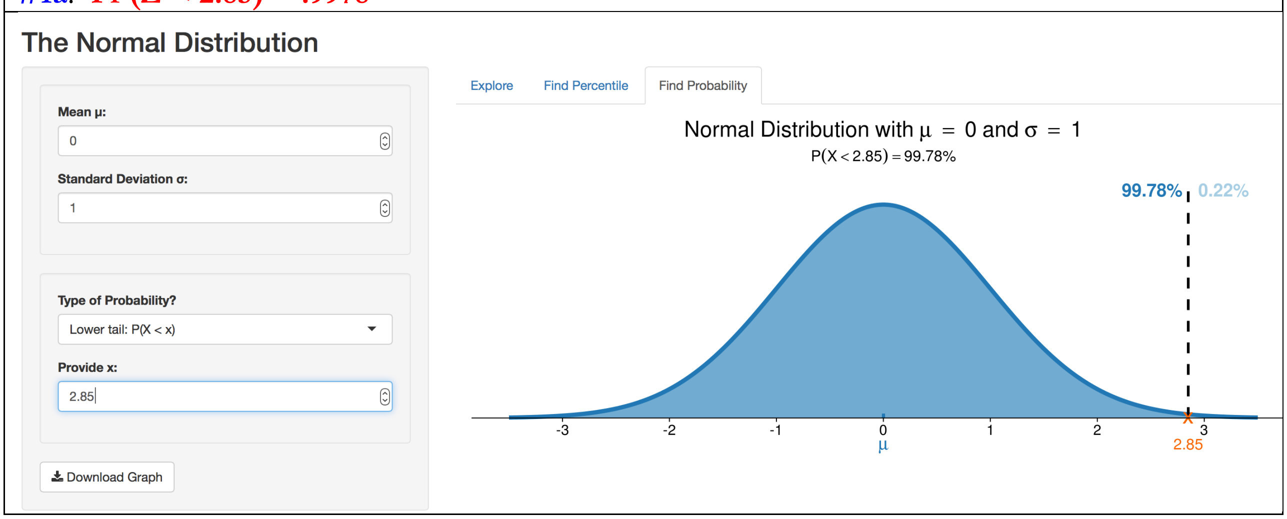
Unit 7 – The Normal Distribution
Homework
Solutions

#1. Find the proportion of observations from a standard normal distribution that satisfies each of the following statements.

a. Z < 2.85	**Pr (Z < 2.85) = .9978**
b. Z > 2.85	**Pr (Z > 2.85) = .0022**
c. Z > -1.66	**Pr (Z > -1.66) = .9515**
d. -1.66 < Z < 2.85	**Pr (-1.66 < Z < 2.85) = .9494**
e. Z < -2.25	**Pr (Z < -2.25) = .0122**
f. Z > -2.25	**Pr (Z > -2.25) = .9878**
g. Z > 1.77	**Pr (Z > 1.77) = .0384**
h. -2.25 < Z < 1.77	**Pr (-2.25 < Z < 1.77) = .9494**

Art of Stat Solution for #1a ONLY (solutions for #1b-#1h are similar)

#1a. Pr (Z < 2.85) = .9978



R Solution

```
# 1a)  Pr[Normal(mean=0, sd=1) <= 2.85]
pnorm(2.85)
## [1] 0.997814

# 1b)  Pr[Normal(mean=0, sd=1) > 2.85]
pnorm(2.85, lower.tail=FALSE)
## [1] 0.002185961

# 1c)  Pr[Normal(mean=0, sd=1) > -1.66]
pnorm(-1.66, lower.tail=FALSE)
## [1] 0.9515428

# 1d)  Pr[-1.66 <= Normal(mean=0, sd=1) <= 2.85]
pnorm(2.85) - pnorm(-1.66)
## [1] 0.9493568

# 1e)  Pr[Normal(mean=0, sd=1) <= -2.25]
pnorm(-2.25)
## [1] 0.01222447

# 1f)  Pr[Normal(mean=0, sd=1) > -2.25]
1 - pnorm(-2.25)
## [1] 0.9877755

# 1g)  Pr[Normal(mean=0, sd=1) > 1.77]
pnorm(1.77, lower.tail=FALSE)
## [1] 0.03836357

# 1h)  Pr[-2.25 <= Normal(mean=0, sd=1) <= 1.77]
pnorm(1.77) - pnorm(-2.25)
## [1] 0.949412
```

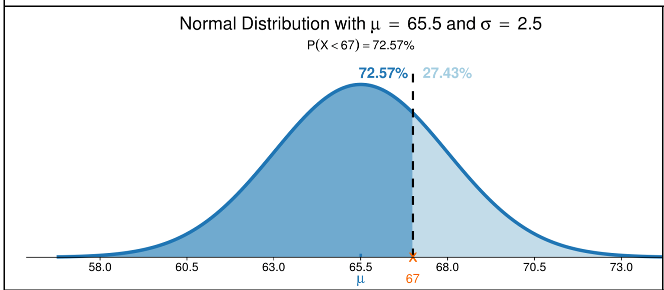
#2. The height, X, of young American women is distributed normal with mean μ=65.5 and standard deviation σ=2.5 inches. Find the probability of each of the following events

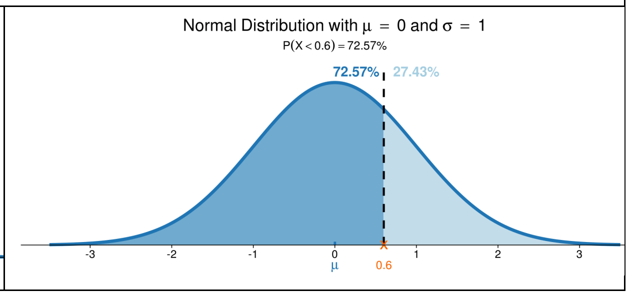
a. X < 67

Art of Stat Solution

#2a. .7257

$$\text{pr}(X<67) = \text{pr}\left[\left(\frac{X-\mu}{\sigma}\right) < \left(\frac{67-\mu}{\sigma}\right)\right]$$
$$= \text{pr}\left[Z < \left(\frac{67-65.5}{2.5}\right)\right]$$
$$= \text{pr}\left[Z < .6\right]$$
$$= .7257$$





R Solution

```
# Pr[Normal(mean=65.5, sd=2.5) < 67]
pnorm(67, mean=65.5, sd=2.5)
## [1] 0.7257469
```

b. 64 < X < 67

Art of Stat Solution

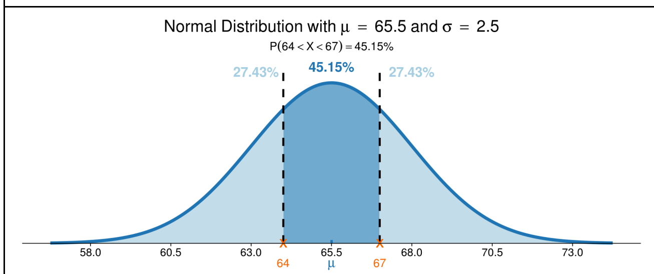
#2b. .4515

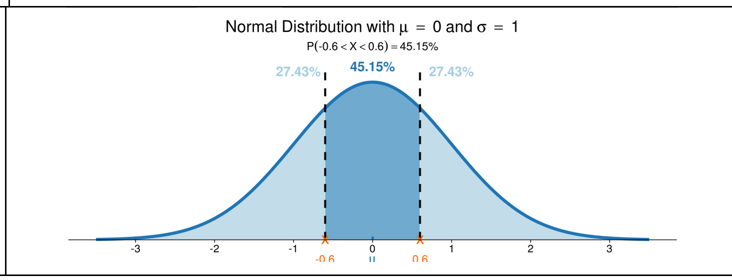
$$\text{pr}\left(64 < X < 67\right) \;=\; \text{pr}\left[\left(\frac{64\text{-}65.5}{2.5}\right) < Z < \left(\frac{67\text{-}65.5}{2.5}\right)\right]$$

$$. \;= \text{pr}\left[-0.6 < Z < +0.6\right]$$

$$=.4515$$

Using X ~ Normal(mean=65.5, sd=2.5)	Using Standardization to Z ~ Normal(mean=0, sd=1)





R Solution

```
# Pr[64 < Normal(mean=65.5, sd=2.5) < 67]
pnorm(67, mean=65.5, sd=2.5) - pnorm(64, mean=65.5, sd=2.5)
## [1] 0.4514938
```

#3. Suppose that, in a certain population, the distribution of GRE scores is normal with mean μ=600 and standard deviation σ=80.

a. What is the probability of a score less than 450 or greater than 750?

Art of Stat Solution

Answer: **.0608**

Solution:

Define the random variable X = GRE score.

Thus, X is distributed normal with mean μ=600 and standard deviation σ=80.

We write this more compactly as X ~ Normal (μ=600, σ=80). →

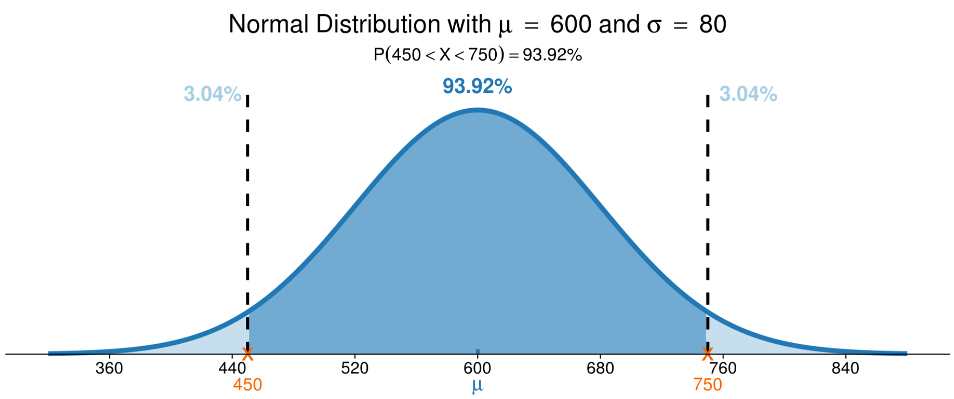
Probability { score < 450 OR score > 750 }

$$= \text{pr}\left[X < 450\right] + \text{pr}\left[X > 750\right]$$

$$= 1 - \text{pr}\,[\,450 < X < 750\,]$$

$$= 1 - .9392$$

$$= .0608$$



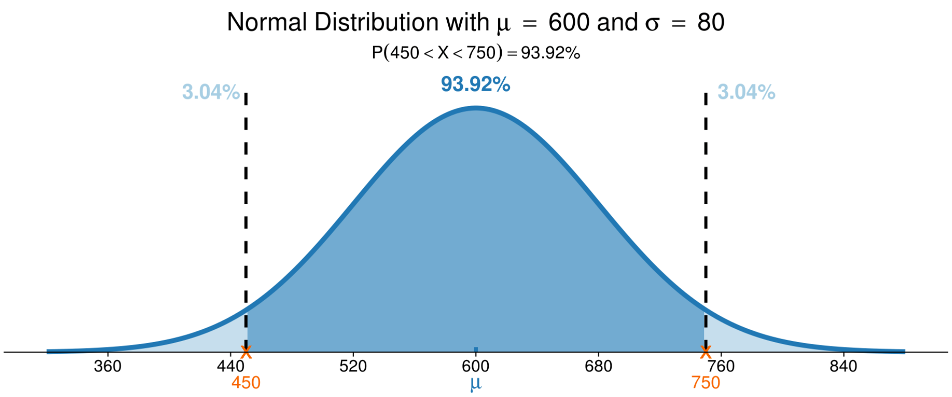
R Solution

```
#  Pr[ Normal(mean=600, sd=80) < 450] + Pr[ Normal(mean=600, sd=80) > 750]
pnorm(450, mean=600, sd=80) + pnorm(750, mean=600, sd=80,lower.tail=FALSE)
## [1] 0.06079272
```

b. What proportion of students has scores between 450 and 750?

Art of Stat Solution

Answer: .9392
Solution: "Proportion" of students with scores between 450 and 750 → we want: $= \text{pr}\left[450 < X < 750\right]$ $=.9392$



R Solution

```
# Pr[ 450 < Normal(mean=600, sd=80) < 750]
pnorm(750, mean=600, sd=80) - pnorm(450, mean=600, sd=80)
## [1] 0.9392073
```

#3 – Continued. The Chapin Social Insight Test evaluates how accurately the subject appraises other people. In the reference population used to develop the test, Chapin Social Insight Test scores are distributed normal with mean μ=25 and standard deviation σ=5.

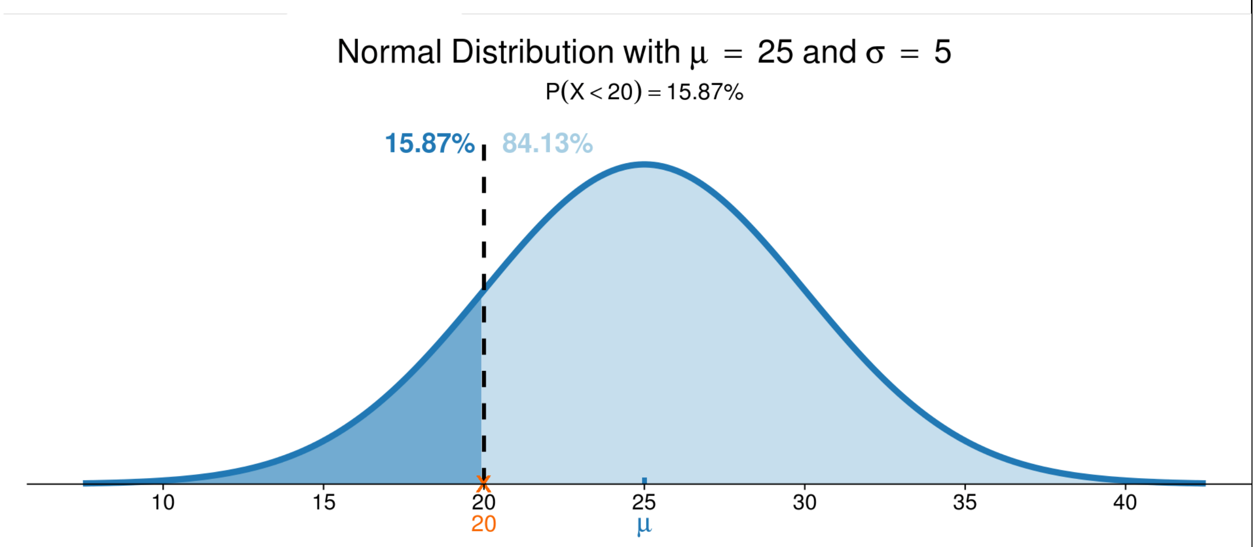
c. What proportion of the population has scores below 20 on the Chapin test?

Art of Stat Solution

Answer: .1587

Solution:
The solution for the "proportion of the population" is a probability calculation.

Define the random variable X = Chapin Social Insight Test Score.
X is distributed Normal (μ=25, σ=5).
Want: $\text{pr}(X < 20) = .1587$



R Solution

```
# Pr[ Normal(mean=25, sd=5) < 20]
pnorm(20, mean=25, sd=5)
## [1] 0.1586553
```

d. What proportion has scores below 10?

Art of Stat Solution

Answer: .0013

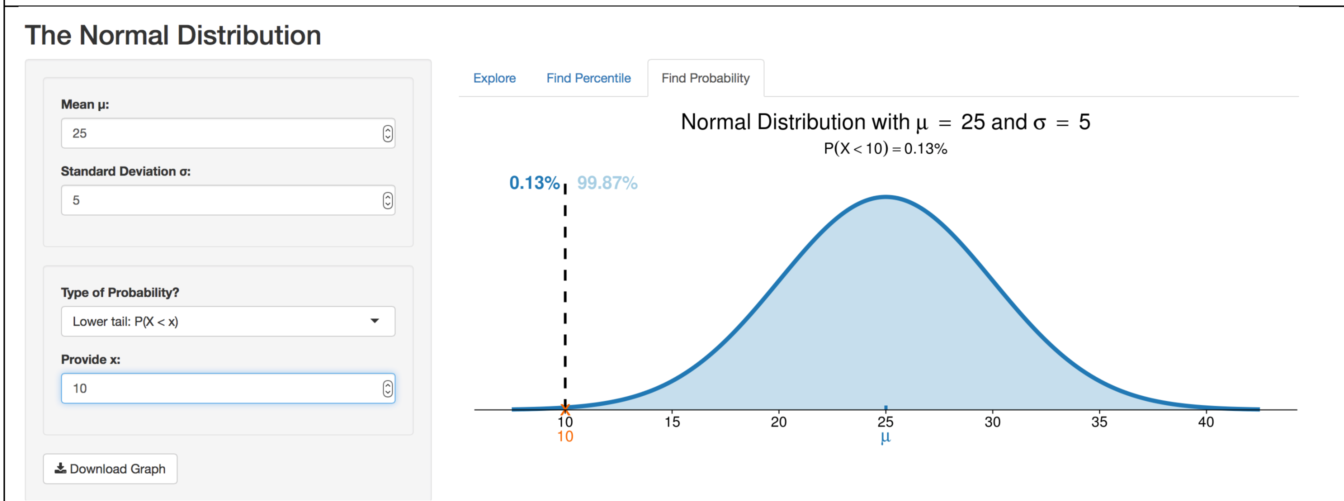
Solution:

This is similar to "a". →

The solution for the "proportion of the population" is a probability calculation.

X is distributed Normal (μ=25, σ=5).

Want: $pr(X < 10) = .0013$



R Solution

```
#  Pr[ Normal(mean=25, sd=5) < 10]
pnorm(10, mean=25, sd=5)
## [1] 0.001349898
```

#4. Consider again the setting in questions #3a and #3b: in a certain population, the distribution of GRE scores is normal with mean μ=600 and standard deviation σ=80.

a. What score is equal to the 95th percentile?

Art of Stat Solution

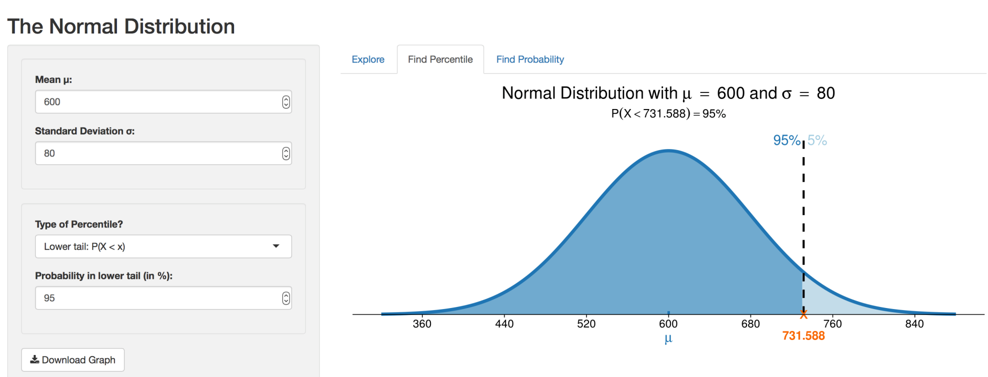
Answer: 731.6

There are at least two solutions to this question:

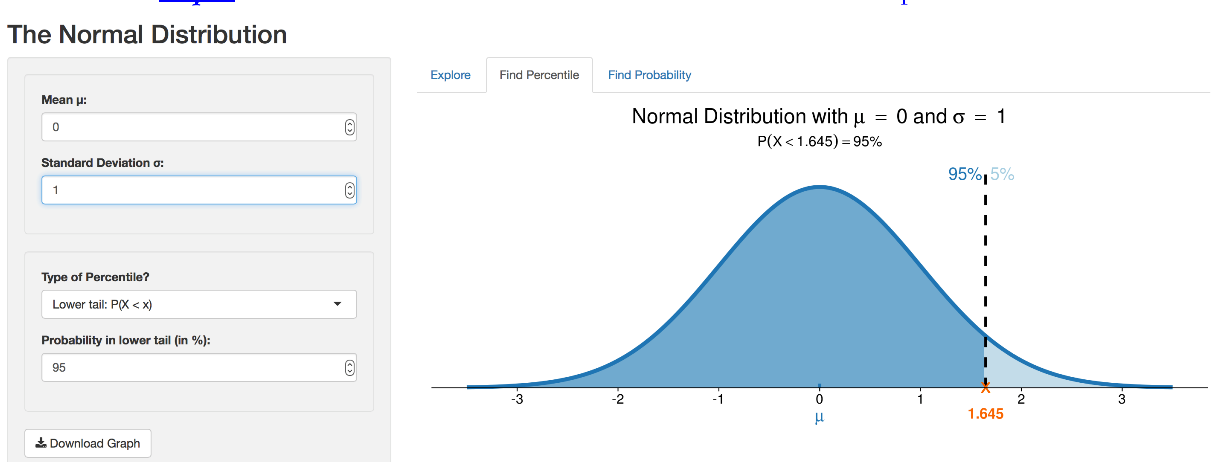
Solution I – Simple "plug in" variety

Solution II – 2 step solution that re-enforces the concepts..
Step 1: Obtain the 95th percentile for Z~Normal(0,1). Call this $Z_{.95}$
Step 2: Use $Z_{.95}$ and the formula on page 26 of the course notes to obtain $X_{.95}$

Solution I: Set mean=600 and standard deviation = 80



Solution II *Step 1*: Set mean=0 and standard deviation = 1 and then solve for the percentile of X



Solution II *Step 2*:
Use the formula on page 26 of the unit 7 notes with the following inputs: (1) $Z_{.95} = 1.645$ (2) $\mu = 600$ and $\sigma = 80$

$$X_{.95} = \sigma Z_{.95} + \mu$$
$$= (80)[1.645] + 600$$
$$= 731.6$$

R Solution

```
#   95th percentile of a Normal(mean=600, sd=80). I added the round( )so as to get just 2 digits
round(qnorm(.95,mean=600,sd=80),digits=2)
## [1] 731.59
```

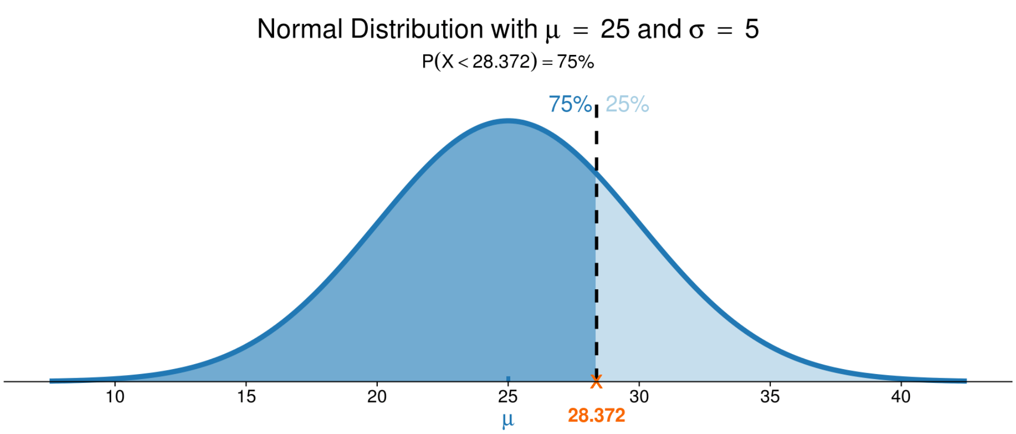
#4 – Continued. Next, consider again the setting of questions #3c and #3d: The Chapin Social Insight Test evaluates how accurately the subject appraises other people. In the reference population used to develop the test, Chapin Social Insight Test scores are distributed normal with mean μ=25 and standard deviation σ=5.

b. How high a score must you have in order to be in the top quarter of the population in social insight?

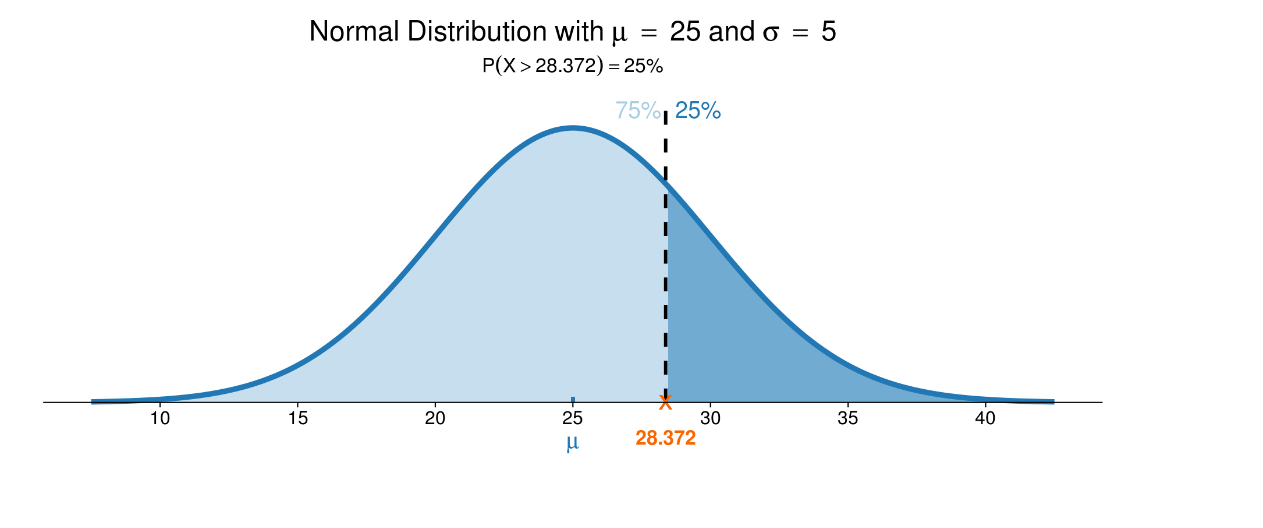
Art of Stat Solution

Answer: 28.37

Solution:
Hone your translation skills here. To be in the "top quarter" your score must be $\geq 75^{th}$ percentile



Of course you can always just do the brute force right tail probability = .25 to get the same answer!!



R Solution

```
#   75th percentile of a Normal(mean=25, sd=5)
round(qnorm(.75,mean=25,sd=5),digits=2)
## [1] 28.37
```

5. A normal distribution has mean μ=100 and standard deviation σ=15 (for example, IQ). Give limits, symmetric about the mean, within which 95% of the population would lie:

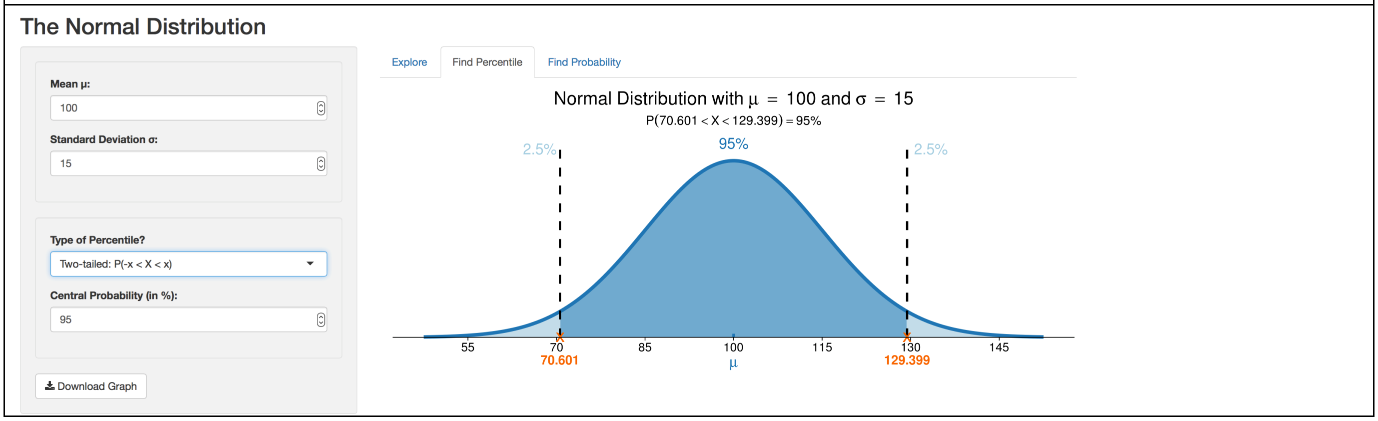
Solution:
This exercise is asking you to work with the following characteristic of the Normal distribution:

If X_1, X_2, …. X_n are a simple random sample, each distributed Normal(μ, σ^2)
Then the sample mean of n observations is distributed Normal((μ, σ^2/n)

Tip!
It is necessary to input the value of $\sqrt{\sigma^2 / n}$ in the box "standard deviation"

a) Individual observations

Answer: 70.6, 129.4
"Individual observations" →   **"mean" = μ=100**   **"standard deviation" = σ = 15.**

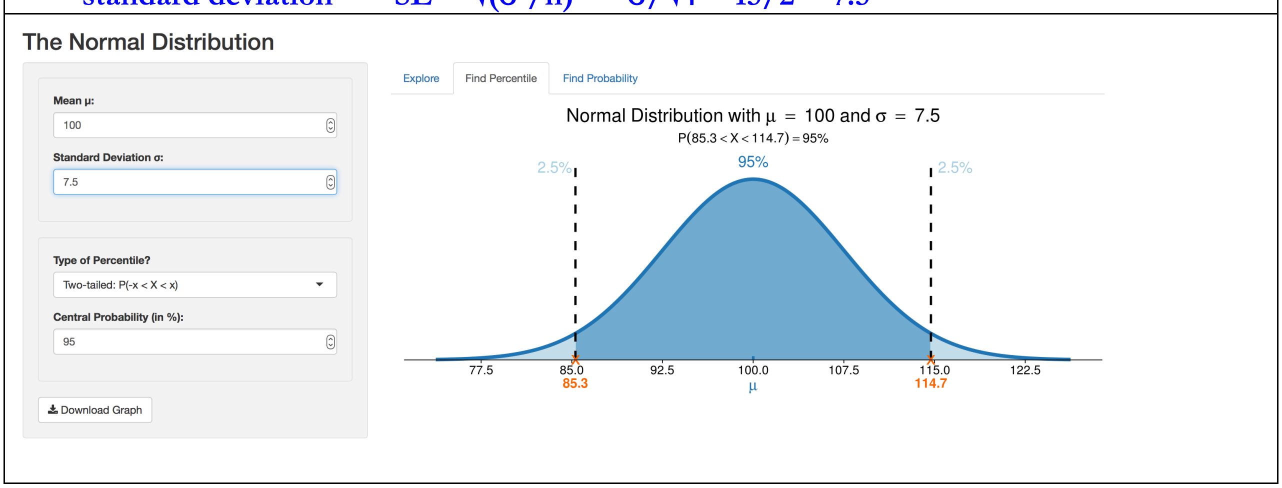


R Solution

```
#   Distribution of Individual Observations:  2.5th and 97.5th percentiles of Normal(mean=100, sd=15)
paste(round(qnorm(.025,mean=100,sd=15),digits=2)," and " ,round(qnorm(.975,mean=100,sd=15),digits=2))
## [1] "70.6  and  129.4"
```

b) Means of 4 observations

Answer: 85.3, 114.7
"Means of 4 observations" →   **"mean" = μ=100**   **"standard deviation" = SE = $\sqrt{(\sigma^2/n)}$ = $\sigma/\sqrt{4}$ = 15/2 = 7.5**



R Solution

```
#   Means of n=4 Observations:  2.5th and 97.5th percentiles of Normal(mean=100, sd=7.5)
paste(round(qnorm(.025,mean=100,sd=7.5),digits=2)," and " ,round(qnorm(.975,mean=100,sd=7.5),digits=2))
## [1] "85.3  and  114.7"
```

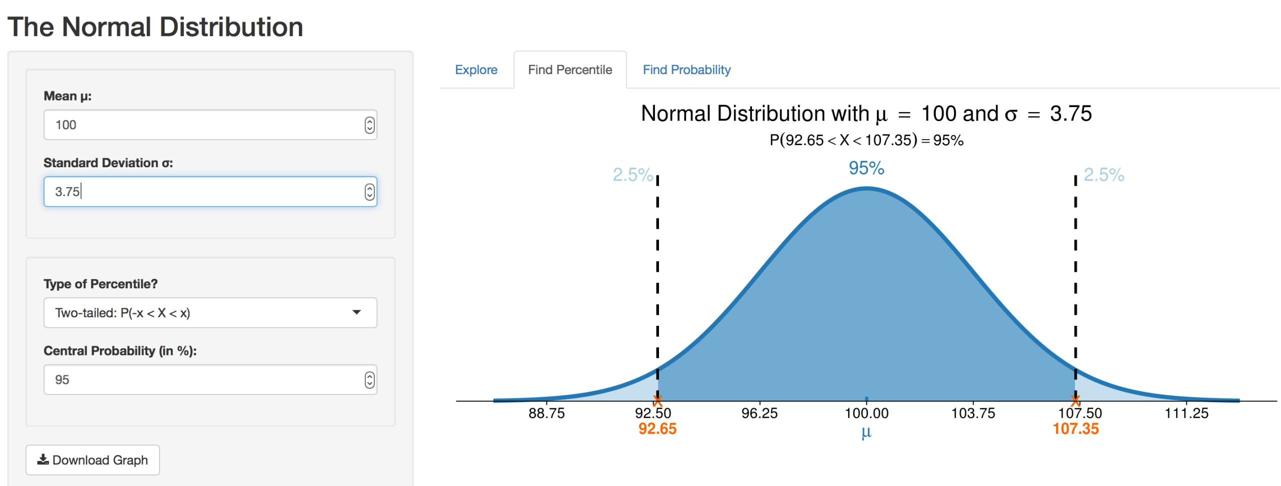
c) Means of 16 observations

Answer: 92.65, 107.35

"Means of 16 observations" →

"mean" = μ=100

"standard deviation = SE = $\sqrt{(\sigma^2/n)}$ = $\sigma/\sqrt{16}$ = 15/4 = 3.75



R Solution

```
#   Means of n=16 Observations:  2.5th and 97.5th percentiles of Normal(mean=100, sd=3.75)
paste(round(qnorm(.025,mean=100,sd=3.75),digits=2)," and " ,round(qnorm(.975,mean=100,sd=3.75),digits=2))
## [1] "92.65  and  107.35"
```

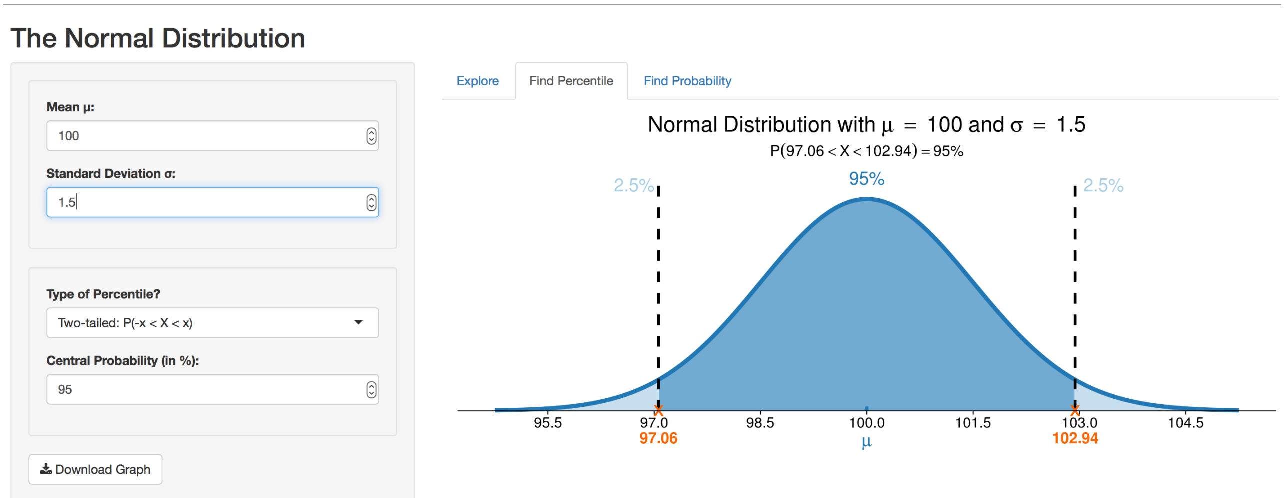
d) Means of 100 observations

Answer: 97.06, 102.94

"Means of 100 observations" →

"mean" = μ=100

"standard deviation" = SE = $\sqrt{(\sigma^2/n)}$ = $\sigma/\sqrt{100}$ = 15/10 = 1.5



R Solution

```
#   Means of n=100 Observations:  2.5th and 97.5th percentiles of Normal(mean=100, sd=1.5)
paste(round(qnorm(.025,mean=100,sd=1.5),digits=2)," and " ,round(qnorm(.975,mean=100,sd=1.5),digits=2))
## [1] "97.06  and  102.94"
```