The Sample Variance S^2 is an Unbiased estimator of σ^2

$$E\left[S^2\right] = E\left[\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{(n\text{-}1)}\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right] E\left[\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right] E\left[\sum_{i=1}^{n}\left(X_i - \mu + \mu \text{-} \bar{X}\right)^2\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right] E\left[\sum_{i=1}^{n}\left([X_i - \mu] \ - \ [\bar{X} \text{-} \mu]\ \right)^2\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right] E\left[\sum_{i=1}^{n}\left([X_i - \mu]^2 \ - \ 2[X_i \text{-} \mu][\bar{X} \text{-} \mu] + [\bar{X} \text{-} \mu]^2\ \right)\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right] E\left[\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 - 2\left(\bar{X} \text{-} \mu\right)\sum_{i=1}^{n}\left(X_i - \mu\right) \ + \ \left(\bar{X} \text{-} \mu\right)^2 \sum_{i=1}^{n}(1)\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right] E\ \left[\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 \ \text{-}\ 2n\left(\bar{X}_n - \mu\right)\left(\bar{X}_n - \mu\right) + n\left(\bar{X} \text{-} \mu\right)^2\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right]\left[\sum_{i=1}^{n} E\left(X_i - \bar{X}\right)^2 - 2n\, E\left(\bar{X} \text{-} \mu\right)^2 \ + n\, E\left(\bar{X} \text{-} \mu\right)^2\right]$$

$$= \left[\frac{1}{(n\text{-}1)}\right]\left[n\sigma^2 \ \text{-}\ (2n)\left(\frac{\sigma^2}{n}\right) \ + \ (n)\left(\frac{\sigma^2}{n}\right)\right]$$

$$= \left[\frac{1}{(n-1)} \right] \left[n\sigma^2 \; - \; 2\sigma^2 \; + \; \sigma^2 \right]$$

$$= \left[\frac{1}{(n-1)} \right] \left[n\sigma^2 \; - \; \sigma^2 \right]$$

$$= \left[\frac{1}{(n-1)} \right] \left[(n-1)\sigma^2 \right]$$

$$= \sigma^2$$