Probability Theory, Part 1

Why probability in Statistics ?

- Results are not certain
- To evaluate how accurate our results are
  - Given how our data were collected, are our results accurate ?
  - Given the level of accuracy needed, how many observations need to be collected ?

When can we talk about probability ?

- When dealing with a process that has an uncertain outcome
- Experiment = any process with an uncertain outcome

Experiments

- The possible results of an experiments are its outcomes
- When an experiment is performed, one and only one outcome is obtained
- S = set of all possible outcomes

Events

- Event = something that may happen or not when the experiment is performed
- Any event can be expressed as a subset of the set of all possible outcomes (S)
- Graphically, events can be represented by Venn Diagrams

Probability of an Event

- Probability of an event E = a number between 0 and 1 representing the proportion of times that event E is expected to happen when the experiment is done over and over again under the same conditions

Properties of Probability

- $0 \leq P(E) \leq 1$
- S = set of all possible outcomes

P(S) = 1

- While choosing at random, talking about probabilities is equivalent to talking about proportions

Probability Distributions

- Probability distribution = a description of the possible outcomes and their probabilities of occurrence

- Using the probability distribution we can compute the probability of any event

Counting

- The MN Rule : If a task takes two steps, there are M ways of doing the first step and N ways of doing the second step, then there are (M.N) ways of doing the task

- Permutations : If we have n objects, in how many ways can we order them ?

n factorial = $n! = n \times (n-1) \times \cdots \times 2 \times 1$

- Combinations : If we have n objects, in how many ways can we choose a group of size s ?

$$C_s^n = \frac{n!}{s! \times (n-s)!}$$

Conditional Probability

- A, B = events

- P (A | B) = Probability of A given B
  = Probability of A conditional on B
  = Given that B has happened, what is the proportion of times that A is expected to happen?

Notation

- P (A and B) = P (A $\cap$ B)

- P (A or B) = P (A $\cup$ B)

Laws of Probability

- P (not A) = 1 – P (A)

- Multiplication Rule : The probability that two things will happen equals the probability that the first will happen multiplied by the probability that the second will happen, given that the first has already happened

$$P(A \text{ and } B) = P(A \cap B) = P(A)\,P(B|A)$$

- Bayes Rule : As a consequence of the multiplication rule,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Addition Rule : $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- Law of Total Probability : P (A) = P (A and B) + P (A and not B)