Unit 2– Introduction to Probability
Week #3 - Practice Problems

SOLUTIONS

1. Let A and B denote two independent genetic traits. Suppose the probability that an individual will exhibit trait A is ½ and the probability that an individual will exhibit trait B is ¾. What is the probability that an individual will exhibit

(a) Both traits?

Answer: .375

Pr[both traits] = Pr[A]Pr[B]=[.50][.75]=.375

(b) Neither trait?

Answer: .125

Pr[neither trait] = Pr[not A]Pr[not B]=[.50][.25]=.125

(c) trait A but not trait B?

Answer: .125

Pr[A and not B] = Pr[A]Pr[not B]=[.50][.25]=.125

(d) trait B but not trait A?

Answer: .375

Pr[not A and B] = Pr[not A]Pr[B]=[.50][.75]=.375

(e) exactly one trait?

Answer: .50

We sum the probabilities of the two mutually exclusive ways that yield "exactly one"

Pr[exactly one] = Pr[(A, not B) or (not A, b)]
=Pr[A, not B]+Pr[not A, B]
=[(.50)(.25)]+[(.50)(.75)] =.125+.375
=.50

2. Suppose you are told that pr (right eye is blue) = 1/3 and pr(left eye is blue) = 1/3. Using the concepts and formulae in the lecture notes for Unit 2 (Introduction to Probability), confirm for yourself what you know by intuition, namely that pr(person is blue eyed = 1/3 by solving for pr(blue right eye and blue left eye).

   Under the assumption that left and right eye colors are always the same,
   Pr[left is blue AND right is blue] is the same as Pr[left is blue] = 1/3

   1/3=Pr[left is blue and right is blue]
   =Pr[right is blue]Pr[left is blue|right is blue]
   =Pr[right is blue]{1}
   =Pr[right is blue] √

3. A physician develops a diagnostic test that is positive for 95% of the patients who have disease and is positive for 10% of the patients who do not have disease. Of patients tested, 20% actually have disease. Suppose you evaluate a patient by administering this diagnostic test and obtain a positive result. Using the information given, calculate the probability that this patient has disease.

   Answer: .7037
   Solution:

We want to calculate Probability (Disease | + test)

- **Probability (+ test | disease) = .95**
- **Probability (+ test | no disease = .10**

Probability (Disease) = .20
Probability (not Disease) = .80

$$\Pr(\text{ disease } | +) = \frac{\Pr(\text{disease and } +)}{\Pr(+)}$$ by definition of conditional Probability

$$= \frac{\Pr(+ | \text{disease}) \Pr(\text{disease})}{\Pr(+)}$$ because we can re-write the numerator this way

$$= \frac{\Pr(+ | \text{disease}) \Pr(\text{disease})}{\Pr(+ | \text{disease}) \Pr(\text{disease}) + \Pr(+ | \text{no disease}) \Pr(\text{no disease})}$$

$$= \frac{(.95)(.20)}{(.95)(.20) + (.10)(.80)} = .7037$$