Unit 1
Summarizing Data
Practice Quiz

SOLUTIONS

1. (b) Discrete data with categories that do not follow a natural sequence.

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2. new mean = $\frac{[\text{old mean - 1000}]}{5}$ new variance = $\frac{[\text{old variance}]}{5^2}$ new sd = $\frac{[\text{old sd}]}{5}$

You could obtain this by brute force, that is by doing the calculations for the old data set and the new data set which is: 1, 2, 3, 4 and 5.

For the interested reader … An alternate solution works through the formulae as follows.

$$\text{new mean} = \frac{1}{n}\sum\left[\frac{(\text{x-1000})}{5}\right]$$
$$= \frac{1}{5n}\left[\sum(\text{x-1000})\right]$$
$$= \frac{1}{5n}\left[\sum \text{x} - \sum 1000\right]$$
$$= \frac{1}{5n}\left[\sum \text{x} - (1000)(\text{n})\right]$$
$$= \frac{\sum \text{x}}{5\text{n}} - \frac{(1000)(\text{n})}{(5)(\text{n})}$$
$$= \frac{1}{5}\left[\frac{\sum \text{x}}{\text{n}} - 1000\right]$$
$$= \frac{1}{5}[\text{old mean - 1000}]$$

$$\text{new variance} = \left[\frac{1}{\text{n-1}}\right]\sum\left(\text{new x - new mean}\right)^2$$

$$= \left[\frac{1}{\text{n-1}}\right]\sum\left(\left[\frac{\text{x-1000}}{5}\right] \text{ - new mean}\right)^2$$

$$= \left[\frac{1}{\text{n-1}}\right]\sum\left(\left[\frac{\text{x-1000}}{5}\right] \text{ - } \left[\frac{\text{old mean-1000}}{5}\right]\right)^2$$

$$= \left[\frac{1}{\text{n-1}}\right]\left[\frac{1}{5}\right]^2\sum\left(\left[\text{x-1000}\right] \text{ -}\left[\text{old mean - 1000}\right]\right)^2$$

$$= \left[\frac{1}{\text{n-1}}\right]\left[\frac{1}{5}\right]^2\sum\left(\text{x-1000-old mean + 1000}\right)^2$$

$$= \left[\frac{1}{\text{n-1}}\right]\left[\frac{1}{5}\right]^2\sum\left(\text{x - old mean}\right)^2$$

$$= \left[\frac{1}{5}\right]^2\left[\frac{1}{\text{n-1}}\right]\sum\left(\text{x - old mean}\right)^2$$

$$= \left[\frac{1}{5}\right]^2\left[\text{old variance}\right]$$

$$\text{new standard deviation} = \sqrt{\text{new variance}}$$

$$= \sqrt{\left[\frac{1}{5}\right]^2\left(\text{old variance}\right)}$$

$$= \frac{1}{5}\sqrt{\text{old variance}}$$

$$= \frac{1}{5}\left[\text{old standard deviation}\right]$$

3. **New sample mean = 53.07**
   New sample standard deviation = 9.137

Solution for new sample mean.

Step 1 – Work with the original mean to obtain the sum of ages of the original 50

$$\text{IF } \overline{X} = \frac{\sum_{i=1}^{50} x_i}{50} \quad \text{THEN } \sum_{i=1}^{50} x_i = (50)(\overline{X}) = (50)(53.87) = 2{,}693.5$$

Step 2 - Use this to solve for the sum of the ages of the original 49 plus the new person

$\sum_{i=1}^{49} x_i = (2{,}693.5 - 82) = 2611.5$ **and** $\sum_{i=1}^{50} x_i = (2611.5+42) = 2653.5$. **Thus,**

$$\overline{X} = \frac{\sum_{i=1}^{50} x_i}{50} = \frac{2653.5}{50} = 53.07$$

Solution for new sample standard deviation.

$$\text{If } S^2 = \left[\frac{1}{n\text{-}1}\right]\sum\left(x_i - \overline{x}\right)^2 \quad \text{Then } S^2 = \left[\frac{1}{n\text{-}1}\right]\left(\left[\sum x_i^2\right] - \left[n\overline{x}^2\right]\right)$$

Step 1 – Work with the original variance to obtain the sum of squared ages of the original 50

Note – Save rounding until the last step (reporting)

$$s = 9.87 \rightarrow$$

$$s^2 = 97.417 \rightarrow$$

$$(n\text{-}1)s^2 = \sum_{i=1}^{50}\left(x_i - \overline{x}\right)^2 = (49)(97.417) = 4773.428 \rightarrow$$

$$4773.428 = \sum_{i=1}^{50} x_i^2 - (n)(\overline{x}^2) \rightarrow$$

$$4773.428 = \sum_{i=1}^{50} x_i^2 - (50)(53.87^2) \rightarrow$$

$$4773.428 = \sum_{i=1}^{50} x_i^2 - 145{,}098.845 \rightarrow$$

$$\sum_{i=1}^{50} x_i^2 = 149{,}872.273$$

<u>Step 2 – Obtain the sum of squared ages of the new 50</u>

$$\text{New} \sum_{i=1}^{50} x_i^2 = \text{Old} \sum_{i=1}^{50} x_i^2 - 82^2 + 42^2$$

$$= 149{,}872.273 - 6724 + 1764$$

$$= 144{,}912.273$$

<u>Step 3 – Use this plus new mean to obtain new S^2</u>

$$\text{New } S^2 = \left[\frac{1}{\text{n-1}}\right]\left[\sum_{i=1}^{50} x_i^2 - (n)\left(\overline{x}^2\right)\right] \rightarrow$$

$$S^2 = \left[\frac{1}{49}\right]\left[144{,}912.273 - (50)(53.07^2)\right]$$

$$= 83.49$$

<u>Last Step – Use new S^2 to obtain new S</u>

$$\text{New S} = \sqrt{\text{new } S^2}$$
$$= \sqrt{83.49}$$
$$= 9.137$$

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4. **False.**
   Removal of an extreme value yields remaining data that are less variable.

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5. **Median = 121.5 IK**

Ordering the data from smallest to largest yields
42 < 89 < 94 < 108 < **115 < 128** < 136 < 149 < 158 < 196
Thus,

$$\text{median} = \frac{1}{2}[115 + 128] = 121.5 \text{ IK}$$

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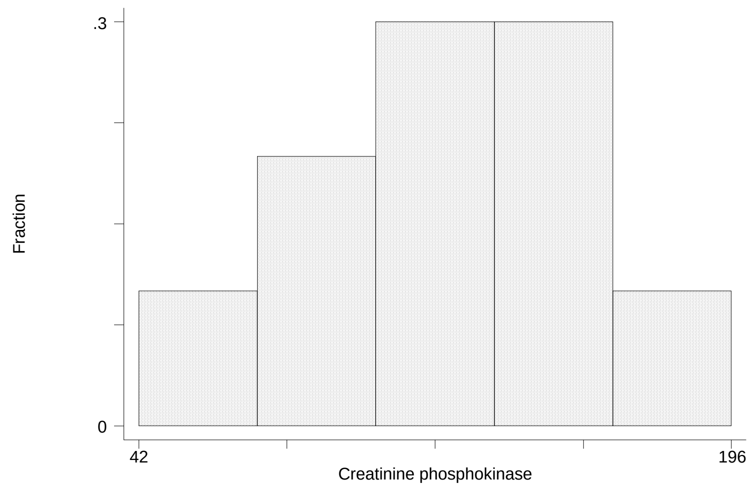
6. Averaging the 10 values yields mean = 121.5
The similarity of the mean and median suggests that the data are **symmetric**.
A histogram confirms this.



. graph cpk, histogram ***source: quiz1num5.wmf***

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7. A box weighs **16** ounces, give or take **0.2** ounces.

Not what you thought, is it! This question is a little more involved than you might have thought and, so, I don't expect that you will have gotten the correct answer the first time.

As the 4 sticks of butter are independent,
Mean of box = Sum of means of 4 sticks = (4) (4 ounces) = 16 ounces
Variance of box = Sum of variances of 4 sticks = (4) (0.1^2 ounces^2) = 0.04 ounces^2
Therefore, standard deviation of box = $\sqrt{0.04 \text{ ounces}^2}$ = 0.2 ounces

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8. $\overline{X}=0.667$
$S=0.471$

$$\overline{X}=\left[\frac{1}{n}\right]\sum_{i=1}^{3000} X_i = \left[\frac{1}{3000}\right]\left[(1000)(0)+(2000)(1)\right] = 0.667$$

$$S^2 = \left[\frac{1}{\text{n-1}}\right]\left[\sum_{i=1}^{3000} X_i^2 - (n)(\bar{X}^2)\right]$$

$$= \left[\frac{1}{2999}\right]\left[(1000)(0^2) + (2000)(1^2) - (3000)(.667^2)\right]$$

$$= \left[\frac{1}{2999}\right]\left[(2000) - (1,333.33)\right]$$

$= 0.222 \rightarrow \rightarrow S=\sqrt{S^2} = \sqrt{0.222} = 0.471$

9.

(9.1) **50**
(9.2) **25**
(9.3) **40**

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10. **5 feet 7.8 inches**

Preliminary
5 feet 8 inches = 68 inches
4 feet 11 inches = 59 inches

Old $\bar{X}$ = 5 feet 8 inches = 68 inches →

$\sum_{i=1}^{49} X_i = (n)(\bar{X})$=(49)(68 inches) = 3332 inches Thus,

$$\text{New } \bar{X} = \left[\frac{1}{n}\right]\sum_{i=1}^{50} X_i = \left[\frac{1}{50}\right]\left[\sum_{i=1}^{49} X_i + 59 \text{ inches}\right] = \left[\frac{1}{50}\right]\left[3332 + 59\right] = 67.82$$