Unit 6
Estimation
Practice Quiz
SOLUTIONS

Some studies of Alzheimer's disease (AD) have shown an increase in $_{14}CO^2$ production in patients with the disease. In one such study, the following $_{14}CO^2$ values were obtained from 16 neocortical biopsy samples from AD patients.

1009	1280	1180	1255	1547	2352	1956	1080
1776	1767	1680	2050	1452	2857	3100	1621

Assume that the population of such values is normally distributed with a standard deviation of $\sigma = 350$.

1. Construct a 95 percent confidence interval for μ.

   Answer: (1576.1, 1919.1)

   Solution:

   (i) point estimate is $\overline{X} = 1747.6$

   (ii) standard error of point estimate is $SE(X) = \sigma/\sqrt{n} = 350/4 = 87.5$

   (iii) solution for confidence coefficient: Since $(1 - \alpha) = 0.95$, $(1-\alpha/2)=0.975$. Get 97.5^{th} percentile of Normal(0,1) = 1.96. Thus the required confidence interval is

   estimate $\pm$ { confidence coefficient} { se of estimate}
   $= 1747.6 \pm \{ 1.96 \} \{87.5 \}$
   $= (1576.1, 1919.1)$

2. If the true population mean is $\mu = 1800$ with $\sigma = 350$, what proportion of patient values would be greater than 1900?

   Answer: .3876

   Solution:

   Since $\mu = 1800$ and $\sigma = 350$, solve as

   $$\begin{aligned} \text{Prob}(X \geq 1900) &= \text{Prob}(Z \geq \{(1900 - 1800)/350\}) \\ &= \text{Prob}(Z \geq 0.2857) \\ &= 0.3876 \end{aligned}$$

3. If the true population mean is μ = 1800 with σ = 350, what proportion of means of size 16 would be greater than 1900? What proportion of means from samples of size 25 would be greater than 1900?

Answer:

Sample Size	Proportion of Means Greater than 1900
16	**.1265**
25	**.0765**

Solution:

$\text{Prob}\ (\ \overline{X}_{n=16} \geq 1900\ ) = \text{Prob}\ (\ Z \geq \{\ (1900 - 1800)/(350/4)\ \}\ )$

$= \text{Prob}\ (\ Z \geq 1.143)$

$= 0.1265$. Conclude that about 12.7% of sample means of size 16 would be greater than 1900.

$\text{Prob}\ (\ \overline{X}_{n=25} \geq 1900\ ) = \text{Prob}\ (\ Z \geq \{\ (1900 - 1800)/(350/5)\ \}\ )$

$= \text{Prob}\ (\ Z \geq 1.429)$

$= 0.0765$. Conclude that about 7.7% of sample means of size 25 would be greater than 1900.

4. Considering the derivation of confidence interval estimates, comment on the role of sample size in the estimation of the unknown population mean parameter.

Solution:

Narrower, or more precise, confidence interval estimates are obtained when obtained from data sets of larger sample sizes.

Recall that the width of a confidence interval for the mean parameter of a normal probability distribution with known variance is:

$$(\ 2\ )\ \{\ \text{critical } z\ \}\ \{\ \text{standard deviation} / \sqrt{n}\ \}$$

where n is the number of observations in the sample.

This means that, for fixed critical z and for fixed standard deviation, as n increases, the quantity $1/\sqrt{n}$ decreases. Consequently, the width of the confidence interval also decreases.

5. Now, assume that the population of such values is normally distributed with unknown mean and ***unknown*** variance. Construct a 95% confidence interval for the population mean. Compare this interval to the interval you got for question #1.

Answer: (1425.6, 2069.61)

Solution:

Confidence intervals constructed utilizing the Student's t-distribution for the determination of the confidence coefficient are larger than those using the Normal (0,1) distribution, all other things equal.

Calculate sample variance s^2 and from this obtain s = 604.65

(i) point estimate is $\overline{X} = 1747.6$

(iv) **<u>sample</u>** standard error of point estimate is $s/\sqrt{n}$ = 604.65/4 = 151.16

(v) solution for confidence coefficient: Since $(1 - \alpha) = 0.95$, $(1-\alpha/2)=0.975$. Get 97.5^{th} percentile of student's t distribution with 15 degrees of freedom = 2.13. Thus the required confidence interval is

estimate $\pm$ { confidence coefficient from student's t} { standard error of estimate}
= 1747.6 $\pm$ { 2.13 } {151.16 }
= (1425.63, 2069.57)