A Reformulated Version of Marx’s Theory of Ground-Rent Shows that there Cannot be any Absolute Rent

Deepankar Basu*

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Abstract

This paper develops a simple theoretical model to analyze Marx’s theory of ground rent. Using the model, I demonstrate two important results. First, if we take capital outlay as exogenously given, then total ground-rent can be decomposed into the three components: differential rent of the first variety (DRI), differential rent of the second variety (DRII), and absolute rent (AR). Second, if we let the amount of capital outlay on each plot of land be determined by the profit-maximizing behaviour of capitalist farmers, then absolute rent becomes zero. Neither low organic composition of capital nor monopoly power of the class of landlords can generate any absolute rent. I conclude that under reasonable behavioural assumptions about landlords and capitalist farmers, there will be no absolute rent in a capitalist economy.

JEL Codes: B51.

Key words: ground rent, differential rent, absolute rent.

*Department of Economics, UMass Amherst. Email: dbasu@econs.umass.edu. Without implicating him in any way in the conclusions of this paper, I would like to thank Debarshi Das and Duncan Foley for extensive and continuing discussions on the Marxist theory of rent. I would also like to thank three reviewers of RRPE for their insightful comments.

1
1 Introduction

The theory of ground-rent is an important but relatively under-theorized part of Marxist political economy. Part of the reason for the lack of theoretical development in this area probably arises from the difficulty scholars face in clearly and precisely defining the meaning of key terms involved in the discussion: ground rent, and the three components into which Marx decomposed it, i.e. differential rent of the first variety (DRI), differential rent of the second variety (DRII) and absolute rent (AR). In this paper, I present a theoretical model to define ground-rent, to decompose it into DRI, DRII and AR, and to analyze in detail the meaning and source of AR.

Marx had worked out his theory of ground-rent mainly through numerical examples and later scholars have largely followed Marx’s approach. The continued use of simple numerical examples and the difficulty of clearly and precisely defining key terms involved in the discussion have limited the development of Marx’s theory of ground-rent. While there is broad agreement about the definition of ground-rent as such, there is much less agreement about the decomposition of ground-rent into its three components: DRI, DRII and AR. This paper offers a theoretical framework to address these disagreements and potentially move the literature ahead.

Two types of disagreements in the extant literature are especially worth highlighting. The first significant disagreement relates to different views about the existence, meaning and source of absolute rent (Fine, 1979; Ramirez, 2009; Fine and Filho, 2010). While some scholars view absolute rent as arising from the class power (or monopoly power) of landlords (Ramirez, 2009), others contend that the cause of absolute rent is the relatively low organic composition of capital in agriculture (Fine, 1979, 2019). The theoretical framework developed in this paper will be able to throw light on this disagreement.

The second disagreement relates to the meaning of DRI and DRII. In addressing this disagreement, it is important to note that there was, for a long time, no theoretical framework available to implement the intuitions about these two components of differential rent in a unified manner. Part of the difficulty in carrying out this decomposition arises from the fact that defining DRI and DRII require us to use different benchmarks. While DRI is defined with respect to the least-productive plot of land, DRII is defined with respect to the least-productive unit of capital. Most scholars, following Marx, have addressed DRI and DRII one at a time, but have not been able
to integrate them together into a single theoretical framework (Fine, 1979; Fine and Filho, 2010).

Das (2018) offered the first coherent theoretical framework to understand the three components of ground-rent in a unified manner. In this paper, I follow and extend the analysis in Das (2018). In particular, I modify two key components of the analysis in Das (2018). Whereas Das (2018) had assumed that increments of capital outlay occur in discrete units, I allow capital outlay to increase continuously. This is important because it is difficult to justify discrete changes in capital outlay. While it is true that agricultural production involve fixed costs that can be lumpy, the variable cost of production is likely to vary continuously. Hence, total capital outlay is more realistically modeled as varying continuously, rather than in discrete amounts.

The second modification is even more important. Where Das (2018) had assumed that the amount of capital outlay on each plot of land is given, I probe deeper and ask: how much capital will be invested on any plot of land? Since plots of land are operated by capitalist farmers, a natural way to answer this question is to see what level of capital outlay will maximize the capitalist farmer’s profit. In this paper, I show that this modification has far reaching consequences. If capitalist farmers choose the level of capital outlay to maximize the profit income they earn, then there will be no absolute rent. This is the most important result of this paper. In general, I show that absolute rent will be zero. The analysis in this paper shows that neither a relatively low organic composition of capital in agriculture Fine (1979) nor the class power of landlords Ramirez (2009) can generate absolute rent.

The overall significance of Marx’s theory of rent, and its clarification in this paper, is two fold. On the one hand, it is a component part of his theory of the distribution of surplus value in capitalist economies. It explains the origin of the source of income of landlords and other resource owners as a fragment of the surplus value produced through the exploitation of labour. Marx understood ground-rent in a capitalist economy to be a part of the surplus value that is appropriated by private owners of non-reproducible resources like land (Marx, 1993). Marx argued that use of the non-reproducible resource can confer benefits on capitalist producers and generate what Marx called ‘surplus profit’, i.e. profit over and above what could be earned at the prevailing uniform (average) rate of profit. Therefore, private owners of non-reproducible resources can withhold the resource from the capitalist producers unless the latter agree to make a monetary payment for its use. In fact, in the process of bargaining between capitalists and
resource owners, capitalist producers who wish to use those resources have to part with the full amount of the surplus profit. This monetary payment is what Marx calls ground-rent. For Marx, therefore, ground-rent is the economic form that the monopoly of private ownership of non-reproducible resources takes in a capitalist economy, and its monetary worth is the surplus profit. While Marx mainly discussed land rent in the context of agricultural production, he was clear that his analysis applied to all cases where a private monopoly of land ownership obtains. The clearest articulation of the idea that rent is surplus profit and that it can be applied widely can be seen from the following quotation.

Whenever natural forces can be monopolized and give the industrialist who make use of them a surplus profit, whether a waterfall, a rich mine, fishing grounds or a well situated building site, the person indicated as the owner of these natural objects, by virtue of his title to a portion of the earth, seizes this surplus profit from the functioning capital in the form of rent. (Marx, 1993, Chapter 46, pp. 908)

On the other hand, it also highlights the total irrationality of rent as an income stream that derives from the private ownership of a ‘portion of the earth’, something which should not be the domain of private property at all.

From the standpoint of a higher socio-economic formation, the private property of particular individuals in the earth will appear just as absurd as the private property of one man in other men. Even an entire society, a nation, or all simultaneously existing societies taken together, are not the owners of the earth. They are simply its possessors, its beneficiaries, and have to bequeath it in an improved state to succeeding generations, as boni patres familias. (Marx, 1993, pp. 911).

The rest of the paper is organized as follows. In section 2, I discuss an example to build intuition and explain key ideas behind Marx’s theory of ground-rent, both with exogenous and endogenous capital outlays. The discussion of the example is followed by a presentation of Marx’s theory of rent in more general terms in the next few sections. In section 3, I discuss the set-up for the general analysis and define the technological conditions of production. In section 4, I define ground-rent and derive explicit expressions for its three components when capital outlays are exogenously given:
In section 5, I endogenize capital outlays using profit maximising behaviour of capitalists and show that, in such a situation, absolute rent will be zero. In section 6, I close the model by determining the price of corn by the interaction of demand and supply of the agricultural product. The final section concludes the discussion.

2 Example

Before I present my argument in general terms, I would like to discuss an example to build intuition. In this example, the agricultural economy is composed of just two plots of land– plot 1 and plot 2 – and the agricultural output is ‘corn’. The marginal product of capital outlay on each plot declines linearly with the level of capital outlay, i.e. as more capital is invested on a plot, the extra amount of output produced declines. This is an intuitively appealing assumption because there is a fixed factor of production: land. As more capital is invested on a given plot of land, output rises but only at a declining rate. This is captured by the assumption that the marginal product of investment, i.e. the increment of output with each dollar of investment, declines as capital investment is increased.

For this example, let us assume that the marginal product of capital on plot 1 is given by $10 - 3k$ and on plot 2 by $5 - k$, where $k$ denotes capital outlay. Note that, on both plots, the marginal product declines with capital outlay. Suppose, finally, that total capital outlay on both plots of land is 2.75 dollars, the economy-wide average rate of profit is 25% and the price of corn is 2 dollar per unit.

Given this configuration of the agricultural economy, I would like to compute the amount of ground-rent on both plots and then decompose total ground-rent into its three components. I will first discuss a situation where the level of capital outlay on both plots of land is given to be 2.75. In the following sub-section, I will discuss the situation when capitalist farmers choose the level of capital outalys to maximize their profit income.

2.1 Exogenous Capital Outlay

Marginal output as a function of capital outlay is depicted for both plots of land in Figure 1. Both graphs have a vertical, broken line at $k = 2.75$, which is the total amount of capital outlay on each plot. This is exogenously given.
The marginal product curves give us some useful information. For plot 1, the curve starts at a value of 10 units of corn and then declines towards zero as more capital is invested; for plot 2, the curve starts at 5 units of corn and then, in a similar manner, declines towards zero. The curves intersect at \( k = 2.5 \). For levels of capital outlay less than 2.5 dollars, plot 1 has higher marginal product; for levels of capital outlay that is larger than 2.5 dollars, plot 2 has higher marginal product.

2.1.1 Which Plot is More Fertile?

My first task is to identify the plot that is more fertile. This is necessary because ground rent of the first variety (DRI) is related to differential fertility of the plots, i.e. a part of the total ground-rent on relatively more fertile plots can be identified as DRI (Marx, 1993, chapter 39). I will associate fertility of a plot of land with the marginal product of investment on that plot. This makes sense because a plot with higher marginal product of investment can be intuitively understood to be more fertile. One immediate issue we face here is that, as we see in Figure 1, the marginal product declines with the level of capital investment. For instance if we chose to use the marginal product at 1 dollar of capital outlay to define fertility, then plot 1 would be deemed more fertile; if, instead, we chose the marginal product at 2.7 dollar of capital outlay to define fertility, then plot 2 would be deemed more fertile. Thus, fertility ranking would change as capital outlays change across plots of land, rendering any analysis infeasible.

To get around this problem and to define a stable fertility ranking among plots of land, a ranking that does not change when the level of capital outlays change, I will choose to define fertility with the marginal product at zero capital outlay. This is the extra amount of output produced when capital investment is just increased beyond zero. Intuitively, this definition takes us close to what we can understand as the intrinsic fertility of a plot of land. If this extra amount of output varies across plots, it must vary because of the intrinsic qualities of the plots of land (impacted no doubt by past investments). Hence, I will identify plots with higher marginal product at zero capital outlay as more fertile than plots with lower marginal product at zero capital outlay. Using this convention, we identify plot 1 as more fertile than plot 2.
Figure 1: Marginal output of capital outlay on two plots of land with different fertility when capital outlay is exogenously fixed at 2.75.

2.1.2 Total Ground-Rent

I will first find the total amount of ground-rent on the two plots of land. According to Marx, total ground-rent is the surplus profit on a plot of land, where surplus profit is the total output less the notional amount that would have been earned if the capital earned the economy-wide average rate of profit (Marx, 1993, chapter 37, 38). On plot 1, total output can be found by adding up the marginal outputs associated with all capital outlays between 0 and 2.75. This is represented in Figure 1 by the area $AKIF$ and is equal to 16.16 units of corn. If the total capital invested on this plot had earned the economy-wide average rate of profit of 25%, it would produce revenue
of $2.75 \times (1 + 0.25)$ dollars. Since the price of each unit of corn is 2 dollars per unit, this is equivalent to 1.72 units of corn. Hence, surplus profit on plot 1 is equal to $14.44(= 16.16 - 1.72)$ units of corn. Since ground-rent is quantitatively equal to surplus profit, we see that the total ground-rent on plot 1 is 14.44 units of corn. Using a similar argument, we find that the total ground-rent on plot 2 is 7.56 units of corn.

### 2.1.3 Absolute Rent

Total ground-rent on plot 1 is represented in Figure 1 by the area $BJIF$, and total ground-rent on plot 2 is represented in Figure 1 by the area $BJHE$. We see immediately that the area $BJIC$ is part of the total ground-rent on both plots of land. Moreover, this area and the portion of ground-rent that it represents, does not get impacted by the *declining* marginal products of capital investments, once total capital outlay is fixed on the two plots of land. This leads me to identify this part of total ground-rent as absolute rent (AR). The reason for this is that the other two components of ground-rent, DRI and DRII, depend, respectively, on fertility differentials and differences of capital outlays - both of which will be impacted by the declining marginal products (Marx, 1993, chapter 38, 39).

At this point, let me highlight an important point. Notice that the height of the rectangle $BJIC$ is given by $JI$. This is equal to the difference between $KI$ and $KJ$. On the one hand, $KI$ represents the marginal product of capital on the plot in which it is lowest when total capital outlay is 2.75; on the other hand, $KJ$ represents the notional revenue, measured in units of corn, that would be earned at the economy-wide average rate of profit for each dollar invested. This is given by $(1 + r)/p$, where $r$ is the economy-wide average rate of profit and $p$ is the price of a unit of corn, and can be thought of as the opportunity cost of investing capital in agriculture. Hence, the amount of absolute rent, being the area of the rectangle $BJIC$, depends crucially on the *difference* between the lowest marginal product of capital outlay among the plots of land and the notional revenue earned by each dollar of investment. As this difference falls (rises), AR falls (rises). Using the numbers in Figure 1, we get the magnitude of absolute rent, on both plot 1 and plot 2, as 3.09 units of corn.
2.1.4 Differential Rent

Let us now turn to plot 1 and compute the two components of differential rent. To compute differential rent of the first variety on plot 1, $DRI_1$, we need to quantify the difference in the fertility levels of the two plots. Using Figure 1, we see that one way to capture this difference is by the length of the segment $EF$. This is the difference in the marginal product at zero capital outlay between plot 1 (more fertile) and plot 2 (less fertile). To use this difference in the marginal product at zero capital outlay to compute $DRI$ on plot 1, note that, starting with with a value of 10, as the magnitude of capital outlay rises on plot 1, its marginal product declines. When capital outlay is 1.67, the marginal product equals 5, which is exactly equal to the marginal product on plot 2 at zero capital outlay. This is represented in Figure 1 by the point $G$.

Since differential rent of the first variety arises from fertility differentials, I define $DRI$ on plot 1 as being represented by the area $EGF$ in Figure 1. This is the extra amount of rent earned on plot 1 due to the higher fertility on that plot in comparison to plot 2. Using the numbers in Figure 1, we get $DRI$ on plot 1 as 4.17 units of corn. This allows us to now compute $DRII$ on plot 1 as total ground-rent less the sum of $AR$ and $DRI$. Hence, $DRII$ on plot 1 is $7.18 = 14.44 - 4.17 - 3.09$. On Figure 1, $DRII$ for plot 1 is represented by the area $CIGE$.

We can compute the components of differential rent on plot 2 in an analogous manner. To start with, note that, by definition, differential rent of the first variety on plot 2, $DRI_2$, is zero. This is because plot 2 has lower fertility than plot 1 and cannot have $DRI$. In fact, plot 2 has been used as the reference to compute $DRI$ on plot 1. Thus, there cannot be any $DRI$ on plot 2. Since we have already computed total ground rent and absolute rent on plot 2, we can get $DRII$ as the difference between them. Hence, $DRII$ on plot 2 is $4.47 = 7.56 - 3.09$ units of corn.

Let me now summarize what I have found. The total ground-rent on plot 1 and plot 2 are 14.44 and 7.56 units of corn, respectively. The components of ground-rent on plot 1 are: $DRI_1 = 4.17, DRII_1 = 7.18, AR_1 = 3.09$; the components of ground-rent on plot 2 are: $DRI_2 = 0, DRII_2 = 4.47, AR_2 = 3.09$. 

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2.2 Capital Outlay Endogenized

I will now discuss the determination and decomposition of ground-rent when capital outlay is endogenized. Let me start with plot 1 and ask how much capital outlay would a profit-maximizing capitalist farmer choose to invest? The answer is straightforward. The capitalist farmer will choose a level of capital outlay that will maximize her profit. This implies that the chosen level of capital outlay will be such that the marginal product of capital is exactly equal to \((1 + r)/p\).\(^1\) If she were to choose a level of capital outlay that is lower than this level, then the marginal product of capital would be higher than \((1 + r)/p\). By increasing capital outlay by a small magnitude, the capitalist farmer would be able to increase her profit income. On the other hand, if the chosen capital outlay was higher than this level, then the marginal product of capital would be lower than \((1 + r)/p\), so that the capitalist could reduce her loss by reducing her capital outlay. The only point where there would be no incentive for the capitalist to change her level of capital outlay would be the level at which the marginal product of capital is exactly equal to \((1 + r)/p\).

Using this logic, we can see from Figure 2 that the profit-maximizing level of capital outlay on plot 1 would be represented by the point \(H\). At this level of capital outlay, the marginal product of capital is exactly equal to \((1 + r)/p\). Using the numbers for the example, we find that this level of capital outlay is 3.125 dollars. Using the same logic as above, we can see that the profit-maximizing level of capital outlay on plot 2 will be represented in Figure 2 by the point \(I\). Using the numbers for the example, this profit-maximizing level of capital outlay on plot 2 is 4.375 dollars.

Let us now compute the total ground-rent on the two plots. On plot 1, this is represented by the area \(BHF\) in Figure 2. Using the numbers for the example, total ground-rent on plot 1, when capital outlays is endogenous, is given by 14.65 units of corn. On plot 2, total ground-rent is represented by the area \(BIE\) in Figure 2. Using the numbers for the example, total ground-rent on plot 2, when capital outlays is endogenous, is therefore given by 9.57 units of corn.

What is the magnitude of AR? Recall that the magnitude of AR depends on the difference between the marginal product of capital on the plot where it has the lowest value at its level of capital outlay and the opportunity cost of investing in agriculture. When profit-maximizing capitalist farmers choose

\(^1\)Recall that \((1 + r)/p\) is the opportunity cost of investing capital in agriculture.
the level of capital outlay to maximize their profit income, it necessarily leads to equality of the marginal product of capital on each plot of land and the opportunity cost of investing in agriculture. Hence, the difference between the marginal product of capital and the opportunity cost of investing in agriculture is zero on each plot. This implies that the magnitude of AR is zero.

What is the magnitude of DRI and DRII on plot 1? From Figure 2, we see that DRI on plot 1 is exactly the same as in Figure 1. Hence, DRI on plot 1 is 4.17 units of corn. Since total ground-rent on plot 1 is 14.65, this immediately tells us that DRII on plot 1 is $10.48 (= 14.65 - 4.17)$ units of corn.

Figure 2: Marginal output of capital outlay on two plots of land with different fertility when capital outlay is chosen by capitalist farmers to maximize profit income.
corn. In Figure 2, DRI and DRII on plot 1 are represented by the areas $EGF$ and $BHGE$, respectively. What is the magnitude of DRI and DRII on plot 2? Just like before, DRI on plot 2 is zero because it is the less fertile plot among the two plots that comprise the agricultural economy in this example economy. Hence, total ground-rent on plot 2 is exactly equal to DRII. Thus, DRII on plot 2 is 9.75 units of corn (the total ground-rent on plot 2). In Figure 2, DRII (as also total ground-rent) on plot 2 is represented by the areas $BIE$.

Let me now summarize my findings. When capitalist farmers choose the level of capital outlay to maximize profit income, the total ground-rent on plot 1 and 2 are 14.65 and 9.57 units of corn, respectively. The components of ground-rent on plot 1 are: $DRI_1 = 4.17$, $DRII_1 = 10.48$, $AR_1 = 0$; the components of ground-rent on plot 2 are: $DRI_2 = 0$, $DRII_2 = 9.57$, $AR_2 = 0$. In the next subsections, I present this same argument in more general terms.

3 Conditions of Production

3.1 The Set-Up

I conceive of the economy as being composed of an industrial sector and an agricultural sector. The industrial sector is composed of $I$ industries, each producing a single product with a given technique of production. The agricultural sector produces a homogenous output, called ‘corn’, using land, labour, and inputs from industry. I work with the implicit assumption that corn is a non-basic product in the following precise sense: corn is not used as a direct means of production in any industrial production process. In such a scenario, the prices of industrial products are determined independently of the conditions of agricultural production. In technical terms, this means that the industrial sector forms a decomposable system of equation - which determines the prices of the industrial products and the average rate of profit (Kurz, 1978, pp. 27). Hence, for the analysis of rent, in this paper, I take the prices of industrial products, the wage rate and the economy-wide average rate of profit, as exogenously given.

In agriculture, following Marx (as stated in the quotation below), I assume that there are three classes: capitalists, workers and landlords. All the agricultural land is owned by the class of landlords. Capitalist orga-
nize production of corn by leasing in land from landlords and purchasing the labour-power of workers. The lease contract between capitalists and landlords specifies a fixed period of time - one production cycle - for which the latter hands over the right to use the relevant plot of land to the capitalist in return for a monetary payment known as ground-rent. The labour contract between capitalists and workers specifies a fixed period of time - one production cycle - for which the latter gives up the use of her labour-power to the capitalist for a monetary payment known as the wage. After production is completed, the capitalist sells the corn on the open market to recoup the wage and rent payments she made earlier and, in addition, make a profit income.

The presuppositions for the capitalist mode of production [in agriculture] are thus as follows: the actual cultivators are wage-labourers, employed by a capitalist, the farmer, who pursues agriculture simply as a particular field of exploitation of capital, as an investment of his capital in a particular sphere of production. At certain specified dates, e.g. annually, this capitalist-farmer pays the landowner, the proprietor of the land he exploits, a contractually fixed sum of money ... for the permission to employ his capital in this particular field of production. This sum of money is known as ground-rent, irrespective of whether it is paid for agricultural land, building land, mines, fisheries, forests, etc. It is paid for the entire period for which the landowner has contractually rented the land to the farmer. Marx (1993, pp. 755-756).

3.2 Technology of Production

Suppose the total available land used in agricultural production is divided into $N$ plots and is indexed by the numbers $i = 1, 2, \ldots, N$. Let a subset of these plots, indexed by $i = 1, 2, \ldots, n$, be currently in use for agricultural production. An important point worth highlighting upfront is that $n$ is a function of the price of corn, $p$, i.e. $n = n(p)$. The reason for this is easy to see. As the price of corn rises, some of the previously unused plots of land are brought under cultivation because they can now generate ‘surplus profit’ and hence provide ground-rent. Since plots of land will not be leased out by landlords unless they receive positive amount of ground-rent, the number of plots that will be used for agricultural cultivation will itself depend on the
price of the agricultural commodity. Thus, the plots of land that are in use are indexed by $i = 1, 2, \ldots, n(p)$, where the index of the last plot, $n(p)$ is a function of $p$, the price of the agricultural commodity.

On the $i$-th plot of land that is in use, let total capital outlay by the capitalist producer of corn be denoted by $k_i = c_i + v_i$, where $c_i$ and $v_i$ are constant and variable capital respectively, measured in monetary units. Here $c_i$ refers to the sum of money used by the capitalist to purchase non-labour inputs into production, and $v_i$ refers to the sum of money used to purchase labour-power (for one production cycle). In examples that he works with, Marx explicitly specifies the relationship between capital outlay and the volume of output on each plot of land. For instance, each row in Table I, II and III in Marx (1993, Chapter 39, pp. 791, 794–95) specifies the quantitative relationship between capital outlay and output.

I express Marx’s numerical examples in a general algebraic form and capture the relationship between total capital outlays and the quantity of output on the $i$-th plot with the function, $f_i(.)$. I would like to point out that what I am using is not a standard production function. In my case, following Marx, the monetary value of inputs, and not their real magnitudes, enter into the function, $f_i(.)$, as inputs. To distinguish my analysis from the standard case, where the arguments of the function are real quantities of inputs, I therefore call $f_i(.)$ a pseudo-production function.

I will assume that the collection of pseudo-production functions have standard concavity properties and note this explicitly.

**Assumption 1.** Output on each plot of land is zero if capital outlay is zero, i.e. $f_i(0) = 0$, for all $i = 1, 2, \ldots, n(p)$. The quantity of output increases with the magnitude of capital outlay, i.e. $f_i'(k) > 0$ for $k \geq 0$; the marginal product of capital outlay is always nonnegative and finite, i.e. $0 \leq f_i'(k) < \infty$ for $k \geq 0$; and, the marginal product diminishes with the magnitude of capital outlay, i.e. $f_i''(k) < 0$ for $k \geq 0$.

Assumption 1 is inspired by classical economics (Pasinetti, 1977, Chapter 1, Section 3). It has three components. First, agricultural output cannot be produced without capital outlay to purchase inputs and labour-power. This is easy to justify because production cannot take place without inputs and without labour. Second, the marginal product of capital outlay, i.e. the extra amount of output produced when capital outlay is increased by a small magnitude, is positive. This means that from any given level of capital outlay, when capital outlay is increased by a small magnitude, there is an
increase in the amount of output produced. For instance, if capital outlay increases from 1 dollars to 1.1 dollars, agricultural output increases by a positive amount. Third, the marginal product of capital outlay is a declining function of capital outlay. This means that as capital outlay is increased from any level, the increment in output associated with a small increase in capital outlay itself decreases with the level of capital outlay. For instance, the increment in output associated with capital outlay increasing from 2 dollars to 2.1 dollars is smaller than the increment in output associated with capital outlay increasing from 1 dollars to 1.1 dollars.

The third assumption is crucial and can be justified as follows. Land cannot be produced by labour. Thus, the amount of land in each plot (or parcel) is fixed, i.e. it cannot be increased with the use of labour. To capture this we can say that land is a non-reproducible resource. Since the amount of land on any plot of land is fixed, as more capital is invested on it, the output increases but only at a declining rate - as captured in Figure 3. The key point is that the non-reproducible nature of land imparts to it the property of diminishing marginal product of capital outlay.

3.3 Worst-Quality Plot

A characteristic feature of agricultural production is that the plots of land are of unequal ‘quality’ (which we measure with the marginal productivity of capital); some plots are more fertile, or have better location, than others (Marx, 1993, pp. 789). While it is true that plots of land are of different quality, we cannot rank order them by productivity without taking account of the amount of capital outlay in each plot, as has been stressed by Sraffian scholars (Kurz, 1978). Why is this the case? Because the area of each plot of land is fixed and the pseudo-production functions are concave by Assumption 1, the marginal product of capital outlay can vary across plots of land because of different amounts of capital outlay across plots. Since we use the marginal product of capital as the measure of land productivity (or its quality), there can be rank ordering reversals when capital outlays is allowed to vary across plots of land. This is highlighted in Figure 4, where we can see that the marginal product curves for the $i$-th and $j$-th plots cross over. For

\footnote{For a similar assumption see Figure 1.4 in Pasinetti (1977, Chapter 1, Section 3).}

\footnote{One of the errors in Ricardo’s analysis of rent was the assumption that plots of land could be ordered according to fertility without taking account of capital investment. Marx had inherited the same error from Ricardo. (Kurz, 1978, pp. 21)}
Figure 3: *This figure depicts the pseudo-production function on the i-th plot of land that is in use for agricultural production. The curve \( f_i(k) \) denotes the output, and \( f'_i(k) \) denotes the marginal product, both as a function of the amount of capital outlay, \( k \).*

...
ital, like irrigation, etc. that is now part of the plot of land. This is because the marginal product of capital at zero capital outlay can be allowed to be implicitly determined by past improvements made on the plot of land. That is why I have put the word intrinsic in quotation in the second sentence of this paragraph.

![Diagram](image)

**Figure 4:** *This figure depicts the marginal product of capital outlay on the *i*-th plot (solid line) and *j*-th plot (dashed line) of land. Note how there is a reversal of marginal productivity ranking of the two plots when we move from low to high capital outlays.*

Using this idea, I will order plots *i* and *j* according to intrinsic quality as follows: I will say that plot *i* is of higher quality than plot *j* if \( f'_i(0) > f'_j(0) \), i.e. if the marginal product of capital at zero capital outlay is higher on plot *i* than on plot *j*. Since there are a finite number of plots, we can always arrange them in diminishing order of quality, as we have defined it here (with strict inequalities). Once we do so, we can renumber the plots and call plot 1 the most fertile, plot 2 the next most fertile, and so on. I state this as the

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4 Previous investment in land improvement would shift the whole marginal product curve upwards.
assumption about the ordering of the intrinsic quality of plots of land.

**Assumption 2.** \( f_1'(0) > f_2'(0) > \cdots > f_{n-1}'(0) > f_n'(0) \), where \( n = n(p) \) and \( p \) is the price of the agricultural commodity.

In the rest of the paper, the analysis of ground-rent will proceed in three steps. In the first step, I will derive expressions for ground-rent and its three components, \( DRI \), \( DRII \) and \( AR \), taking the capital outlays on each plot of land, \( k_i \), the price of corn, \( p \), and the economy-wide average rate of profit, \( r \), as exogenously given. In the second step, I will endogenize the capital outlay, \( k_i \), by positing profit-maximising behaviour of capitalist farmers. In the third, and final, step, I will close the model by endogenizing the price of corn, \( p \), by allowing the interaction of demand and supply of corn to clear the market for corn. The economy-wide average rate of profit, \( r \), will remain exogenous throughout the analysis.

## 4 Ground-Rent With Exogenous Capital Outlays

Taking the capital outlays on each plot of land, \( k_i \), the price of corn, \( p \), and the economy-wide rate of profit, \( r \), as exogenously given, I would like to determine the magnitude of ground-rent on each plot of land and decompose it into \( DRI \), \( DRII \) and \( AR \).

### 4.1 Total Ground-Rent

Since the price of corn is given by \( p \), and the economy-wide rate of profit is denoted by \( r \), I can define what Marx calls the ‘surplus profit’ on the \( i \)-th plot of land as \( pf_i(k_i) - (1+r)k_i \), where \( pf_i(k_i) \) is the revenue actually earned and \( (1+r)k_i \) is the counterfactual revenue that would have been earned if the total capital outlay, \( k_i \), had earned the economy-wide average rate of profit. The key insight of Marx’ analysis of rent is that private ownership of land by the class of landlords - which he refers to as the monopoly of landed property - allows them to appropriate the surplus profit as ground-rent (Marx, 1993, Chapter 37).

Implementing the definition of ground-rent in my model, I see that its
magnitude on the $i$-th plot of land, \textit{measured in units of corn}, is given by

\[ GR_i = f_i(k_i) - \frac{(1 + r)k_i}{p} = \int_0^{k_i} f'_i(k)dk - \frac{(1 + r)k_i}{p}. \]  

(1)

Since $f_i(k_i)$ is the total output on the $i$-th plot with capital outlay, $k_i$, and $(1 + r)(k_i/p)$ is revenue that would have been earned, in real terms, on the same capital employed elsewhere in the economy (since the economy-wide average rate of profit is $r$), the surplus profit is given by the difference between the two. Using the fact that $f_i(k_i) = \int_0^{k_i} f'_i(k)dk$, which follows from assumption 1, and especially that $f_i(0) = 0$, we then get the expression in (1).

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vertical axis against the amount of capital outlay, \( k \), on the horizontal axis. The total amount of capital outlay on this plot of land is given by \( k_i \). Hence, total output of corn is given by the area under the marginal product curve, i.e. \( DC\!HO \), which is the first term in (1). The area \( GA\!HO \) represents the amount of corn that would be needed to ensure the economywide rate of profit, \( r \), on the total capital outlay, \( k_i \). This is the second term in (1). Hence, the total ground-rent on the \( i \)-th plot of land is represented by the area \( DC\!AG \).

In writing the expression for total ground-rent in (1) and in constructing the corresponding visual representation in figure 5, I have implicitly assumed that \( f_i'(k_i) > (1 + r)/p \) for \( i = 1, 2, \ldots, n \). This assumption means that on each plot of land that is in use, the marginal product of capital outlay, \( f_i'(k_i) \), is greater than the opportunity cost of investing capital elsewhere in the economy, \( (1 + r)/p \). This is a crucial assumption for the analysis of absolute rent and I will come back to it several times in the rest of the paper.

### 4.2 Marginal-Capital Plot

I would now like to decompose the magnitude of total rent given in (1) into two components: differential rent and absolute rent. To do so, I need to identify the ‘marginal’ unit of capital outlay, which is defined as the unit of capital outlay that is least productive, and then use this benchmark to define differential rent. Note that each plot of land has diminishing marginal productivity of capital outlay by Assumption 1. Hence, to identify the marginal capital, I just need to find the minimum of the marginal product on each plot of land at their \( \text{given} \) capital outlay levels. Recall that the amount of capital outlay on the \( i \)-th plot of land is given by \( k_i \). Hence, I can define the output associated with the marginal unit of capital as

\[
y^m = y^m(p) = \min_{i \in \{1, 2, \ldots, n(p)\}} f_i'(k_i),
\]

where \( y^m \) is the marginal output associated with the least productive unit of capital outlay used among all plots of land. Note that \( y^m \) is a function of \( p \), the price of the agricultural commodity, because it is the minimum of \( f_i'(k_i) \) where \( i \) ranges over \( \{1, 2, \ldots, n(p)\} \). Since \( n = n(p) \) is a function of \( p \), \( y^m \) is also a function of \( p \). To make this explicit, we have specified \( y^m = y^m(p) \).

Let us identify this plot of land with the index \( m \), where \( 1 \leq m \leq n(p) \), and call it the \textit{marginal-capital plot of land}. Thus, the \( m \)-th plot of land
has the lowest marginal product of capital outlay for the last unit of capital, i.e. \( y^m = f'_m(k_m) \), among all the plots of land that are currently under cultivation. Note that, in general, \( m \neq n(p) \), i.e. the worst plot of land in terms of quality that has been identified in assumption 2 is not, in general, the plot of land with the ‘marginal’ unit of capital outlay.

### 4.3 Differential and Absolute Rent

Now that I have identified the minimum marginal product of capital, I can decompose total rent as,

\[
GR_i = DR_i + AR_i,
\]

where the first component on the right hand side in (3) is total differential rent,

\[
DR_i (p) = \int_0^{k_i} \left[ f'_i(k) - y^m(p) \right] dk,
\]

and the second component on the right hand side in (3) is absolute rent,

\[
AR_i (p) = k_i \left[ y^m(p) - \frac{(1 + r)}{p} \right].
\]

An important point to keep in mind is that both \( DR_i \) and \( AR_i \) are functions of \( p \), the price of the agricultural commodity, and \( r \), the economy-wide rate of profit. This is easy to see from (3) and (5).

To build intuition about the definitions of differential and absolute, we can turn to Figure 6. Total ground rent is represented by area \( DCAG \). Differential rent is represented by the area \( DCBF \) (which is a representation of (4), and absolute rent is represented by the area \( FBAG \) (which is a representation of (5)).

The area \( DCBF \) is called differential rent because it represents the addition of all the extra output produced on any plot of land over and above the benchmark level, \( y^m \) (the minimum marginal product), i.e. sum of \( f'_i(k) - y^m \) over all units of capital. This sum depends on productivity differences across plots of land arising both from its intrinsic productivity and from differences in magnitudes of capital outlay - the two together determine the marginal product of capital curve for any plot of land.

I define the difference between the total ground-rent (the area \( DCAG \)) and the total differential rent (the area \( DCBF \)) as absolute rent. Algebraically, this is given by (5) and visually this is represented by the area.
Figure 6: Ground rent in agriculture on the i-th plot of land measured in units of corn. The horizontal axis measures total capital outlay; the vertical axis measures the marginal output as a function of capital outlay, $f'_i(k)$. The price of corn is $p$ and the economy-wide rate of profit is $r$. Total ground rent on the i-th plot of land is given by the area $DCAG$, differential rent by $DCBF$ and absolute rent by $FBAG$.

$FBAG$. I call this absolute rent because it is what remains after I have removed total differential rent from total ground-rent. By definition, it does not depend on differences of productivity across plots of land.

Let us think a little more about the definition of absolute rent. From (5), we see that the absolute rent is the product of two terms: (a) the level of capital outlay, $k_i$, and (b) the difference between $y^m$ and $(1 + r)/p$. We know that $y^m$ is the minimum marginal product of capital across all plots of land and $(1 + r)/p$ is what a unit of capital could earn, in real terms, if it were invested outside agriculture. Thus, for the existence of absolute rent, it is necessary that $y^m > (1 + r)/p$.

Since there is large disagreement in the extant literature about absolute rent, let me discuss it in some more detail. Marx had conceived of absolute...
rent as the total rent earned on a new plot of land that is brought under cultivation to satisfy rising demand for corn (Marx, 1993, Chapter 45). Implicit in Marx’s analysis is the idea that the new land brought under cultivation is also the least fertile plot of land, a fact that has been emphasized by Fine (1979). In the extant literature, there are two interpretations for the existence of absolute rent.

According to one interpretation, the source of absolute rent is the relatively lower organic composition of capital in agriculture compared to the rest of the economy. Fine (1979, pp. 263) has an algebraic expression to capture this idea. Following this claim, if the organic composition of capital rises in the rest of the economy even as the organic composition of capital in agriculture remains unchanged, there should be an increase in absolute rent, as Marx (1993) claimed and Fine (1979) emphasized. In my model, this effect is easy to capture. The relative rise in the organic composition of capital in the rest of the economy *ceteris paribus* leads to a fall in the uniform rate of profit, \( r \). From the expression in (5), we can see that this will lead to a rise in absolute rent if \( y^m \) and \( p \) does not change.

Another interpretation of Marx’s texts point out that the source of absolute rent is the ‘monopoly power’ of the class of landlords (Ramirez, 2009). The class of landlords exercise their monopoly of ownership of land by withholding land from the lease market unless they are assured absolute rent.\(^5\) The mechanism for this is straightforward. By withholding land from the lease market, landlords can reduce the supply of corn and thereby ensure that the price of corn rises. The rise in the price of corn can then allow them to bargain for higher ground-rent, the extra being understood as absolute rent. In my model, this is easy to capture. From the expression in (5), we can see that if the price of corn rises *ceteris paribus* then \( (1 + r)/p \) will fall, and this will lead to a rise in absolute rent.

I would like to point out at this point that the model I have proposed in this paper is more general than existing work on Marx’s rent theory. This is because my model can accommodate both interpretations for the existence of absolute rent that have been previously offered by Marxist scholars, the low organic composition explanation and the monopoly power explanation. What both interpretations seem to have missed is that the existence of absolute rent depends crucially on the *gap* between the minimum marginal product, \( y^m \), and the opportunity cost of capital outlay, \( (1+r)/p \). As long as \( y^m > (1+r)/p \),

\(^5\)I will explore another channel for the exercise of landlord power in the next section.
there will be positive absolute rent; if \( y^m = (1+r)/p \) there will be no absolute rent, no matter if agriculture has a lower organic composition of capital or if the class of landlords have monopoly power. This is precisely where the implicit assumption that \( f'_i(k_i) > (1+r)/p \) becomes important. Since \( f'_i(k_i) > (1+r)/p \) for \( i = 1, 2, \ldots, n \), this implies that the minimum of \( f'_i(k_i) \), i.e. \( y^m \), is also larger than \( (1+r)/p \). Hence, \( y^m > (1+r)/p \). This ensures that there is positive absolute rent on each plot of land.

4.4 Differential Rent I and II

The second step of the decomposition is to break up total differential rent in (4) into differential rent of the first variety, \( DRI \), and differential rent of the second variety, \( DRII \). To do so, I will use the least productive plot, indexed by \( n \) according to assumption 2, as the benchmark to define \( DRI_i \). To implement this decomposition let us define a level of capital outlay on the \( i \)-th plot of land, \( k^*_i(p) \), such that the marginal product of capital outlay on the \( i \)-th plot of land at this level of capital outlay is exactly equal to the marginal product of capital on the least productive plot at zero capital outlay, \( f'_n(p)(0) \), i.e.

\[
f'_i(k^*_i) = f'_n(p)(0).
\]

(6)

Assumption 1 and 2 guarantees that a positive value of \( k^*_i(p) \) always exists (as long as all plots in use have positive capital outlays). With this definition of \( k^*_i(p) \), I can define differential rent of the first variety as,

\[
DRI_i(p) = \int_0^{k^*_i(p)} f'_i(k)dk - k^*_i(p) f'_n(p)(0)
= \int_0^{k^*_i(p)} [f'_i(k) - f'_n(p)(0)] dk.
\]

(7)

Note that assumption 2 ensures that \( DRI_i(p) \) is positive. If one weakens assumption 2 and uses weak inequalities, then one will be ensured a nonnegative \( DRI_i(p) \). Thus, we will always have \( DRI_i(p) \geq 0 \).

What is the intuition for the definition of \( DRI_i(p) \)? Using the worst-quality plot of land as the benchmark, the definition is identifying all units of capital outlay on any plot of land that have higher marginal product, i.e. productivity, than the ‘intrinsic’ quality of the worst-quality plot (marginal product of capital of the least productive plot of land at zero capital outlay). When we add up all the additional output produced by the units of capital
so identified, we get differential rent of the first variety, \( DRI_i(p) \), as defined in (7). Thus, this definition captures Marx’s intuition that differential rent of the first variety arises due to differences in the quality (or fertility) of plots of land - the extensive margin of David Ricardo - with respect to a benchmark plot of land (the least productive plot). Using Figure 7, this definition would translate into identifying differential rent of first variety, \( DRI_i(p) \), as the area \( DIJ \).

I can now define the second component of differential rent as the difference of total differential rent and differential rent of the first variety, i.e.

\[
DRII_i(p) = \int_0^{k_i} [f_i'(k) - y^m(p)] \, dk - DRI_i(p).
\]  

(8)

Using Figure 7, differential rent of second variety would be identified with the area \( IJCBF \), which is the difference in total differential rent, \( DCBF \), and differential rent of the first variety, \( DIJ \). Since \( y^m(p) \leq f_i'(k_i) < f_n'(p)(0) \), where the last (strict) inequality is true if \( k_i > 0 \) for all plots, we can see that the area \( IJCBF \) will always be nonnegative. Hence, \( DRII_i(p) \geq 0 \).

What is the intuition for \( DRII_i(p) \)? We have seen earlier that total differential rent \( DR_i(p) \), defined in (4), arises from a combination of productivity differences that come from differences in quality of plots of land and differences in the magnitude of capital outlay. We have also seen that \( DRI_i(p) \), defined in (7), arises from differences in quality of a plot with respect to the benchmark worst-quality plot of land. When we remove \( DRI_i(p) \) from \( DR_i(p) \), we are left with the component of differential rent that arises due to differences in magnitude of capital outlay. Hence, \( DRII_i(p) \), as defined in (8), captures Marx’s intuition that differential rent of the second variety arises from differences in the magnitude of capital outlay - the intensive margin of David Ricardo.

I can now bring together the above discussion to see that Marx’s ideas about ground-rent, including his claim about its decomposition into three parts, can be established. Defining total ground-rent as the transformation of surplus profit, I have shown that it can be decomposed on any plot of land into differential rent of first variety, differential rent of second variety, and absolute rent, i.e.

\[
GR_i(p) = DRI_i(p) + DRII_i(p) + AR_i(p),
\]

where \( GR_i(p) \) is defined in (1), \( DRI_i(p) \) is defined in (7), \( DRII_i(p) \) is defined in (8), and \( AR_i(p) \) is defined in (5). It is important to note that not only
the total ground-rent, but each component of ground-rent as well, depends on the price of the agricultural commodity, p.

4.5 Two Special Plots of Land

The analysis of rent presented above uses two special plots of land as reference plots, the \( m \)-th plot (the worst capital plot), and the \( n \)-th plot of land (the worst quality plot of land, according to the convention captured in Assumption 2). What can we say about the components of rent on these two reference plots of land?

On the worst capital plot, \( y^m = f'_m(k_m) \), i.e. \( y^m \) is the marginal product on the worst capital plot. The decomposition of total rent on the worst capital
plot is depicted in Figure 8. There is no qualitative difference between the \( m \)-th plot and any other plot of land. Much like on any other plot, total rent on the \( m \)-th plot is also the sum of \( DRI, DRII \) and \( AR \). A more interesting case is presented by the worst quality plot, i.e. the plot indexed with \( n = n(p) \).

On the worst quality plot, which is depicted in Figure 9, the graph of the marginal product curve starts at \( f'_n(0) \). From (6), we see that for the \( n \)-th plot, \( k^*_n = 0 \). This implies, using the expression in (7), that \( DRI_n = 0 \). By the concavity of the pseudo-production function, we have \( f''(,) < 0 \). Hence \( DRII_n > 0 \), which is given by the area \( DCBF \) in Figure 9. We also know that \( y^m > (1 + r)/p \), which implies that \( AR_n > 0 \) (depicted as the area \( FBAG \) in Figure 9). Thus, on the worst quality plot, total rent is composed of \( DRII \) and \( AR \); there is no \( DRI \). This is an important conclusion and is worth commenting on. Marx thought that total rent on the worst quality plot of land could be identified with absolute rent, i.e. there would be no differential rent on the worst quality plot (Marx, 1993, Chapter 45). My analysis shows that that is not correct. On the worst quality plot of land, total rent is composed of both \( AR \) and \( DRII \). Since \( DRII \) arises from the concavity of the pseudo-production function, i.e. the diminishing marginal product of capital outlay, Marx’s conclusion seems to derive from his ignoring this latter factor when analyzing rent on the worst quality plot of land.

There is a deeper problem in the analysis presented so far. I have completely ignored the decision making process of the capitalist farmers. In more concrete terms, by assuming a given amount of capital outlay on each plot of land, I have ignored the process and implications of the behaviour of capitalist farmers. This is a serious shortcoming because, once we allow a reasonable behaviour of capitalist farmers, there will be a serious implication for the analysis of rent. But before I turn to that, let me use an example to illustrate my model.

5 Ground-Rent With Endogenous Capital Outlays

Thus far, the analysis of ground-rent has treated the amount of capital outlays on each plot of land as exogenously given. I would now like to endogenize capital outlay by asking: how much capital would be advanced on any plot of land? I provide a simple answer. On each plot of land, capitalist produc-
ers will choose the amount of capital outlay that will maximize the surplus profit they can earn vis-a-vis what they can earn if they were to employ their capital elsewhere in the economy. The competitive process in capitalist economies will force capitalist farmers to choose this level of capital outlay. Let us understand why and then see what this implies for absolute rent.

5.1 There is No Absolute Rent

Since the economy-wide rate of profit is exogenously given to be $r$, capitalist producers can always earn the total revenue of $(1 + r)k_i$ by employing their capital, $k_i$, elsewhere in the economy. If, on the other hand, they choose to employ the capital in agricultural production on the $i$-th plot of land, then they can expect to earn total revenue of $pf_i(k_i)$, if the price of the agricultural commodity is $p$. If total rent is a lump-sum monetary payment given by $GR_i$, then the amount of revenue they can expect to earn by employing their capital in agriculture is $pf_i(k_i) - GR_i$. Hence, the extra revenue a capitalist farmer can earn by investing her capital in agriculture is given by

\[ f'_m(k/m) \]

**Figure 8:** Ground rent in agriculture on the $m$-th plot (i.e. worst capital plot) of land measured in units of corn.
Figure 9: Ground rent in agriculture on the n-th plot (i.e. worst quality plot) of land measured in units of corn. Note that DRI is zero.

\[ pf_i(k_i) - GR_i - (1 + r)k_i. \] I posit that capitalist producers choose the level of capital outlay, \( k_i \), on the \( i \)-th plot to maximize this surplus, i.e. her choice problem on the \( i \)-th plot of land can be represented as follows:

\[
\max_{k_i} pf_i(k_i) - GR_i - (1 + r)k_i.
\]

The first order condition of this maximisation problem gives us the optimal choice of capital outlay as \( k_i^* \), where,

\[
f_i'(k_i^*) = \frac{1 + r}{p}. \tag{9}
\]

This condition means that on the \( i \)-th plot of land, the amount of capital outlay that will be chosen by profit-maximising capitalist farmers will be such that the marginal product of capital outlay will be equalized to \((1 + r)/p\). If she were to choose a level of capital outlay, \( k_i \), that is lower than \( k_i^* \), then the marginal product of capital would be higher than \((1 + r)/p\). By increasing capital outlay by a small magnitude, \( \Delta k_i \), the capitalist farmer would be able
to increase her profit income because

\[ pf'(k_i)\Delta k_i > (1 + r)\Delta k_i. \]

Hence, a level of capital outlay that is lower than \( k_i^* \) would not be chosen by a capitalist farmer. On the other hand, if she chose a level of capital outlay, \( k_i \), that was higher than \( k_i^* \), then the marginal product of capital would be lower than \((1 + r)/p\). Thus, the capitalist farmer could increase her surplus by moving a small amount of her capital, \( \Delta k_i \), out of agriculture because

\[ pf'(k_i)\Delta k_i < (1 + r)\Delta k_i. \]

Hence, a level of capital outlay that is higher than \( k_i^* \) would not be chosen by a capitalist farmer. This implies that, given competitive pressures operating in capitalist economies, a capitalist farmer would choose her level of capital outlay which would ensure that the marginal product of capital was exactly equal to \((1 + r)/p\).

Note that the same economy-wide average rate of profit, \( r \), and the same market price of corn, \( p \), are faced by capitalist farmers on each plot of land. Since competitive pressures force capitalist farmers to behave in the same way on each plot of land, the marginal product of capital will be equalized at \((1 + r)/p\) on each plot of land.

The condition represented in (9) has an important implication for the decomposition of ground-rent. Turning to the previous definition of the marginal-capital plot of land in (2), we can see that once we allow capitalist farmers to determine the level of capital outlays using the principle of surplus maximization, all plots become marginal-capital plots of land. This is because \( f'(k_i) \) is equal for each plot. Hence, \( y^m = f'_i(k_i) = (1 + r)/p \). Using (9), we see further that \( y^m = f'_i(k_i) = (1 + r)/p \). From the expression in (5) it follows that absolute rent, \( AR_i \), is zero. In terms of Figure 6, this means that \( OF = OE = OG \), so that \( AR_i = 0 \).

As I have noted above, in the extant literature, there are two interpretations of Marx’s arguments about the existence of absolute rent. One strand of Marxist literature argues, in line with Marx’s argument in Chapter 45 of Volume III of *Capital*, that the source of absolute rent is the low organic composition of capital in agriculture relative to the rest of the economy (Fine, 1979; Fine and Filho, 2010; Fine, 2019). Another strand, also using textual evidence in its favour, argues that absolute rent arises from the monopoly of the class of landlords (Ramirez, 2009). The analysis in this section shows
that both interpretations are incorrect once we allow capitalist farmers to choose the level of capital outlay on their plots of land.

On the one hand, if the organic composition of capital were to rise in the rest of the economy relative to agriculture, as used in the argument in Fine (1979), then the uniform rate of profit, $r$, would fall to $r'$, say. Capitalist farmers would then choose the new level of capital outlay to take this into account. Their choice would ensure that the marginal product of capital, $f'_i(k_i)$, is equal to $(1 + r')/p$. Hence, absolute rent would still be zero. The relatively lower organic composition of capital in agriculture does not generate any absolute rent. On the other hand, if the class of landlords use their monopoly power to withhold land from the lease market, this will raise the price of corn from $p$ to $p'$, say. Capitalist farmers would then choose the level of capital outlay after taking this into account. Their choice will ensure
that \( f_i'(k_i) \) is equal to \((1 + r)/p'\). Thus, absolute rent would still be zero. Monopoly power of landlords cannot give rise to absolute rent, either.

I have visually represented the configuration of ground-rent and its decomposition, when capital outlay is chosen by capitalist farmers to maximize their surplus, in Figure 10. The total amount of ground-rent on plot \( i \) is represented by the area \( DCE \). This is, as before, the surplus profit. \( DRI_i(p) \) is represented, just like in Figure 5, by the area \( DJI \). This is the part of ground-rent that can be attributed to quality differentials across plots of land. \( DRII_i(p) \) is now represented by the area \( IJCE \). There is no \( AR \). Thus, we now have a two-part decomposition of ground-rent on the \( i \)-th plot as

\[
GR_i(p) = DRI_i(p) + DRII_i(p),
\]

where

\[
DRI_i(p) = \int_{k^*_i(p)}^{k^*_i} \left[f_i'(k) - f_n'(0)\right] dk,
\]  

(10)

and

\[
DRII_i(p) = \int_{k_i}^{k^*_i} \left[f_i'(k) - \frac{1 + r}{p}\right] dk - DRI_i(p).
\]  

(11)

Let us now determine the components of ground-rent on the two plots. On both plots of land, we know that the marginal product of capital is exactly equal to \((1 + r)/p\). Hence, \( y^m \), the minimum marginal product of capital at the given levels of capital outlay on the two plots is exactly equal to \((1 + r)/p\). Using (5), we see, therefore, that the absolute rent is equal to zero on both plots of land.

5.2 Can Absolute Rent be Positive?

Let me now return to the condition that I had implicitly assumed when analysing rent with exogenous capital outlays: \( f_i'(k_i) > (1 + r)/p \). This assumption was crucial for generating positive amount of absolute rent on the \( i \)-th plot of land (see equation 5).\(^6\) The importance of this assumption can be visually seen from Figure 6 and 7. This assumption implies that there is a ’gap’ between the marginal product of capital outlay, \( f_i'(k_i) \), and the opportunity cost of capital outlay, \((1 + r)/p\). Thus, only when there is a gap between \( f_i'(k_i) \) and \((1 + r)/p\), can the \( i \)-th plot generate positive \( AR \). If

\(^6\)I had assumed that \( f_i'(k_i) > (1 + r)/p \) for \( i = 1, 2, \ldots, n \). This ensured that \( y^m \), which is the minimum \( f_i'(k_i) \) across the \( n \) plots, was also larger than \((1 + r)/p\).
there are mechanisms available in a capitalist economy that can put a wedge between the marginal product of capital outlay and the opportunity cost of capital outlay, then it can generate positive absolute rent.

The two main contenders for explaining absolute rent are the relatively low organic composition of capital and the class power of landlords. In a previous sub-section, I have argued that a relatively low organic composition of capital cannot generate any wedge between the marginal product of capital outlay and the opportunity cost of capital outlay. Hence, a relatively low organic composition of capital cannot be the source of absolute rent. I have also argued that if we understand the exercise of the monopoly power of landlords as consisting in their ability to withhold land from the lease market, even then we will not have any absolute rent. Now, I would like to investigate another channel for the exercise of the class power of landlords: influencing taxation of capital by the State.

Suppose the state taxes capital outlays at the rate of $t > 0$. Then, the capitalist farmers choice problem would become

$$\max_{k_i} pf_i(k_i) - GR_i - tk_i - (1 + r)k_i,$$

so that the optimal choice of capital outlay would satisfy the condition that

$$f'_i(k^*_i) = \frac{1 + r + t}{p}.$$ (12)

Since $t > 0$, this would ensure that $f'_i(k^*_i) > (1 + r)/p$. Hence, using (5), we can see that there is positive absolute rent. Moreover, we can also see that the absolute rent is just be the tax revenue collected by the state: $AR_i = tk^*_i/p$. If the class of landlords were strong enough to ensure that the state redistributes the tax revenue, $tk^*_i/p$, to them, then we could consider it to be absolute rent. Thus, there seems to be some justification for the claim that class power of landlords can generate absolute rent. But this is only apparently correct.

To see the problem with the class power argument, let $k^*_{i,T}$ and $k^*_{i,NT}$ denote the optimal capital outlays on the $i$-th plot of land with and without taxation of capital outlay, respectively. The first order condition in (9) determines $k^*_{i,NT}$ as, $f'_i(k^*_{i,NT}) = (1 + r)/p$; and the first order condition in (12) determines $k^*_{i,T}$ as $f'_i(k^*_{i,T}) = (1 + r + t)/p$. The concavity of $f_i(.)$ and the fact that $t > 0$ implies that $k^*_{i,NT} > k^*_{i,T}$. Thus, when there is taxation of capital outlays, it will lead to a lower level of capital outlay. This would, in turn, lead to a loss of ground-rent for the class of landlords. This loss is
represented by the area $JCF$ in Figure 11. Therefore, it seems likely that if the class of landlords were strong enough to impact public policy, they would in fact push for removing the taxation of capital outlays, rather than ensuring the redistribution of the tax revenue towards themselves (and call it absolute rent). We already know, of course, that if there is no taxation of capital outlays, then absolute rent would be zero.

In the mid-19th century, when Marx turned his attention to the problem of absolute rent, feudal obligations were still enforced in many parts of Germany. One could perhaps think of these feudal obligations as taxes on capital, which would then explain why Marx theorised about absolute rent. It might also be argued that once landlords became capitalist revenue maximizers, they realized that the feudal dues were in fact reducing the total ground-rent by creating a wedge between the marginal rate of profit in industry and agriculture. This might have been one of the important factors that led to the outlawing of feudal obligations.\footnote{I would like to thank Duncan Foley for pointing this out.} In contemporary capitalist societies, where capitalists are the dominant element in the ruling class, there do not seem to be any ground for the existence of absolute rent.

6 Price of Corn

So far I have taken the price of the agricultural commodity is given. To complete the analysis, I need to investigate how this price is determined. To do so I look at the market for corn and consider factors that affect demand and supply. Suppose the demand function for corn is given by $D(p; \gamma)$, where $\partial D/\partial p < 0$, $p$ is the price of corn, and $\gamma$ captures factors like population growth, urbanization, regulatory aspects of the corn market, etc. that shift the whole demand curve. The total supply of corn can be expressed as

$$S(p) = \sum_{i=1}^{n(p)} f_i(k_i(p)).$$

(13)

Note that total supply of corn is an upward sloping function of price for two reasons. First, the actual number of plots in use for agricultural production is a function of $p$, i.e. $n = n(p)$. I had commented on this assumption right at the outset, and it is now clear why this was important. The dependence of the number of plots of land in use is a function of the price of corn because as
the price of corn rises, it makes worse plots of land profitable to bring under cultivation. Second, on any plot in use, we know that $k_i$ is an (increasing) function of $p$, so that output is an (increasing) function of price, i.e. $f_i(k_i(p))$. Hence, total supply, $S(p)$ is an (increasing) function of $p$.

The equilibrium price of corn is the level of $p = p^*$ which brings supply and demand into balance, i.e.

$$D(p^*; \gamma) = S(p^*) = \sum_{i=1}^{n(p^*)} f_i(k_i(p^*)).$$

(14)

To ensure that my model is properly closed, let me conduct a simple counting argument. On the one hand, when there is no taxation of capital outlays, the model is captured by (12) and (14). There are $n + 1$ equations because (9) gives us $n$ equations and (14) gives us one more. The model has $n + 1$ endogenous variables, $k_1, \ldots, k_n, p^*$, and one exogenous variables, $r$. Hence, the equation system can be solved to arrive at the equilibrium magnitudes of all the endogenous variables. On the other hand, when the state taxes capital
outlays, the model is captured by (12) and (14). There as \( n + 1 \) equations and \( n + 2 \) endogenous variables, \( k_1, \ldots, k_n, t, p^* \). Once we choose a value of \( t = t^* \), the model is closed and we can solve for all the endogenous variables.

I can now present the condition under which any plot of land will be in use in a capitalist economy with landed property, i.e. private ownership of land by the class of landlords. In such a context, any plot of land will be in use if it can be leased in profitably by a capitalist farmer. It can be leased in profitably only when it pays a positive amount of ground-rent to the landlord in addition to ensuring the uniform rate of profit for capital investment. Let us consider the most extreme case when the demand for corn is so low that even the ‘best’ plot of land cannot be profitably used. In this situation, there will be no corn production and hence there will be zero ground-rent. This immediately identifies the minimum threshold for the price, if rent is to be positive, as

\[
p^{**} = 1 + \frac{r}{f_1'(0)},
\]

where, it is to be recalled from Assumption 2 that plot 1 is the best quality plot. If population growth, urbanization, etc. leads to growth of demand for the agricultural product such that \( p > p^{**} \), this will bring land under cultivation. As soon as any land is brought under cultivation, this will generate positive ground-rent. As demand rises further, progressively more (worse plots) land will come under cultivation and total ground-rent will increase.

7 Conclusion

In this paper I have offered a simple mathematical framework to conceptualize Marx’s ideas about ground-rent. In developing the analysis of this paper, I have extended the discussion in two recent, noteworthy attempts to clarify Marx’s theory of ground-rent, Basu (2018, 2021) and Das (2018). Previous attempts to discuss Marx’s theory of ground-rent has been marked by disagreements and confusions partly because of the difficulty in clearly defining the meaning of the terms involved in the discussion, especially the components of ground-rent (Fine, 1979; Ramirez, 2009; Fine and Filho, 2010). In this respect, Basu (2018) and Das (2018) advanced the literature by trying to clearly specify Marx’s ideas, avoid unnecessary exegetical debate, and formalize the ideas in simple mathematical models.

In this paper, I have extended the analysis further by endogenising the
capital outlay on each plot of land using a simple and intuitive profit maximising principle to model the behaviour of capitalist farmers. This simple extension has far reaching conclusions. My analysis shows that total ground-rent can be decomposed into differential rent of the first variety, differential rent of the second variety and absolute rent only when capital outlays are taken to be exogenously given. As soon as we endogenize capital outlays using the principle of profit maximisation, the decomposition of ground-rent changes. While we always have differential rent of the first and second varieties, the existence of absolute rent now depends on whether the marginal product of capital outlay across all plots of land is greater than the opportunity cost of investing capital outside agriculture. In general, competitive pressure will force capitalist farmers to be profit maximizers. When capitalist farmers base their decision about how much to invest on a plot of land so as to maximize their surplus, they will choose a level of capital outlay that equates the marginal product of capital outlay and the opportunity cost of investing capital in agriculture. Thus, absolute will be zero. I have argued in this paper that neither a relatively low organic composition of capital in agriculture nor the class power of landlords can, in general, ensure positive absolute rent.

References


