions* are applied only to functions, and the priority of complete judgments over functions is not only paid lip service but actually explained. The changes Russell made to his philosophy under Wittgenstein's influence were an unfortunate move away from this position. Russell's views were on more solid footing in 1910, and indeed, in many ways his views then were more Wittgensteinian. However, it would be more accurate to say that Wittgenstein's early views preserved a core insight of early Russellianism that later Russell himself lost sight of, viz., that form should be put before function.

Acknowledgements

Abbreviations for Primary Sources

BLC: Frege, "Boole's Logical Calculus and the Concept-Script," in *PW*, pp. 9-46.

BuG: Frege, "Über Begriff und Gegenstand," translated as "On Concept and Object," in *CP*, pp. 182-194.

CF: Russell, "Complexes and Functions," in *CPBR* v. 4, pp. 69-73.

CP: Frege, *Collected Papers on Mathematics, Logic and Philosophy*, ed. B. McGuinness. New York: Blackwell, 1984.

CPBR: Russell, *Collected Papers of Bertrand Russell*. Vol. 2. *Philosophical Papers 1896-99*, ed. N. Griffin and A. Lewis (London: Unwin Hyman, 1990); Vol. 3. *Towards the "Principles of Mathematics" 1900-02*, ed. G.H. Moore (London: Routledge, 1993); Vol. 4. *Foundations of Logic 1903-05*, ed. A. Urquhart (London: Routledge, 1994); Vol. 6. *Logical and Philosophical Papers 1909-1913*, ed. J. Slater (London: Routledge, 1992).

CSM: Frege, "Comments on Sense and Meaning," in *PW*, pp. 118-125.

CT: Frege, "Gedankengefüge," translated as "Compound Thoughts," in *CP*, pp. 390-406.

DV: Frege, "Der Verneinung," translated as "Negation," in *CP*, pp. 373-389.

EA: Russell, *Essays in Analysis*, ed. Douglas Lackey. New York: George Braziller, 1973.

FCR: Russell, "On Functions, Classes and Relations," in *CPBR* v. 4, pp. 85-95.

FN: Russell, "Fundamental Notions," in *CPBR* v. 4, pp. 111-259.

FO: Russell, "Functions and Objects," in *CPBR* v. 4, pp. 50-52.

FuB: Frege, *Funktion und Begriff,* translated as "Function and Concept," in *CP*, pp. 137-156.

GG: Frege, *Grundgesetze der Arithmetik*, translated in part as *The Basic Laws of Arithmetic,* ed. M. Furth. Berkeley: University of California Press, 1964.

GL: Frege, *Die Grundlagen der Arithmetik*, translated as *The Foundations of Arithmetic,* trans. J. L. Austin. Oxford: Blackwell, 1950.

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KAKD: Russell, "Knowledge by Acquaintance and Knowledge by Description," *Proceedings of the Aristotelian Society* 11, pp. 108-128. Reprinted in *CPBR* v. 6, pp. 147-161.

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LM: Frege, "Logic in Mathematics," in *PW*, pp. 203-250.

LPL: Russell, "Les paradoxes de la logique," *Revue de Métaphysique et de Morale* 14 (1906), pp. 627-50. English version published as "On 'Insolubilia' and their Solution by Symbolic Logic," in *EA*, pp. 190-214.

ML: Russell, "Mathematical Logic as Based on the Theory of Types," *American Journal of Mathematics* 30 (1908), pp. 222-262. Reprinted in *LK*, pp. 59-102.

MPD: Russell, *My Philosophical Development*. London: Routledge, 1959 (1995 paperback edition).

NB: Wittgenstein, *Notebooks 1914-1916*, 2nd ed., ed. G. H. von Wright and G. E. M. Anscombe. Chicago: University of Chicago Press, 1979.

NF: Russell, "On the Nature of Functions," in *CPBR* v. 4, pp. 265-72.

NLD: Frege, "Notes for Ludwig Darmstädter," in *PW*, pp. 235-257.

NT: Russell, "The Nature of Truth," in *CPBR*, vol. 4, pp. 490-506.

OD: Russell, "On Denoting," *Mind* 14 (1905), pp. 479-493. Reprinted in *CPBR* v. 4, pp. 414-427.

OF: Russell, "On Fundamentals," in *CPBR* v. 4, pp. 359-413.

OFns: Russell, "On Functions," in *CPBR* v. 4, pp. 96-110.

OKEW: Russell, *Our Knowledge of the External World*. New York: Mentor Books, 1914 (1960 paperback edition).

OMD: Russell, "On Meaning and Denotation," in *CPBR* v. 4, pp. 314-358.

OMDP: Russell, "On the Meaning and Denotation of Phrases," in *CPBR* v. 4, pp. 284-296.

ONT: Russell, "On the Nature of Truth," *Proceedings of the Aristotelian Society* 7 (1907), pp. 28-49.

ONTF: Russell, "On the Nature of Truth and Falsehood," in *Philosophical Essays*. New York: Longmans & Green, 1910. Reprinted in Russell *CPBR* v. 6, pp. 115-124.

OP: Russell, "On Propositions: What They Are and How They Mean," reprinted in *LK*, pp. 283-320.

OS: Russell, "On Substitution," dated 22 December 1905; ms. located at the Russell Archives, McMaster University.

OSL: Russell, "Outlines of Symbolic Logic," in *CPBR* v. 4, pp. 77-84.

PL: Russell, "The Paradox of the Liar," dated September 1906; ms. located at the Russell Archives, McMaster University.

PLA: Russell, "The Philosophy of Logical Atomism," reprinted in *The Philosophy of Logical Atomism*. Chicago: Open Court, 1985.

PM: Whitehead and Russell, *Principia Mathematica*, 2nd ed. Cambridge: Cambridge University Press, 1927.

PMC: Frege, *Philosophical and Mathematical Correspondence*, trans. Hans Kaal. Chicago: University of Chicago Press, 1980.

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