Unit 3 – Probability Basics
Solutions

Question 1.

Let A and B denote two independent genetic traits. Suppose the probability that an individual will exhibit trait A is ½ and the probability that an individual will exhibit trait B is ¾. What is the probability that an individual will exhibit

(a) Both traits?
Answer: .375
Because "A" and "B" are independent,
Pr[both traits] = Pr[A]Pr[B]=[.50][.75]=.375

(b) Neither trait?
Answer: .125
"Neither trait" happens only if both "not A" occurs and "not B" occurs. By independence,
Pr[neither trait] = Pr[not A] Pr[not B] = [.50] [.25] = .125

(c) trait A but not trait B?
Answer: .125
Pr[A and not B] = Pr[A] Pr[not B] = [.50] [.25] = .125

(d) trait B but not trait A?
Answer: .375
Pr[not A and B] = Pr[not A] Pr[B] = [.50] [.75] = .375

(e) exactly one trait?
Answer: .50
There are two scenarios that yield "exactly one" trait: 1st = A and NOT B, 2nd = NOT A and B. Since the occurrence of each scenario means that it is impossible for the other scenario to occur, the two scenarios are mutually exclusive. So their probabilities ADD. Then, within each scenario, because "A" and "B" are independent,

Pr[exactly one] = Pr[(A, not B) or (not A, B)]
= Pr[A, not B] + Pr[not A, B]
= [(.50)(.25)] + [(.50)(.75)] = .125 + .375
= .50

Question 2.

Suppose you are told that pr (right eye is blue) = 1/3 and pr(left eye is blue) = 1/3. Now suppose I tell you that you may assume that a person's two eyes are ALWAYS the same color. Confirm what you know by intuition, namely that pr(person is blue eyed) = 1/3. To do this, you need to solve pr(blue right eye and blue left eye) = ??

So as not to have to write out long hand expressions, let's say:

"A" is event that the left eye is blue and
"B" is the event that the right eye is blue

If we want to solve for the probability that a person is blue eyed, the required calculation is:

Probability [person is blue eyed] = Probability [A and B]

Now what?
To get you on your way, take a look again at page 28 of the unit 3 notes.
There you'll find the definition of <u>conditional probability, Pr [B | A]</u>.
Recall that Probability [B | A] = Probability [B occurs given that we know that A has occurred].
And we saw on page 28 that the solution for a conditional probability says to limit your look to where "A" has occurred.

$$Probability\ [B \mid A] = \frac{Probability[\ B\ and\ A]}{Probability\ [A]} \rightarrow$$

Happily, we can use this definition, together with a little bit of moving things around, to get what we need. In particular, if we multiply both sides by Probability [A], we get the following:

Probability [B and A] = Probability [A] x Probability [B | A]

Thus,

Probability [Person is blue eyed] = Probability [B and A]
= Probability [A] x Probability [B | A]
= 1/3 x 1
= 1/3 hooray ….

Because:

Probability [A] = Probability [Left eye is blue] = 1/3
Probability [B | A] = Probability [Right eye is blue GIVEN that left eye is blue] = 1

Question 3.

A physician develops a diagnostic test that is positive for 95% of the patients who have disease and is positive for 10% of the patients who do not have disease. Of patients tested, 20% actually have disease. Suppose you evaluate a patient by administering this diagnostic test and obtain a positive result. Using the information given, calculate the probability that this patient has disease.

Answer: .7037

The solution is a Bayes rule calculation. So be sure to have familiarized yourself with this rule before studying this example!

We want to calculate Probability (Disease | + test)

- Probability (+ test | disease) = .95 Probability (Disease) = .20
- Probability (+ test | no disease = .10 Probability (not Disease) = .80

$$\Pr(\text{ disease} \mid +) = \frac{\Pr(\text{disease and } +)}{\Pr(+)}$$ by definition of conditional probability

$$= \frac{\Pr(+ \mid \text{disease}) \Pr(\text{disease})}{\Pr(+)}$$ because we can re-write the numerator this way

$$= \frac{\Pr(+ \mid \text{disease}) \Pr(\text{disease})}{\Pr(+ \mid \text{disease}) \Pr(\text{disease}) + \Pr(+ \mid \text{no disease}) \Pr(\text{no disease})}$$

$$= \frac{(.95)(.20)}{(.95)(.20) + (.10)(.80)} = .7037$$