Direct numerical simulation of laboratory experiments in isotropic turbulence

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Massively parallel computers are now large enough to support accurate direct numerical simulations (DNSs) of laboratory experiments on isotropic turbulence, providing researchers with a full description of the flow field as a function of space and time. The high accuracy of the simulations is demonstrated by their agreement with the underlying laboratory experiment and on checks of numerical accuracy. In order to simulate the experiments, requirements for the largest and smallest length scales computed must be met. Furthermore, an iterative technique is developed in order to initialize the larger length scales in the flow. Using these methods, DNS is shown to accurately simulate isotropic turbulence decay experiments such as those of Comte-Bellot and Corrsin [J. Fluid Mech. **48**, 273 (1971)]. © *1998 American Institute of Physics.* [S1070-6631(98)02709-3]

In direct numerical simulations (DNSs), the Navier Stokes equations (and possibly scalar transport equations) are solved numerically with no modeling to yield a complete description of the flow field as a function of space and time. Typically, databases created using DNS have then been used to study specific characteristics of turbulent flows.[1–3] It has long been recognized that the limited length and time scales available in DNS will restrict the technique for use as a research tool for the foreseeable future, a fact that has not changed even with the advent of massively parallel computers. The new computer technology, however, has put classical laboratory experiments of decaying isotropic turbulence within reach of DNS.

Accurate simulations of laboratory experiments provide two benefits to researchers. First, the DNSs augment the data reported by the experimentalists with a complete picture of an equivalent flow field. Second, the DNS results gain credibility from the fact that they match experimental data. In the past, the results of numerous DNSs of isotropic decaying turbulence have been presented, but none attempt to match laboratory data, and in fact all display features (e.g., decay rates) different from the data. In this note, an accurate simulation of the isotropic turbulence experiment of Comte-Bellot and Corrsin[4] is presented. The numerical resolution requirements and simulation initialization technique are discussed.

In the laboratory experiment, nearly isotropic turbulence decays downstream of a grid of spacing M oriented normal to a uniform, steady flow. Statistical data were collected at downstream locations $x/M=42$, 98, and 171. The Reynolds number based on the Taylor length scale and the rms velocity at the first station is 71.6. The numerical simulations are performed by a pseudospectral code using a 512^3-point periodic domain considered to be moving with the mean flow, and are in dimensional units with no scaling between the laboratory and simulation parameters. Taylor's hypothesis is envoked to relate simulated time to laboratory coordinates. Simulations are initialized to match the laboratory kinetic energy spectrum at $x/M=42$. In the computer code, Fourier pseudospectral methods are used to approximate spatial derivatives, and a second-order Adams–Bashforth scheme with projection is used for time stepping.

The largest length scale that can be represented in a simulation is determined by the size of the computational domain. If L_D is the length of one side of the domain (here taken to be cubic) then the corresponding minimum wave number, $k_{\min}$, is equal to the $2\pi/L_D$. Furthermore, given the domain size and the number of grid points, the largest wave number, $k_{\max}$, and hence the smallest resolved length scale in the simulation, is prescribed.

The full range of length scales in a turbulent flow of even modest Reynolds number spans several orders of magnitude, and it is not feasible to capture them all in a numerical simulation. Taking a direct numerical simulation to be one in which the effect of the unresolved scales of motion on the resolved scales is negligibly small, this leads to the conclusion that the smallest resolved scale must be related to the Kolmogorov length, η. Eswaran and Pope[5] determined that $\eta k_{\max}>1$ is necessary for the small scales to be adequately resolved, and this criterion has been verified by other researchers.[1,3]

In order to maximize the Reynolds number of the simulated flow, it has been customary to make $k_{\min}$ as large as possible compared to $1/L$, where L is an integral length (i.e., to make L_D/L as small as possible) without having the numerical boundary conditions unduly influencing the solution. Typically, $Lk_{\min}\simeq 1$ in simulations of isotropic turbulence.[1,3,5] One can show by dimensional reasoning, however, that the evolution of the velocity field will depend strongly on the shape of the kinetic energy spectrum at low wave numbers[6] (large length scales), and Chasnov demonstrates this point using large-eddy simulations in which $Lk_{\min}\approx 0.01$.[7] To determine the value of $k_{\min}$ needed for an accurate simulation of the experiment of Comte-Bellot and Corrsin, a series of direct numerical simulations was made

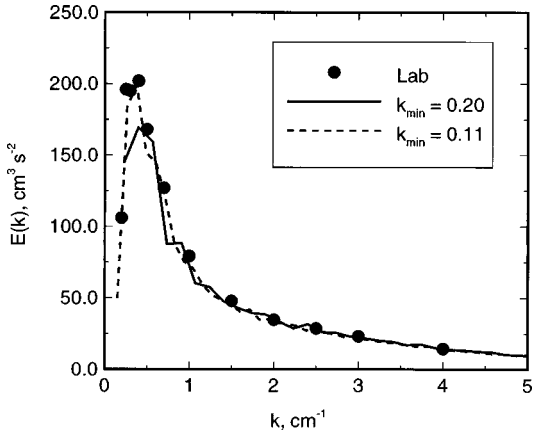

FIG. 1. Energy spectra at $x/M=98$ for low wave numbers.

over a range of values of $Lk_{\min}$. Figure 1 shows the kinetic energy spectrum for two such simulations which have been advanced from the initial location $x/M=42$ to $x/M=98$. It is clear that, in the larger domain ($Lk_{\min}=0.27$ initially), the spectrum evolves in almost exactly the same fashion as in the laboratory flow. In the smaller domain ($Lk_{\min}=0.48$ initially), energy is removed from the large scales too rapidly, and analysis of several realizations of the simulations shows that the peak of the spectrum moves to the left more slowly than that of the laboratory flow.

A more direct approach than trial and error for determining the required computational domain size is to examine the spectral energy transfer function, $T(k,x)$, at $k=k_{\min}$. In Fig. 2, $T(k_{\min},x)$, normalized by the maximum negative value of $T(k,x)$, is negligibly small for the larger domain; this implies that scales larger than the computational domain, if they existed, would not be involved in significant interactions with the smaller scales. In the smaller domain, the transfer rate is not negligibly small at the lowest wave number, indicating that energy is being transferred out of the largest scales in the simulation and presumably would be transferred from even larger scales if they existed. The conclusion is that, for a simulation of decaying, isotropic turbulence to evolve correctly, the energy transfer rate for the largest scales in the simulation must be negligibly small, which for this flow requires approximately $Lk_{\min}<0.3$. This large-scale resolution requirement is consistent with that used in large-eddy simulations of this laboratory experiment by several researchers.[8,9]

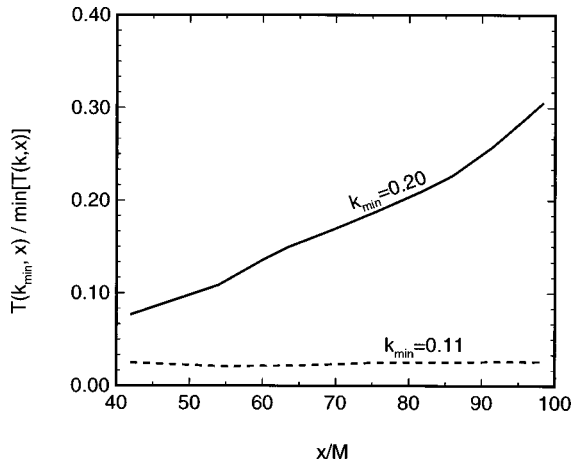

FIG. 2. Energy transfer rate at $k_{\min}$, $T(k_{\min},x)$, normalized by the minimum (most negative) $T(k,x)$.

In the initialization process, it is important to not only match the initial kinetic energy spectrum, and therefore the appropriate velocity and length scales, but also that the flow develop as it would in a laboratory experiment. Initialization of the Fourier modes using a random number generator will result in a value of zero for the velocity derivative skewness, S, and hence no initial spectral transfer.[10] For this reason, researchers have often allowed S to build up to values typical of laboratory experiments, and then use the resulting flow field as an initial condition.[11] It has been found, however, that in order to simulate laboratory experiments, matching S is not sufficient; it is necessary to advance the simulations much farther in time in order to allow the turbulent structures to develop.

Requiring the initial spectrum to match that of a laboratory flow complicates such an initialization process since, in allowing the structures to develop, the energy spectrum drifts away from the desired one. A method which permits both the matching of the initial spectrum and the proper development of the larger-scale structures is the following. The flow field is advanced in time, and then the average amplitude of each wave number band is adjusted to match the desired spectrum. This process is repeated until the statistics of the field are the same at the end of two successive advances of the field. The most sensitive test of the statistical equality of two fields is found to be $T(k,x)$, since it measures the nonlinear interactions between different Fourier components of the velocity field. Typically, when the process of advancing the field for half an integral time scale and rescaling the Fourier amplitudes is repeated ten or more times, a trend will be apparent in $T(k,x)$ when it is observed at the same value of x in each iteration. This trend will become less pronounced with each iteration until the difference in $T(k,x)$ between consecutive iterations is very small, at which point the field is considered to be ready for initializing the simulation. This process is very inefficient in a large DNS. In order to create initial fields for the current simulations, large eddy simulations (LESs) were used. The LES code is identical to the DNS code, except that the viscosity includes a subgrid-scale term which is computed dynamically as a function of space and time.[12] The fields were advanced and rescaled until $T(k,x)$ became consistent in a 64^3 LES domain before inserting higher wave number bands (randomly generated) to form a 128^3 initial LES field. The progression was continued until a 512^3 DNS field had been developed. Of the five or more large-eddy turnover times required to initialize the field, all but about one can be done in the 64^3 LES domain, since the small length scales included at higher resolutions adjust in a time which is smalled compared with the integral time scale.

Figure 1 demonstrates that the simulations accurately predict the energy containing scales, and so it is expected that the kinetic energy will decay correctly in the simula-

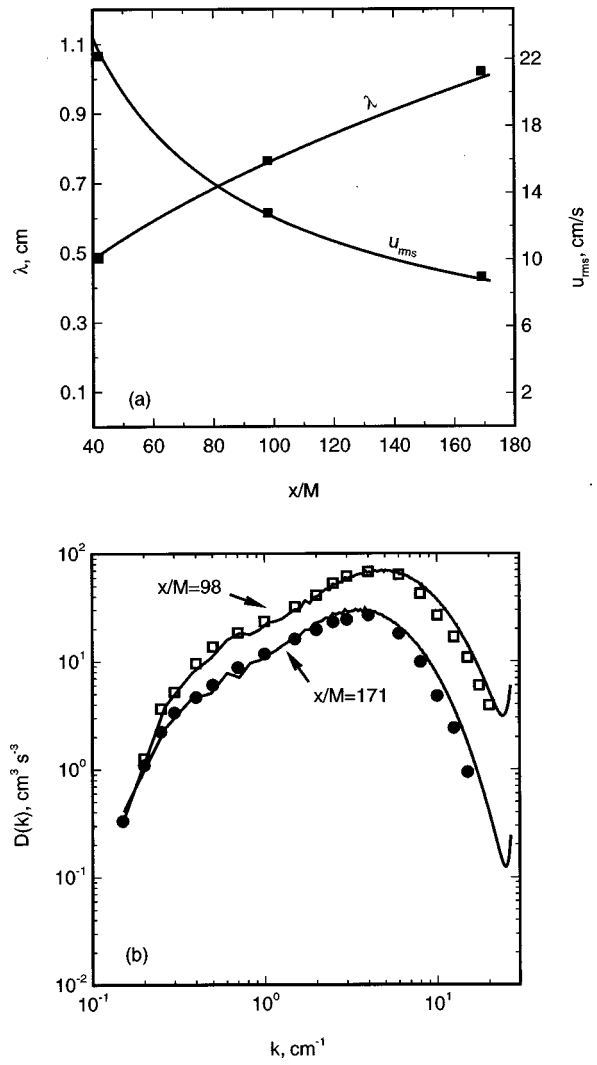

FIG. 3. Comparison of DNS (lines) and laboratory data (symbols). (a) Taylor length scale and rms velocity vs time. (b) Dissipation rate spectra.

tions. Figure 3(a) shows this to be the case. The behavior of the dissipation spectrum is exhibited in Fig. 3(b). The agreement is very good, although slight aliasing errors at the smallest length scales and failure to resolve the Kolmogorov length, i.e., to satisfy $\eta k_{\max}=2\pi$, are evidenced by the upturn at the highest wave numbers and by the slight overprediction of the dissipation rate at the higher wave numbers. The velocity derivative skewness ranges from -0.44 to -0.51 and the flatness is about 4.6, which are consistent with values observed in numerous laboratory experiments.[6,13]

Massively parallel computers are now large enough to support direct numerical simulations of isotropic turbulence measured in laboratory experiments. Such simulations provide researchers with a full description of the flow field, as a function of space and time, which is known to be physically accurate. A large scale resolution requirement of $Lk_{\min}<0.3$ must be met, and a new iterative technique is used to initialize the flow. Using these methods, DNS can accurately simulate simple flows such as those of Comte-Bellot and Corrsin.[4]

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