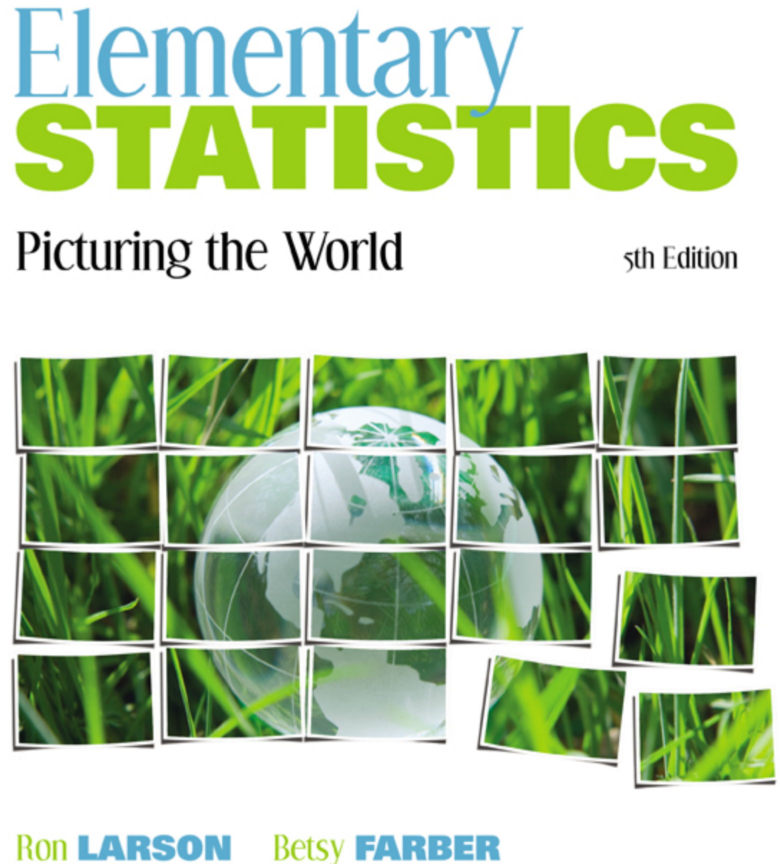
Chapter 5

Normal Probability Distributions



Chapter Outline

- 5.1 Introduction to Normal Distributions and the Standard Normal Distribution
- 5.2 Normal Distributions: Finding Probabilities
- 5.3 Normal Distributions: Finding Values
- 5.4 Sampling Distributions and the Central Limit Theorem
- 5.5 Normal Approximations to Binomial Distributions

Section 5.1

Introduction to Normal Distributions

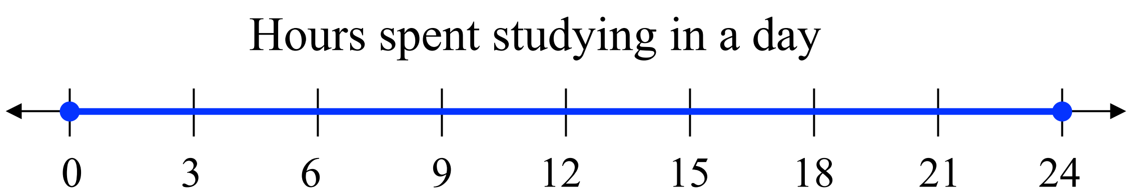
Section 5.1 Objectives

- Interpret graphs of normal probability distributions
- Find areas under the standard normal curve

Properties of a Normal Distribution

Continuous random variable

- Has an infinite number of possible values that can be represented by an interval on the number line.



The time spent studying can be any number between 0 and 24.

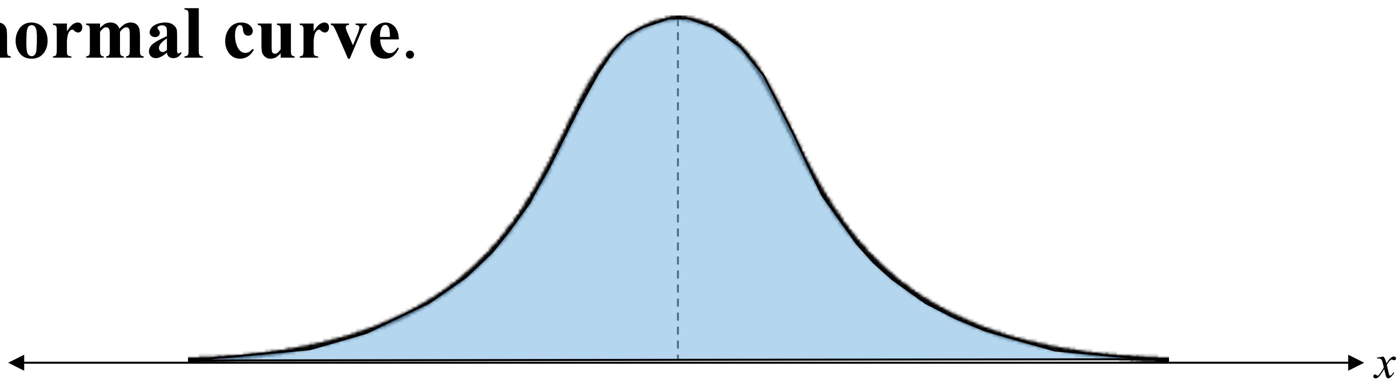
Continuous probability distribution

- The probability distribution of a continuous random variable.

Properties of Normal Distributions

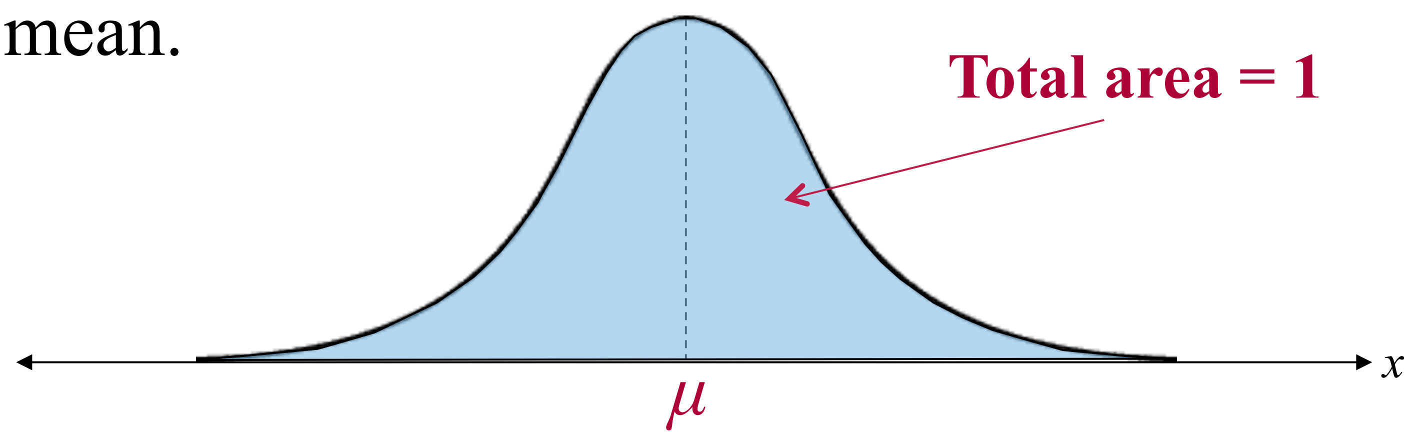
Normal distribution

- A continuous probability distribution for a random variable, x.
- The most important continuous probability distribution in statistics.
- The graph of a normal distribution is called the **normal curve**.



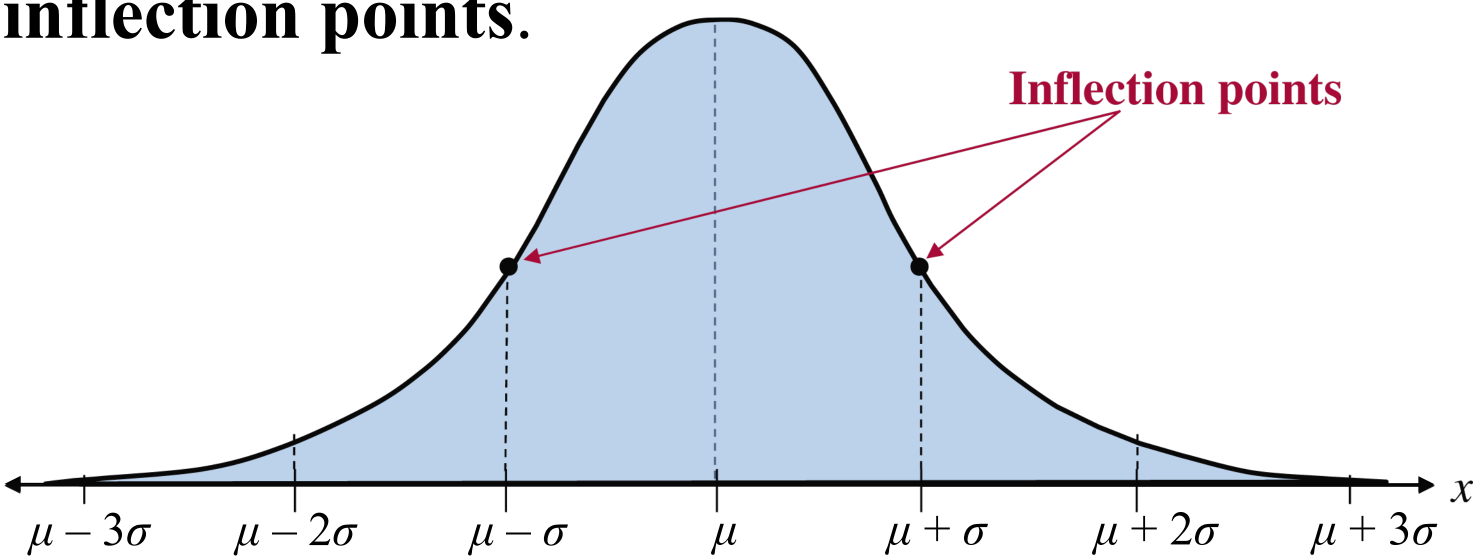
Properties of Normal Distributions

1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and is symmetric about the mean.
3. The total area under the normal curve is equal to 1.
4. The normal curve approaches, but never touches, the x-axis as it extends farther and farther away from the mean.



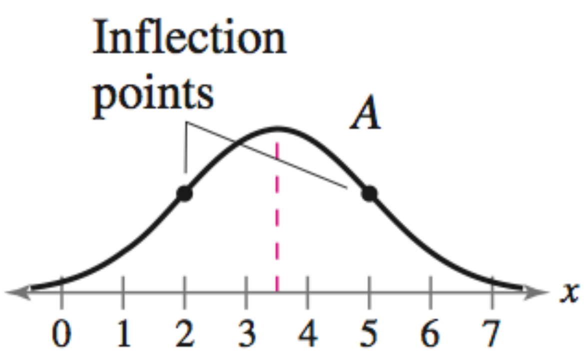
Properties of Normal Distributions

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the **inflection points**.

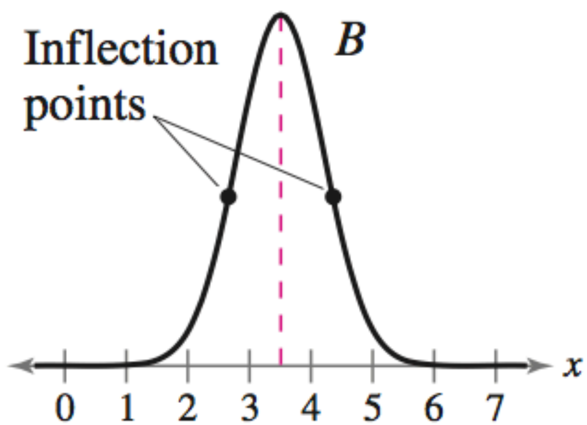


Means and Standard Deviations

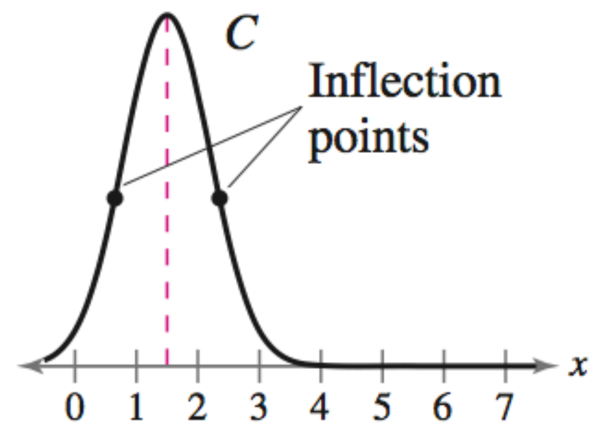
- A normal distribution can have any mean and any positive standard deviation.
- The mean gives the location of the line of symmetry.
- The standard deviation describes the spread of the data.



$\mu = 3.5$
$\sigma = 1.5$



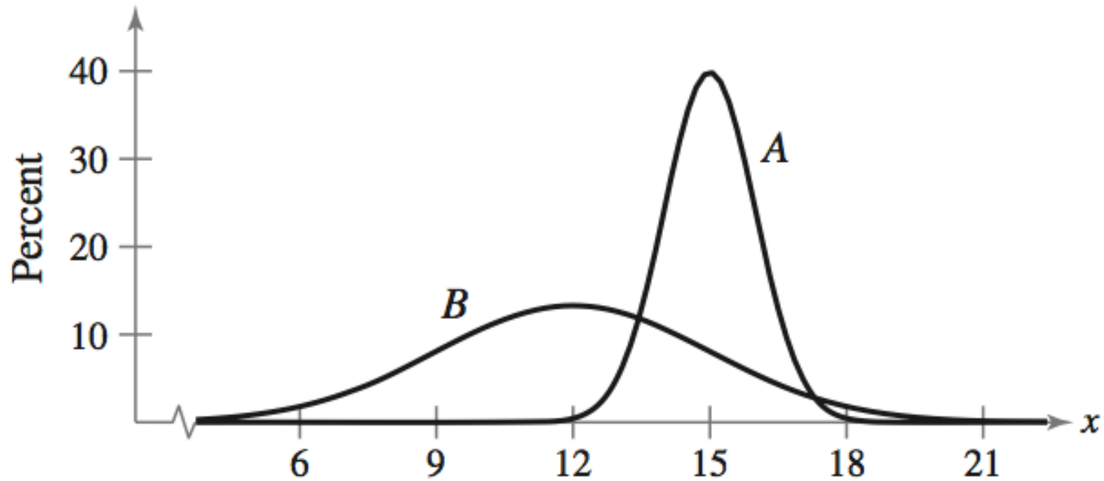
$\mu = 3.5$
$\sigma = 0.7$



$\mu = 1.5$
$\sigma = 0.7$

Example: Understanding Mean and Standard Deviation

1. Which normal curve has the greater mean?

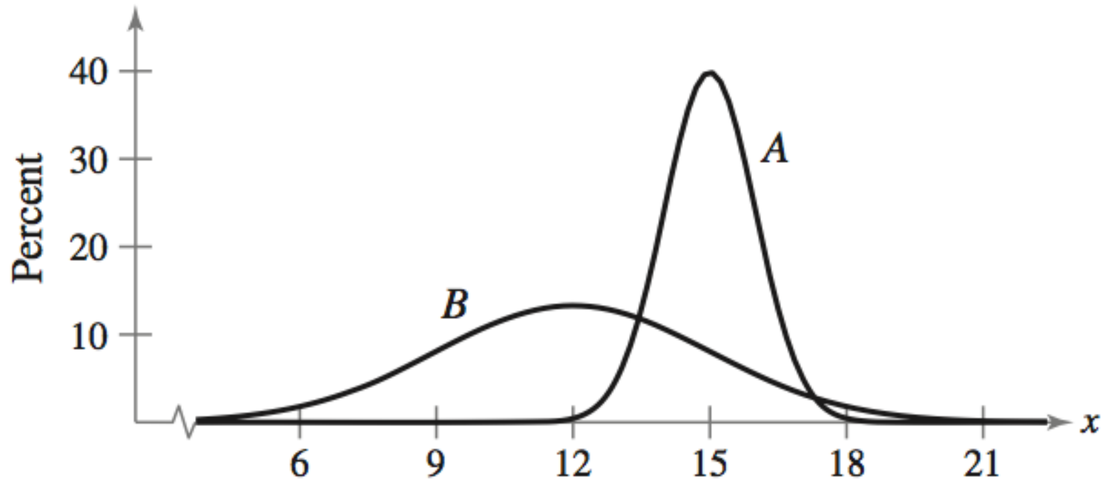


Solution:

Curve *A* has the greater mean (The line of symmetry of curve *A* occurs at $x = 15$. The line of symmetry of curve *B* occurs at $x = 12$.)

Example: Understanding Mean and Standard Deviation

2. Which curve has the greater standard deviation?



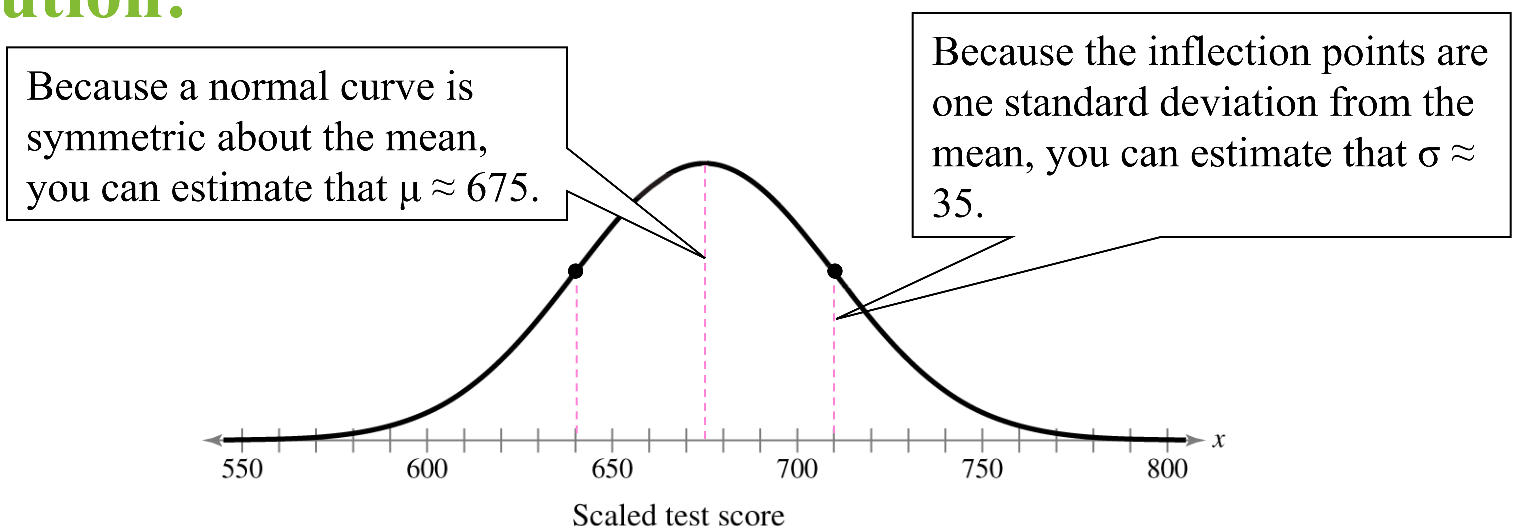
Solution:

Curve *B* has the greater standard deviation (Curve *B* is more spread out than curve *A*.)

Example: Interpreting Graphs

The scaled test scores for the New York State Grade 8 Mathematics Test are normally distributed. The normal curve shown below represents this distribution. What is the mean test score? Estimate the standard deviation.

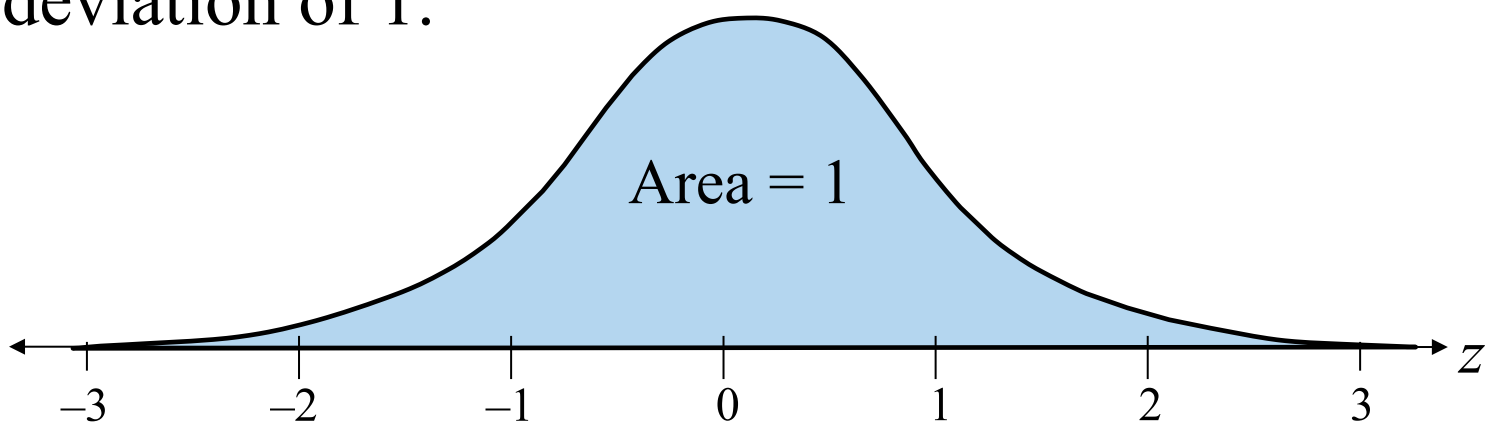
Solution:



The Standard Normal Distribution

Standard normal distribution

- A normal distribution with a mean of 0 and a standard deviation of 1.

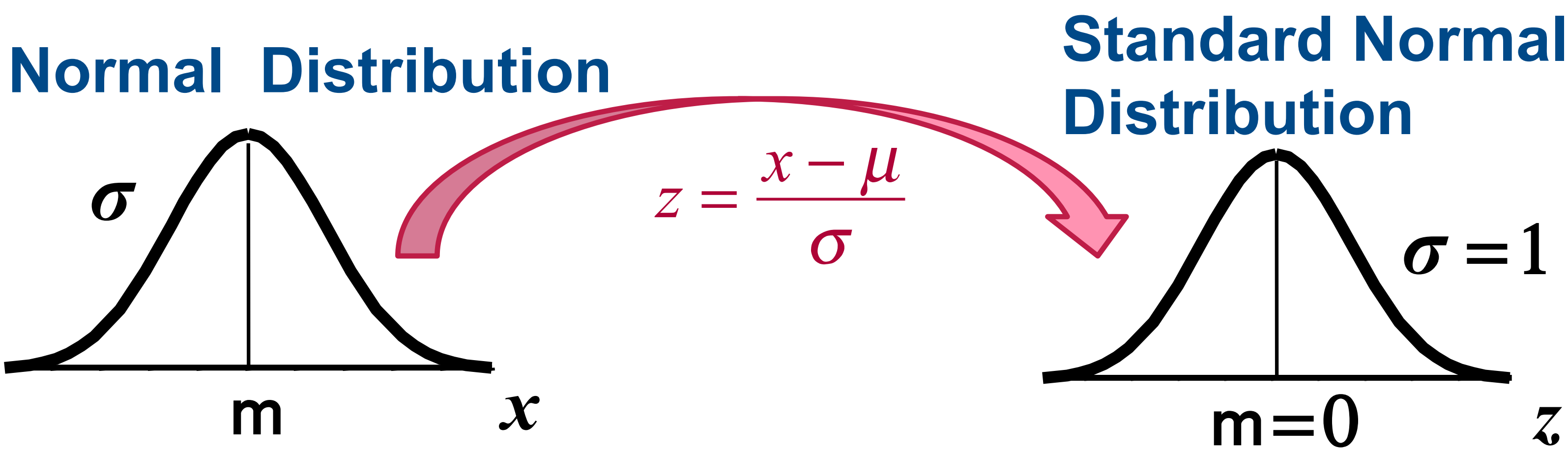


- Any x-value can be transformed into a z-score by using the formula

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma}$$

The Standard Normal Distribution

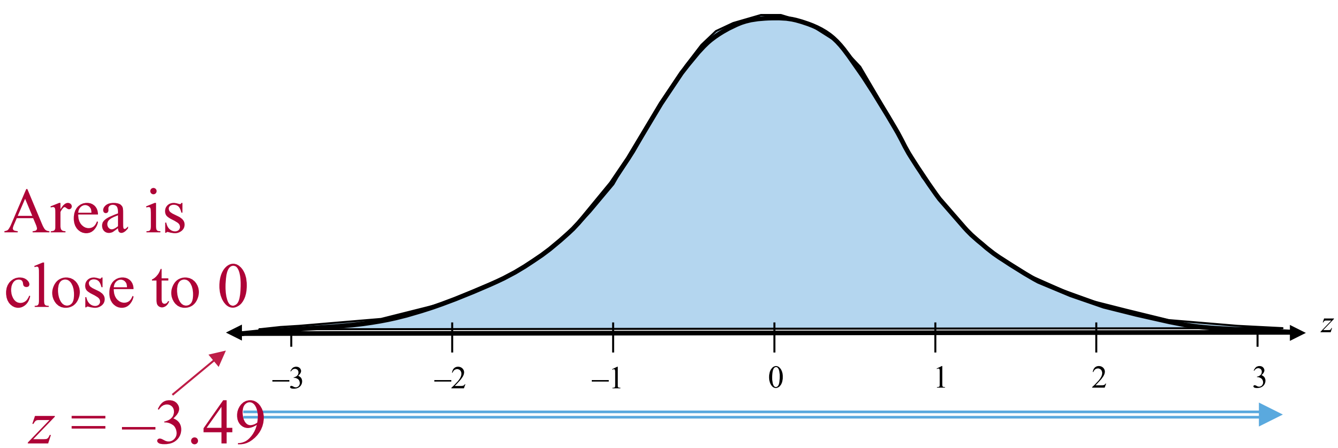
- If each data value of a normally distributed random variable x is transformed into a z-score, the result will be the standard normal distribution.



- Use the Standard Normal Table to find the cumulative area under the standard normal curve.

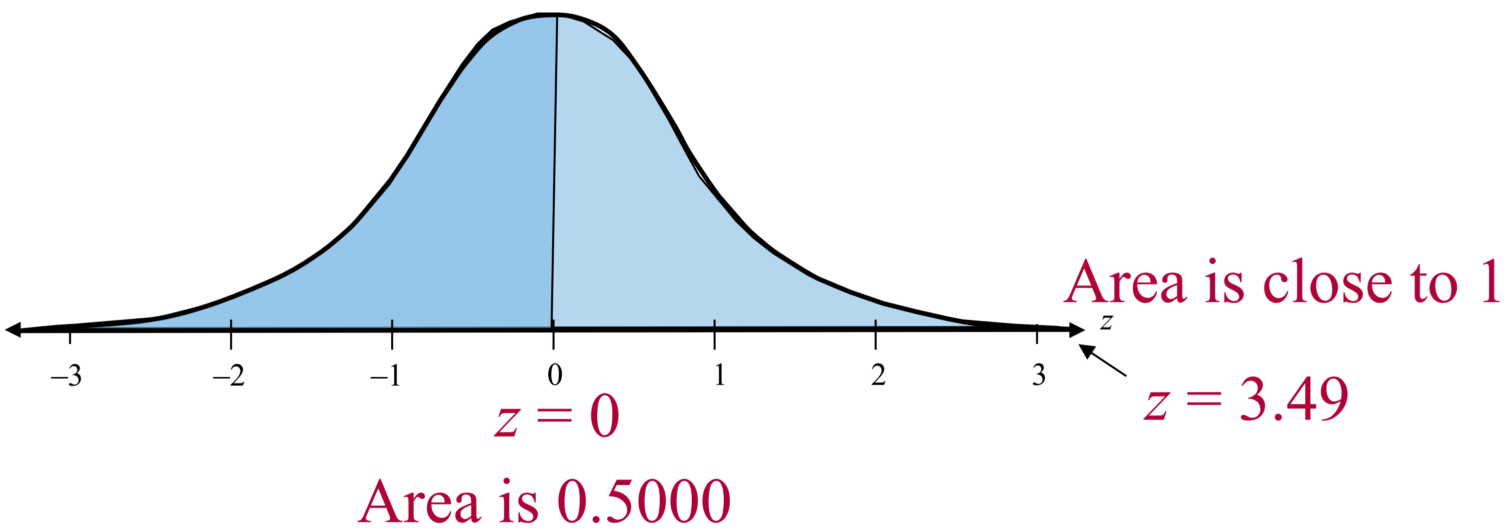
Properties of the Standard Normal Distribution

1. The cumulative area is close to 0 for *z*-scores close to $z = -3.49$.
2. The cumulative area increases as the *z*-scores increase.



Properties of the Standard Normal Distribution

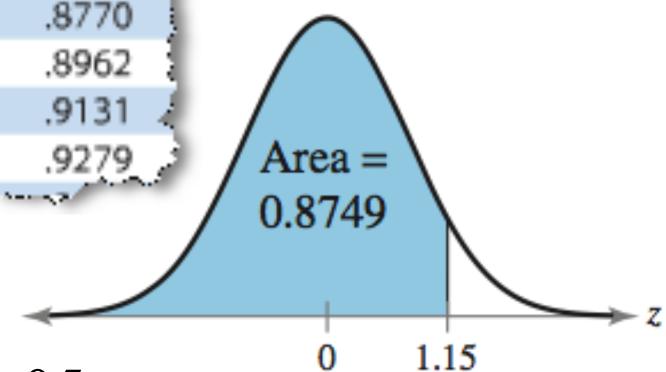
3. The cumulative area for $z = 0$ is 0.5000.
4. The cumulative area is close to 1 for z-scores close to $z = 3.49$.



Example: Using The Standard Normal Table

Find the cumulative area that corresponds to a z-score of 1.15.

z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279



Solution:

Find 1.1 in the left hand column.

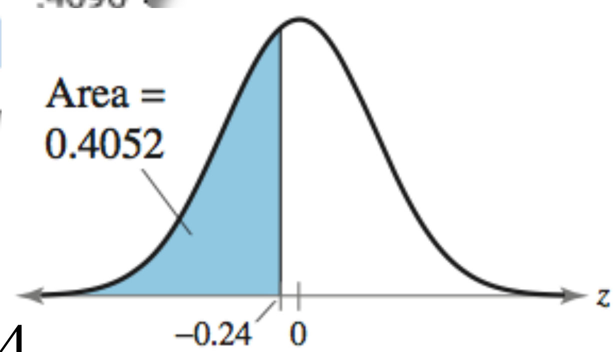
Move across the row to the column under 0.05

The area to the left of $z = 1.15$ is 0.8749.

Example: Using The Standard Normal Table

Find the cumulative area that corresponds to a *z*-score of –0.24.

z	.09	.08	.07	.06	.05	.04	.03
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	



Solution:

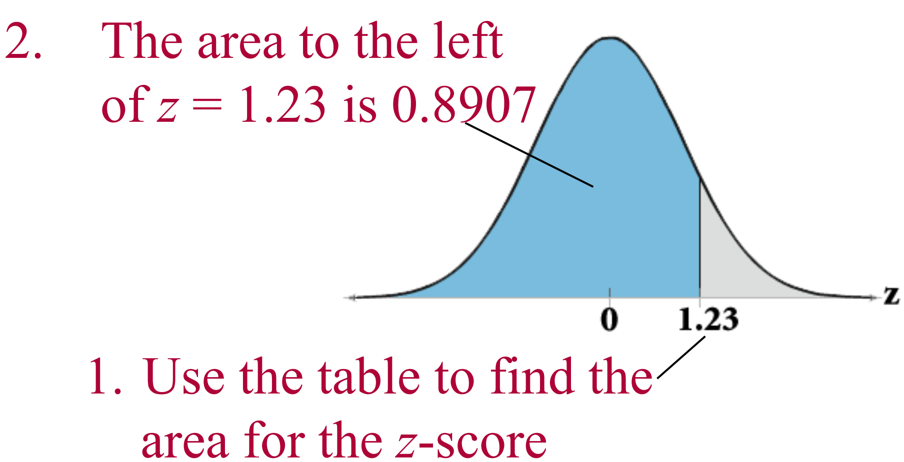
Find –0.2 in the left hand column.

Move across the row to the column under 0.04

The area to the left of *z* = –0.24 is 0.4052.

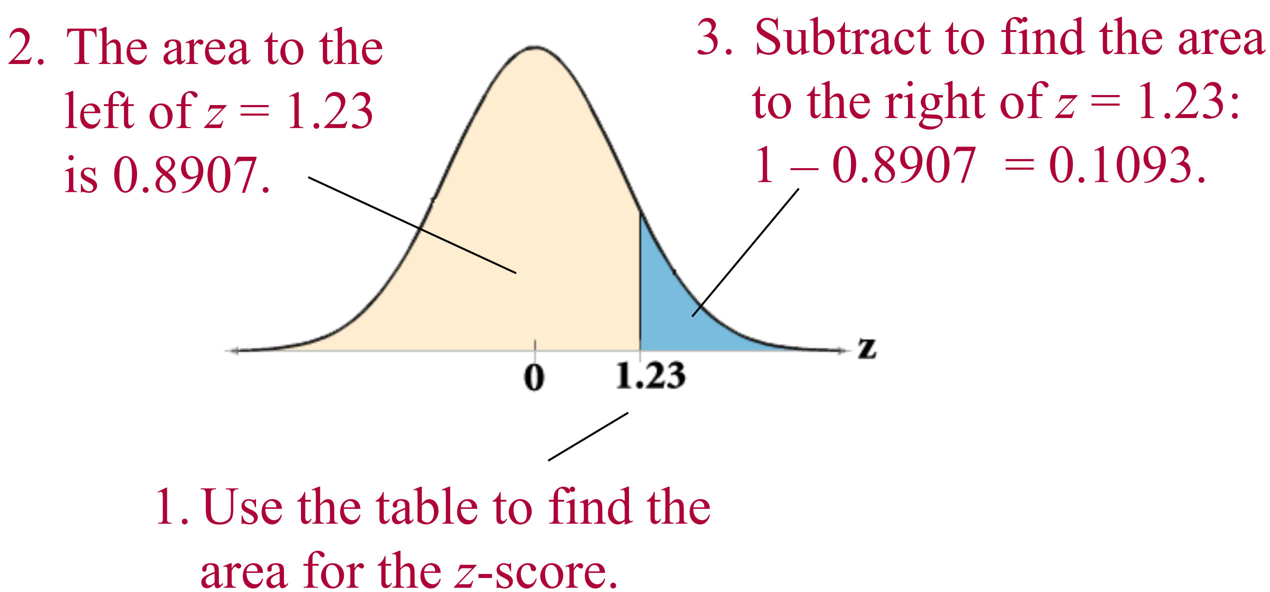
Finding Areas Under the Standard Normal Curve

1. Sketch the standard normal curve and shade the appropriate area under the curve.
2. Find the area by following the directions for each case shown.
   a. To find the area to the *left* of z, find the area that corresponds to z in the Standard Normal Table.



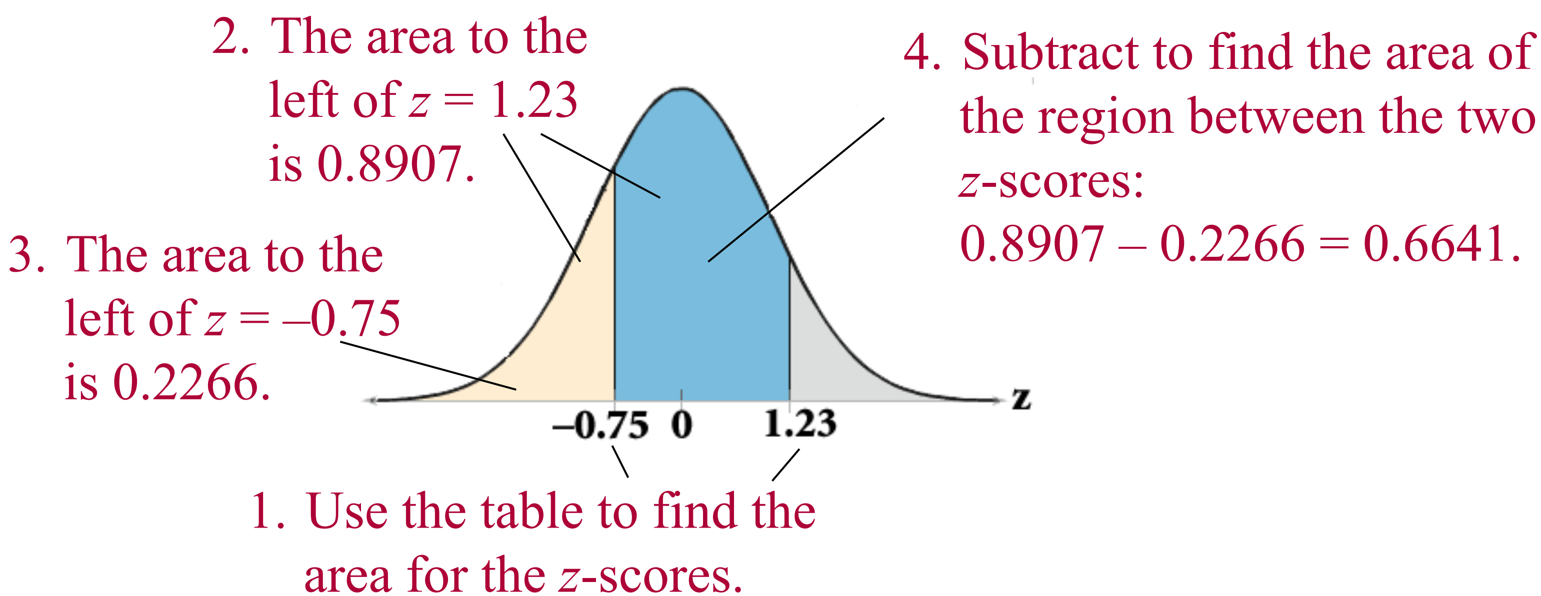
Finding Areas Under the Standard Normal Curve

b. To find the area to the *right* of z, use the Standard Normal Table to find the area that corresponds to z. Then subtract the area from 1.



Finding Areas Under the Standard Normal Curve

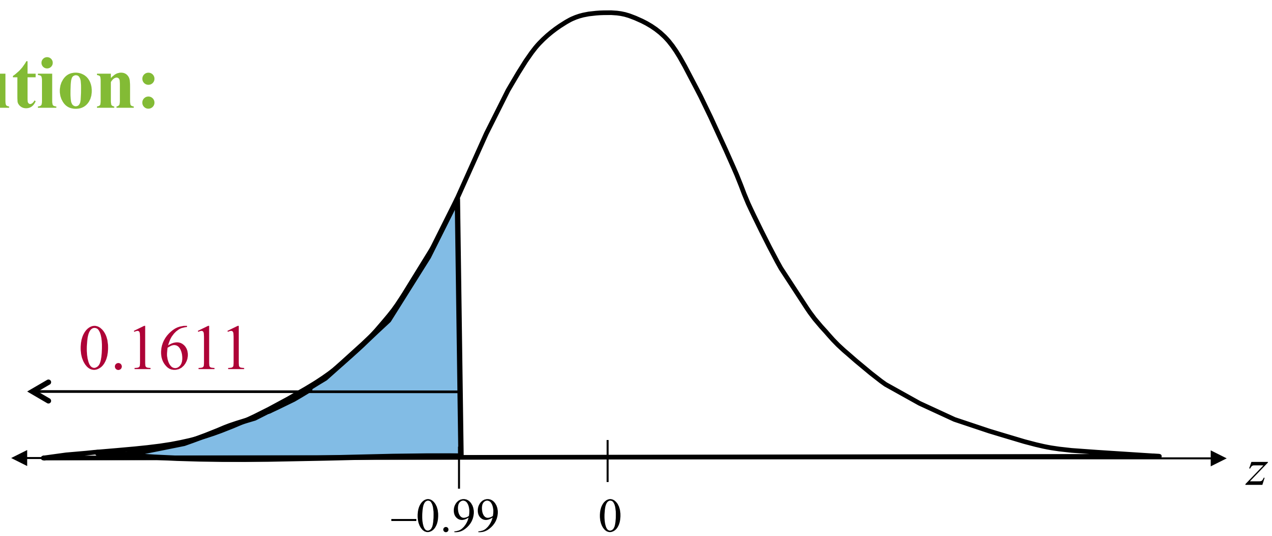
c. To find the area *between* two *z*-scores, find the area corresponding to each *z*-score in the Standard Normal Table. Then subtract the smaller area from the larger area.



Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve to the left of $z = -0.99$.

Solution:

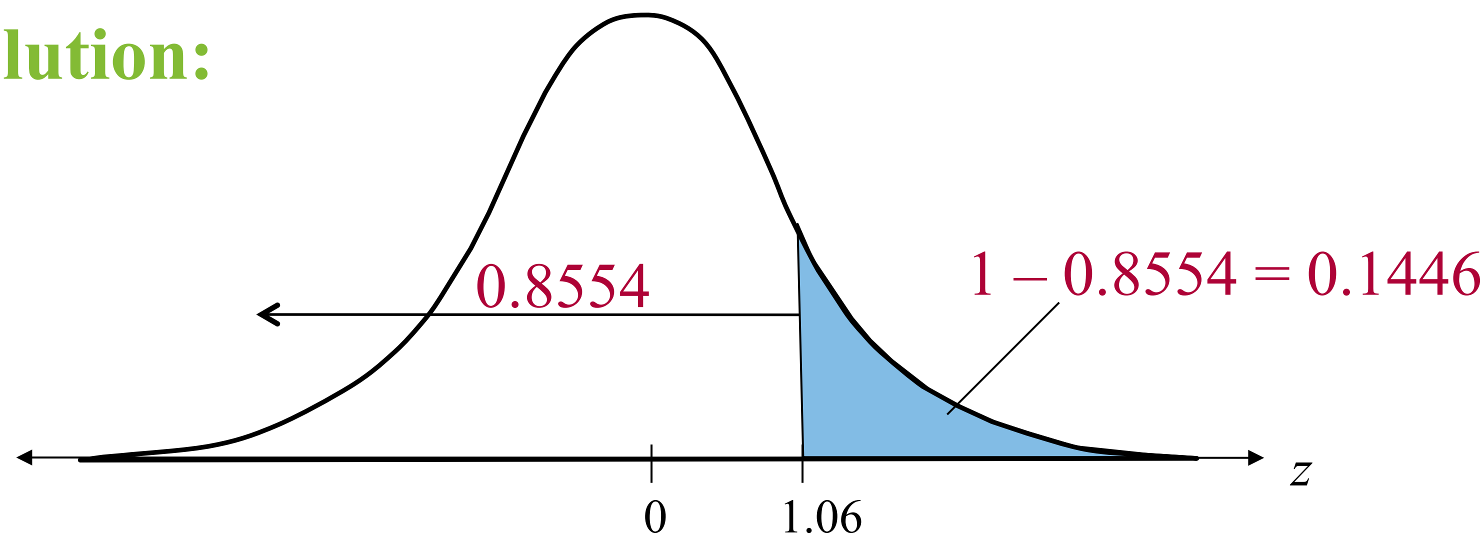


From the Standard Normal Table, the area is equal to 0.1611.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve to the right of $z = 1.06$.

Solution:

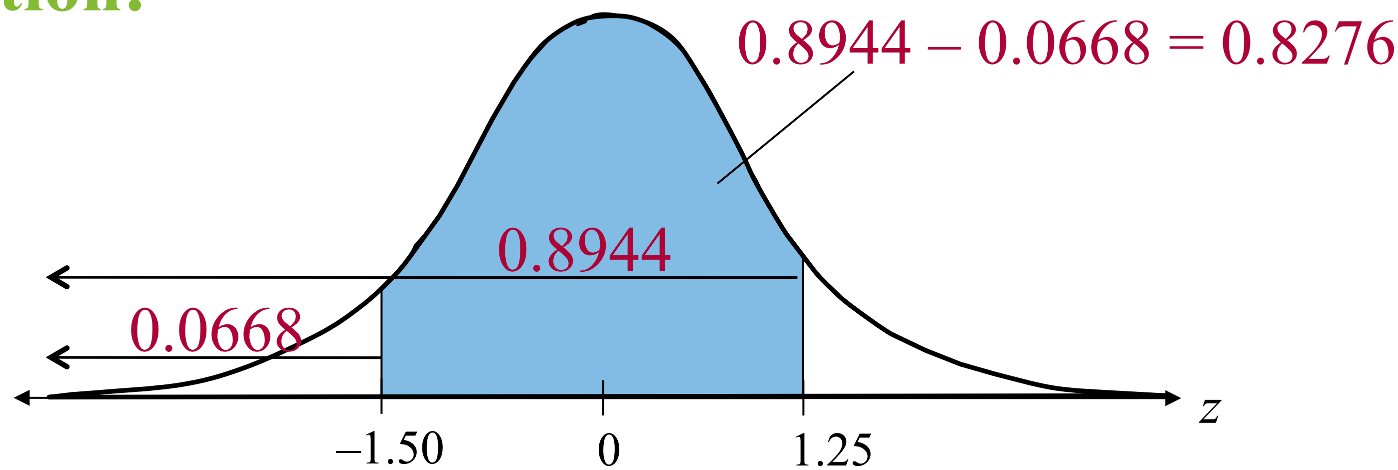


From the Standard Normal Table, the area is equal to 0.1446.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve between $z = -1.5$ and $z = 1.25$.

Solution:



From the Standard Normal Table, the area is equal to 0.8276.

Section 5.1 Summary

- Interpreted graphs of normal probability distributions
- Found areas under the standard normal curve

Section 5.2

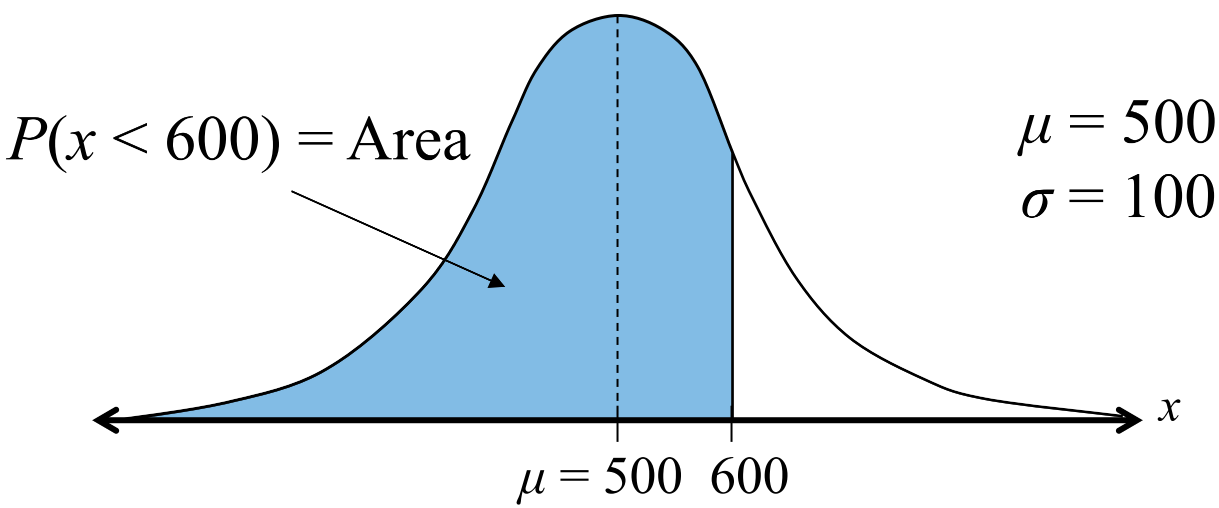
Normal Distributions: Finding Probabilities

Section 5.2 Objectives

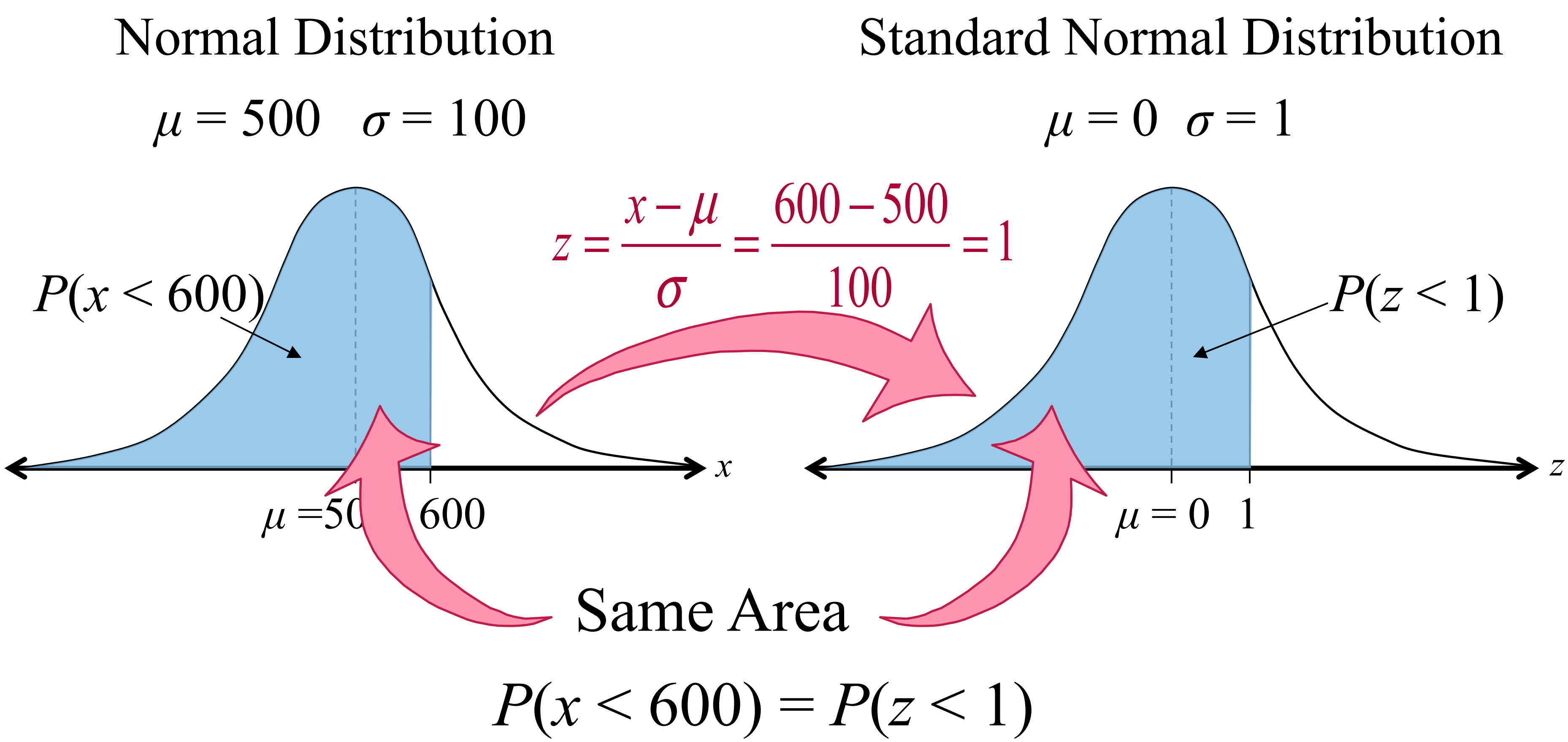
- Find probabilities for normally distributed variables

Probability and Normal Distributions

- If a random variable x is normally distributed, you can find the probability that x will fall in a given interval by calculating the area under the normal curve for that interval.



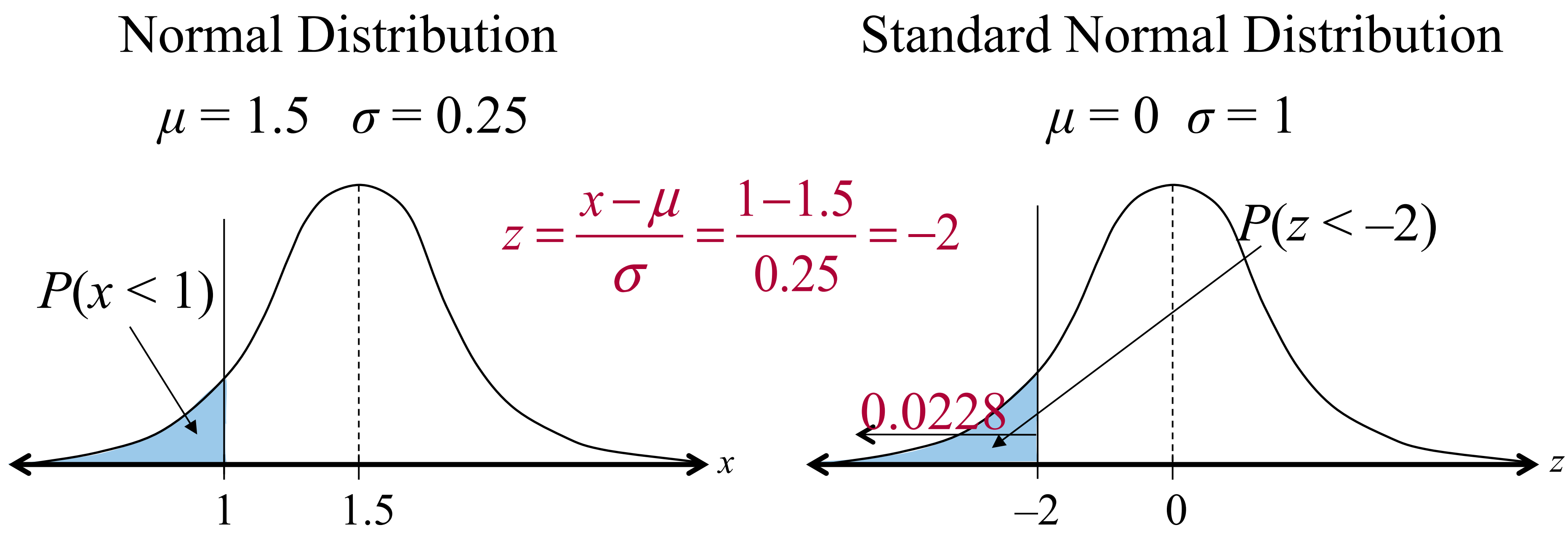
Probability and Normal Distributions



Example: Finding Probabilities for Normal Distributions

A survey indicates that people use their cellular phones an average of 1.5 years before buying a new one. The standard deviation is 0.25 year. A cellular phone user is selected at random. Find the probability that the user will use their current phone for less than 1 year before buying a new one. Assume that the variable x is normally distributed. *(Source: Fonebak)*

Solution: Finding Probabilities for Normal Distributions



$$z = \frac{x - \mu}{\sigma} = \frac{1 - 1.5}{0.25} = -2$$

$$P(x < 1) = \mathbf{0.0228}$$

Example: Finding Probabilities for Normal Distributions

A survey indicates that for each trip to the supermarket, a shopper spends an average of 45 minutes with a standard deviation of 12 minutes in the store. The length of time spent in the store is normally distributed and is represented by the variable x. A shopper enters the store. Find the probability that the shopper will be in the store for between 24 and 54 minutes.

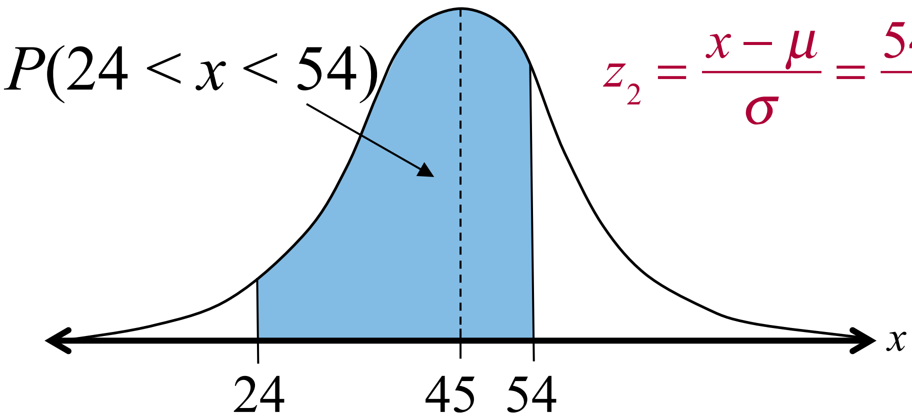
Solution: Finding Probabilities for Normal Distributions

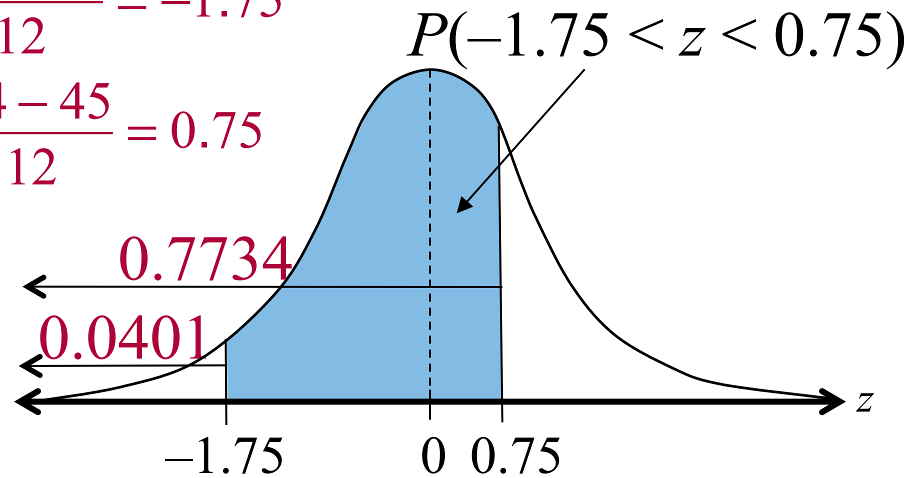
Normal Distribution
$\mu = 45 \quad \sigma = 12$

Standard Normal Distribution
$\mu = 0 \quad \sigma = 1$

$$z_1 = \frac{x-\mu}{\sigma} = \frac{24-45}{12} = -1.75$$

$$z_2 = \frac{x-\mu}{\sigma} = \frac{54-45}{12} = 0.75$$





$$P(24 < x < 54) = P(-1.75 < z < 0.75)$$
$$= 0.7734 - 0.0401 = \mathbf{0.7333}$$

Example: Finding Probabilities for Normal Distributions

If 200 shoppers enter the store, how many shoppers would you expect to be in the store between 24 and 54 minutes?

Solution:

Recall $P(24 < x < 54) = 0.7333$

$200(0.7333) = 146.66$ (or about 147) shoppers

Example: Finding Probabilities for Normal Distributions

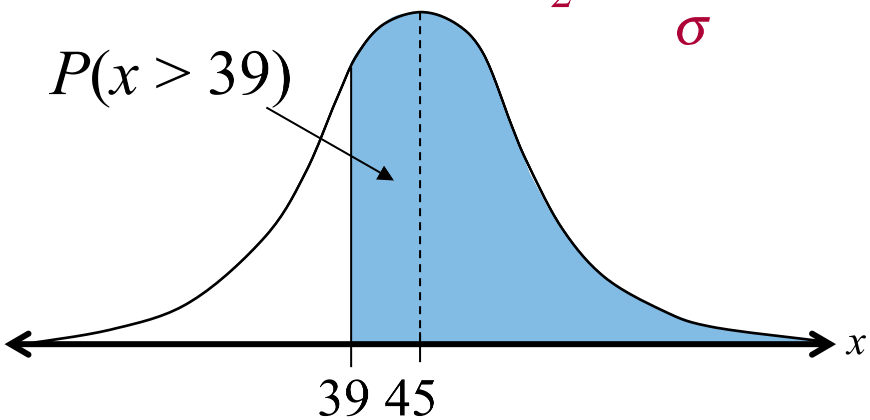
Find the probability that the shopper will be in the store more than 39 minutes. (Recall μ = 45 minutes and σ = 12 minutes)

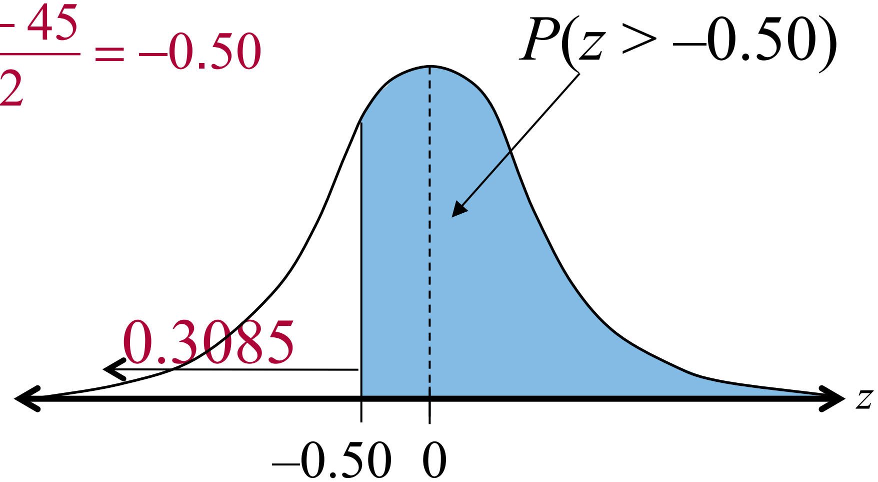
Solution: Finding Probabilities for Normal Distributions

Normal Distribution
$\mu = 45 \quad \sigma = 12$

Standard Normal Distribution
$\mu = 0 \quad \sigma = 1$

$$z = \frac{x - \mu}{\sigma} = \frac{39 - 45}{12} = -0.50$$





$$P(x > 39) = P(z > -0.50) = 1 - 0.3085 = \mathbf{0.6915}$$

Example: Finding Probabilities for Normal Distributions

If 200 shoppers enter the store, how many shoppers would you expect to be in the store more than 39 minutes?

Solution:

Recall $P(x > 39) = 0.6915$

$200(0.6915) = 138.3$ (or about 138) shoppers

Example: Using Technology to find Normal Probabilities

Triglycerides are a type of fat in the bloodstream. The mean triglyceride level in the United States is

134 milligrams per deciliter. Assume the triglyceride levels of the population of the United States are normally distributed with a standard deviation of

35 milligrams per deciliter. You randomly select a person from the United States. What is the probability that the person's triglyceride level is less than 80? Use a technology tool to find the probability.

Solution: Using Technology to find Normal Probabilities

Must specify the mean and standard deviation of the population, and the *x*-value(s) that determine the interval.

MINITAB

Cumulative Distribution Function

Normal with mean = 134 and standard deviation = 35

x	P(X <= x)
80	0.0614327

TI-83/84 PLUS

normalcdf(-10000,80,134,35)
.0614327356

EXCEL

	A	B	C
1	NORMDIST(80,134,35,TRUE)		
2			0.06143272

Section 5.2 Summary

- Found probabilities for normally distributed variables

Section 5.3

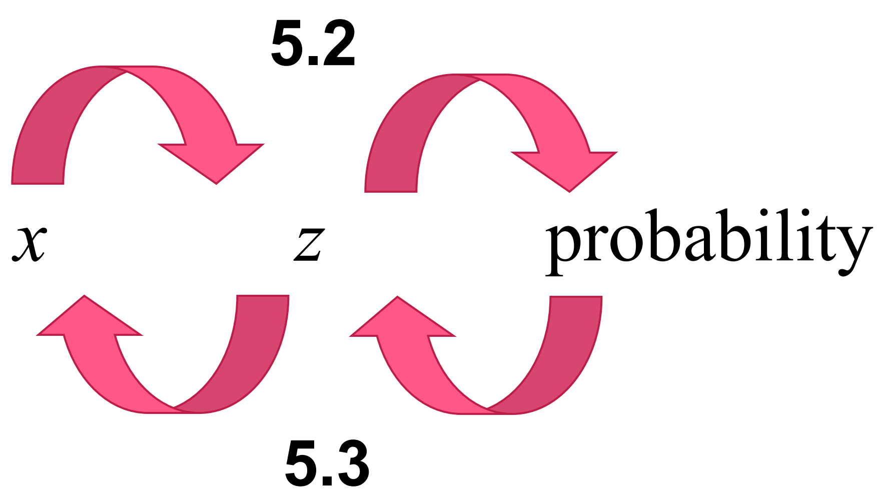
Normal Distributions: Finding Values

Section 5.3 Objectives

- Find a z-score given the area under the normal curve
- Transform a z-score to an x-value
- Find a specific data value of a normal distribution given the probability

Finding values Given a Probability

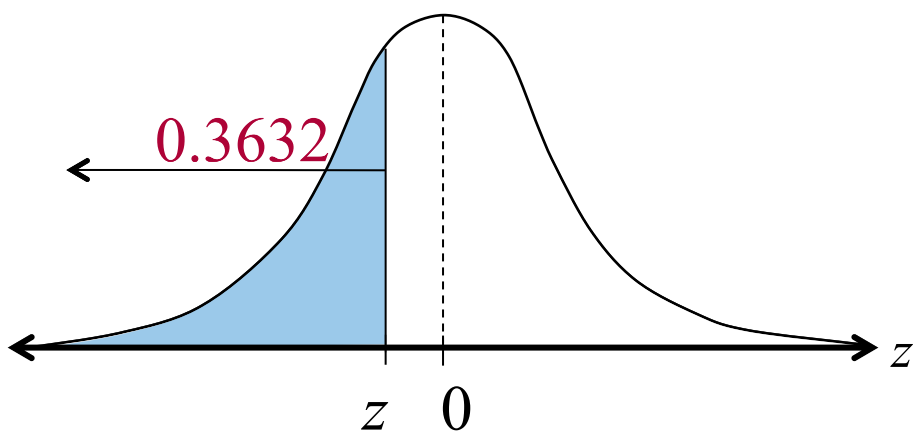
- In section 5.2 we were given a normally distributed random variable x and we were asked to find a probability.
- In this section, we will be given a probability and we will be asked to find the value of the random variable x.



Example: Finding a *z*-Score Given an Area

Find the *z*-score that corresponds to a cumulative area of 0.3632.

Solution:



Solution: Finding a *z*-Score Given an Area

- Locate 0.3632 in the body of the Standard Normal Table.

z	.09	.08	.07	.06	.05	.04	.03
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880

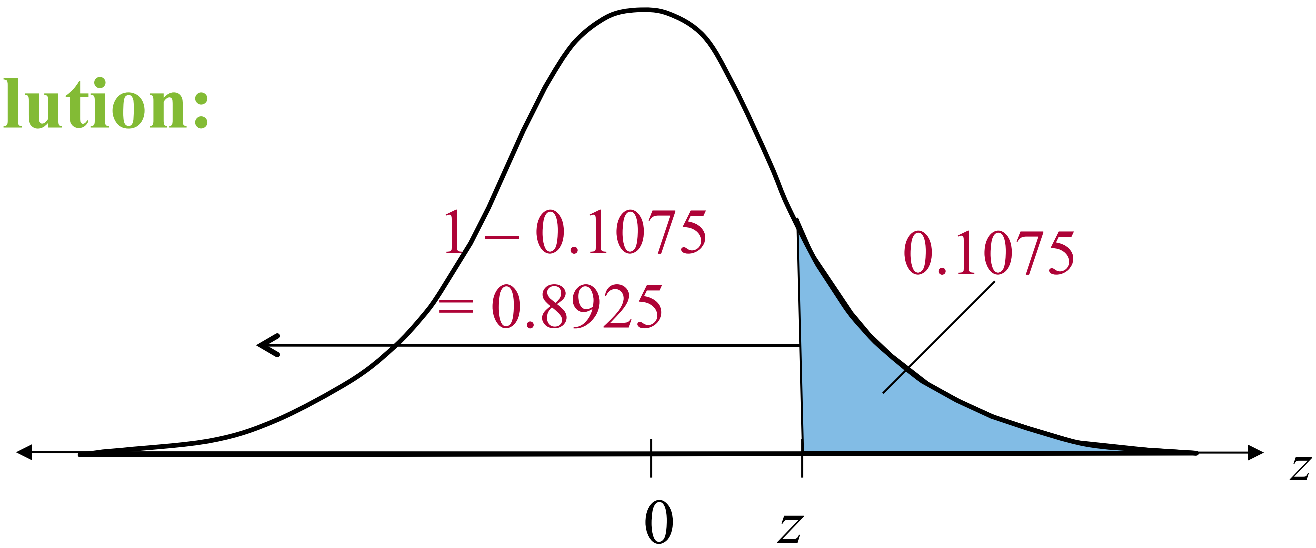
The *z*-score is –0.35.

- The values at the beginning of the corresponding row and at the top of the column give the *z*-score.

Example: Finding a *z*-Score Given an Area

Find the *z*-score that has 10.75% of the distribution's area to its right.

Solution:



Because the area to the right is 0.1075, the cumulative area is 0.8925.

Solution: Finding a *z*-Score Given an Area

- Locate 0.8925 in the body of the Standard Normal Table.

z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279

The *z*-score is 1.24.

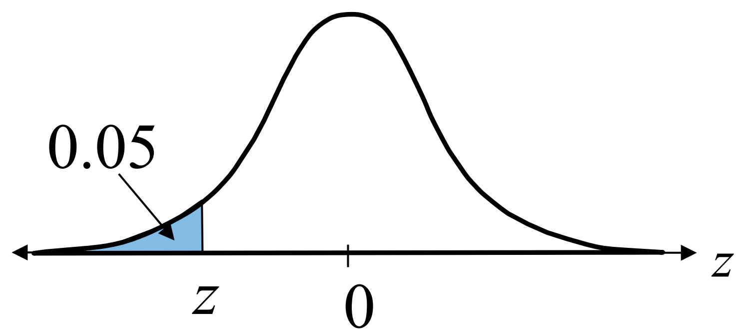
- The values at the beginning of the corresponding row and at the top of the column give the *z*-score.

Example: Finding a z-Score Given a Percentile

Find the z-score that corresponds to P_5.

Solution:

The z-score that corresponds to P_5 is the same z-score that corresponds to an area of 0.05.



The areas closest to 0.05 in the table are 0.0495 (z = –1.65) and 0.0505 (z = –1.64). Because 0.05 is halfway between the two areas in the table, use the z-score that is halfway between –1.64 and –1.65. **The z-score is –1.645**.

Transforming a *z*-Score to an *x*-Score

To transform a standard z-score to a data value x in a given population, use the formula

$$x = \mu + z\sigma$$

Example: Finding an x-Value

A veterinarian records the weights of cats treated at a clinic. The weights are normally distributed, with a mean of 9 pounds and a standard deviation of 2 pounds. Find the weights x corresponding to z-scores of 1.96, –0.44, and 0.

Solution: Use the formula $x = \mu + z\sigma$

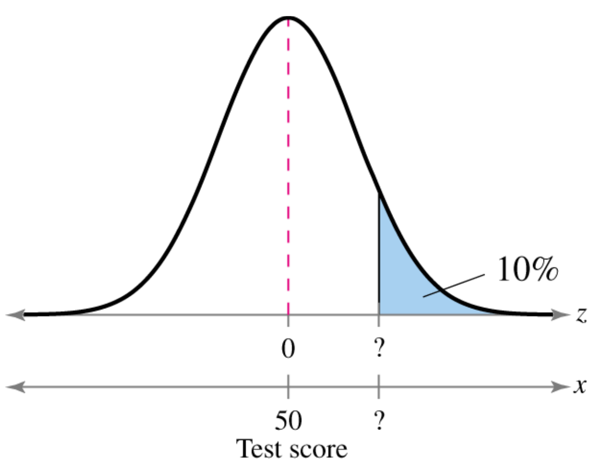
- $z = 1.96$: $x = 9 + 1.96(2) = 12.92$ pounds
- $z = -0.44$: $x = 9 + (-0.44)(2) = 8.12$ pounds
- $z = 0$: $x = 9 + (0)(2) = 9$ pounds

Notice 12.92 pounds is above the mean, 8.12 pounds is below the mean, and 9 pounds is equal to the mean.

Example: Finding a Specific Data Value

Scores for the California Peace Officer Standards and Training test are normally distributed, with a mean of 50 and a standard deviation of 10. An agency will only hire applicants with scores in the top 10%. What is the lowest score you can earn and still be eligible to be hired by the agency?

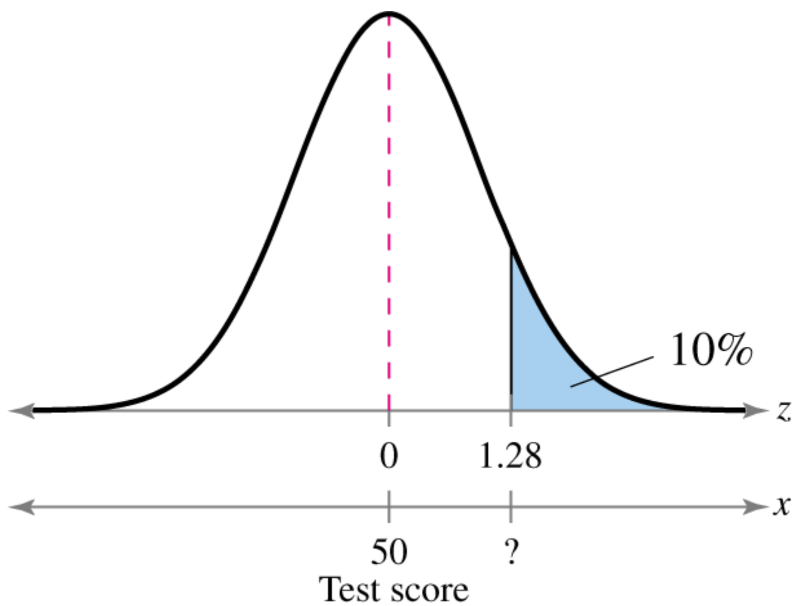
Solution:



An exam score in the top 10% is any score above the 90th percentile. Find the *z*-score that corresponds to a cumulative area of 0.9.

Solution: Finding a Specific Data Value

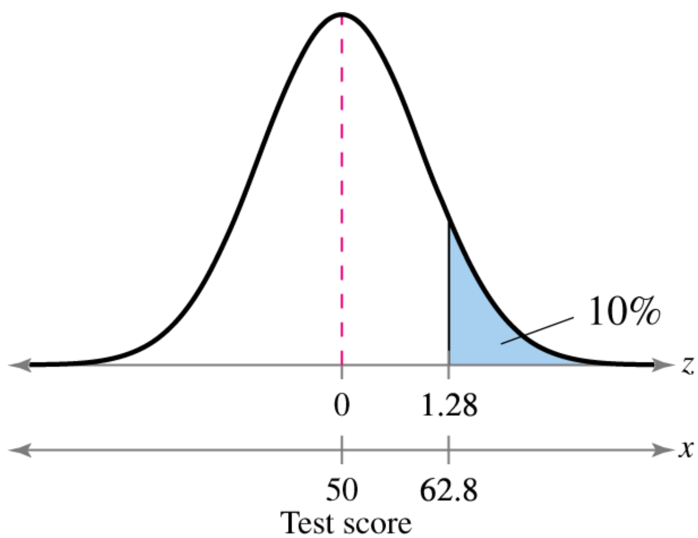
From the Standard Normal Table, the area closest to 0.9 is 0.8997. So the z-score that corresponds to an area of 0.9 is $\boldsymbol{z = 1.28}$.



Solution: Finding a Specific Data Value

Using the equation $x = \mu + z\sigma$

$$x = 50 + 1.28(10) = 62.8$$



The lowest score you can earn and still be eligible to be hired by the agency is about 63.

Section 5.3 Summary

- Found a z-score given the area under the normal curve
- Transformed a z-score to an x-value
- Found a specific data value of a normal distribution given the probability

Section 5.4

Sampling Distributions and the Central Limit Theorem

Section 5.4 Objectives

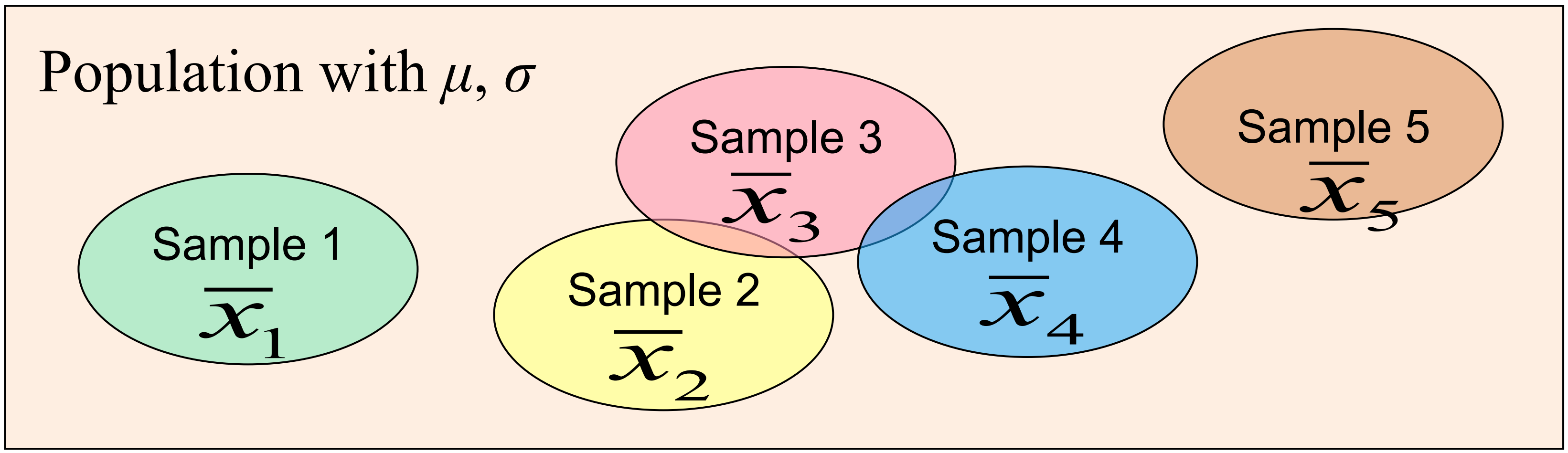
- Find sampling distributions and verify their properties
- Interpret the Central Limit Theorem
- Apply the Central Limit Theorem to find the probability of a sample mean

Sampling Distributions

Sampling distribution

- The probability distribution of a sample statistic.
- Formed when samples of size n are repeatedly taken from a population.
- e.g. Sampling distribution of sample means

Sampling Distribution of Sample Means



The sampling distribution consists of the values of the sample means, $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \bar{x}_5, \ldots$

Properties of Sampling Distributions of Sample Means

1. The mean of the sample means, $\mu_{\bar{x}}$, is equal to the population mean μ.

$$\mu_{\bar{x}} = \mu$$

2. The standard deviation of the sample means, $\sigma_{\bar{x}}$, is equal to the population standard deviation, σ, divided by the square root of the sample size, n.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- Called the **standard error of the mean**.

Example: Sampling Distribution of Sample Means

The population values {1, 3, 5, 7} are written on slips of paper and put in a box. Two slips of paper are randomly selected, with replacement.

a. Find the mean, variance, and standard deviation of the population.

Solution:

Mean: $\mu = \dfrac{\Sigma x}{N} = 4$

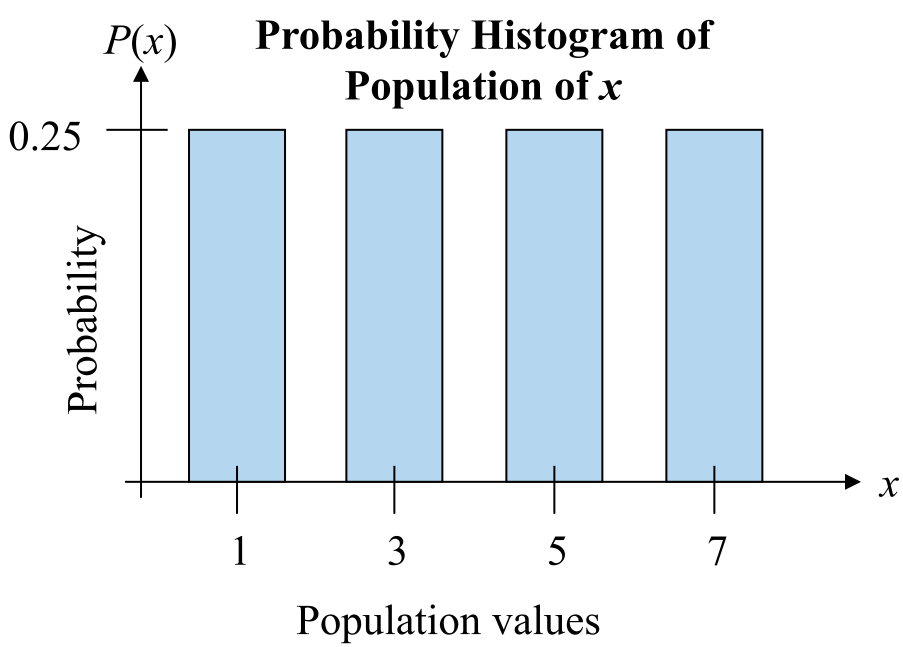
Variance: $\sigma^2 = \dfrac{\Sigma(x-\mu)^2}{N} = 5$

Standard Deviation: $\sigma = \sqrt{5} \approx 2.236$

Example: Sampling Distribution of Sample Means

b. Graph the probability histogram for the population values.

Solution:



All values have the same probability of being selected (uniform distribution)

Example: Sampling Distribution of Sample Means

c. List all the possible samples of size $n = 2$ and calculate the mean of each sample.

Solution:

Sample	$\bar{x}$
1, 1	1
1, 3	2
1, 5	3
1, 7	4
3, 1	2
3, 3	3
3, 5	4
3, 7	5

Sample	$\bar{x}$
5, 1	3
5, 3	4
5, 5	5
5, 7	6
7, 1	4
7, 3	5
7, 5	6
7, 7	7

These means form the sampling distribution of sample means.

Example: Sampling Distribution of Sample Means

d. Construct the probability distribution of the sample means.

Solution:

$\bar{x}$	f	Probability
1	1	0.0625
2	2	0.1250
3	3	0.1875
4	4	0.2500
5	3	0.1875
6	2	0.1250
7	1	0.0625

Example: Sampling Distribution of Sample Means

e. Find the mean, variance, and standard deviation of the sampling distribution of the sample means.

Solution:

The mean, variance, and standard deviation of the 16 sample means are:

$$\mu_{\bar{x}} = 4 \qquad \sigma_{\bar{x}}^2 = \frac{5}{2} = 2.5 \qquad \sigma_{\bar{x}} = \sqrt{2.5} \approx 1.581$$

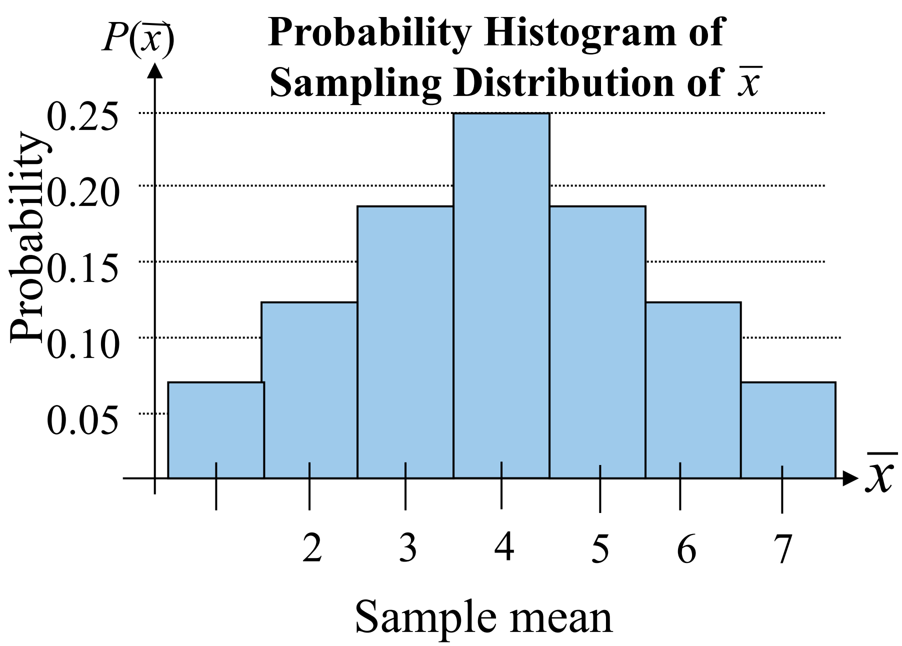
These results satisfy the properties of sampling distributions of sample means.

$$\mu_{\bar{x}} = \mu = 4 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{5}}{\sqrt{2}} \approx \frac{2.236}{\sqrt{2}} \approx 1.581$$

Example: Sampling Distribution of Sample Means

f. Graph the probability histogram for the sampling distribution of the sample means.

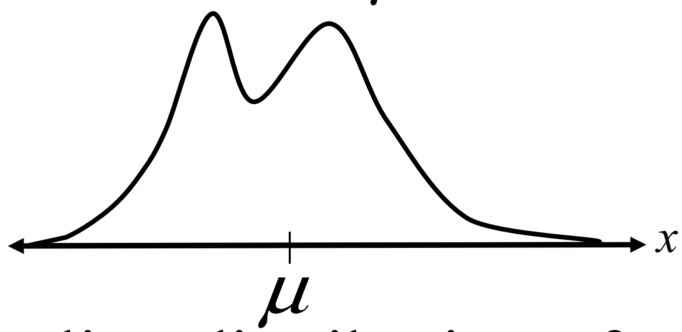
Solution:



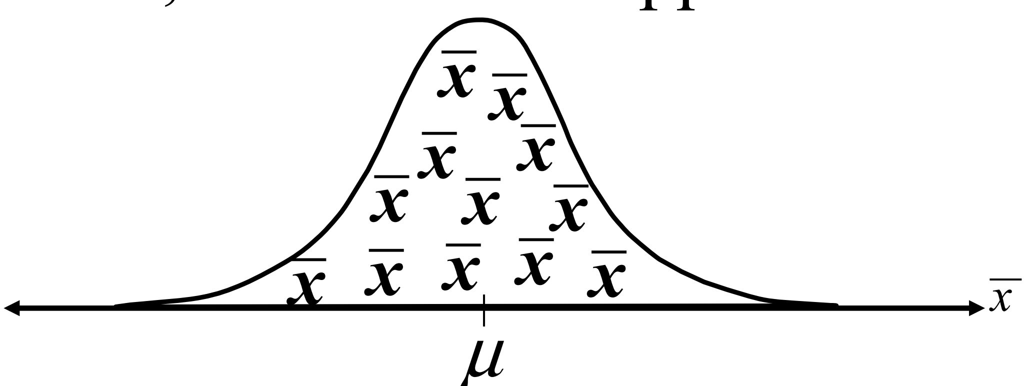
The shape of the graph is symmetric and bell shaped. It approximates a normal distribution.

The Central Limit Theorem

1. If samples of size $n \geq 30$ are drawn from any population with mean = μ and standard deviation = σ,

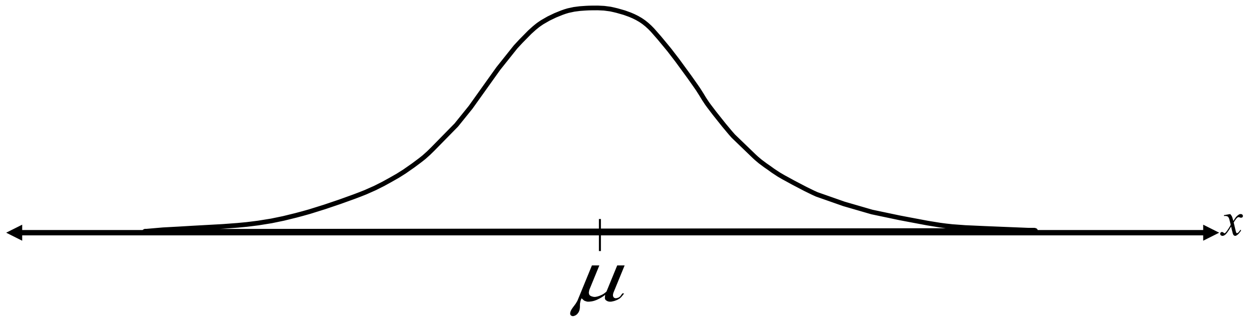


then the sampling distribution of sample means approximates a normal distribution. The greater the sample size, the better the approximation.

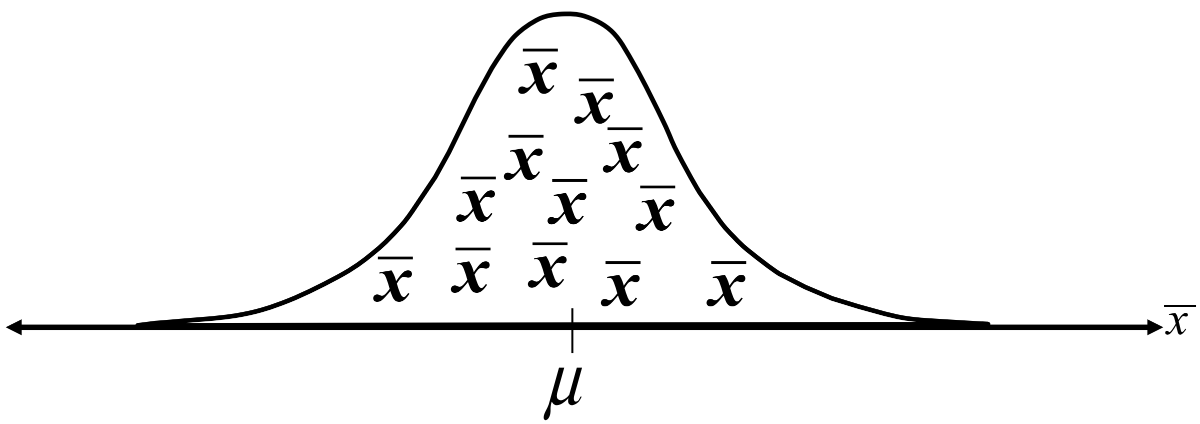


The Central Limit Theorem

2. If the population itself is normally distributed,



then the sampling distribution of sample means is normally distribution for ***any*** sample size *n*.



The Central Limit Theorem

- In either case, the sampling distribution of sample means has a mean equal to the population mean.

$$\mu_{\bar{x}} = \mu \qquad \text{Mean}$$

- The sampling distribution of sample means has a variance equal to $1/n$ times the variance of the population and a standard deviation equal to the population standard deviation divided by the square root of n.

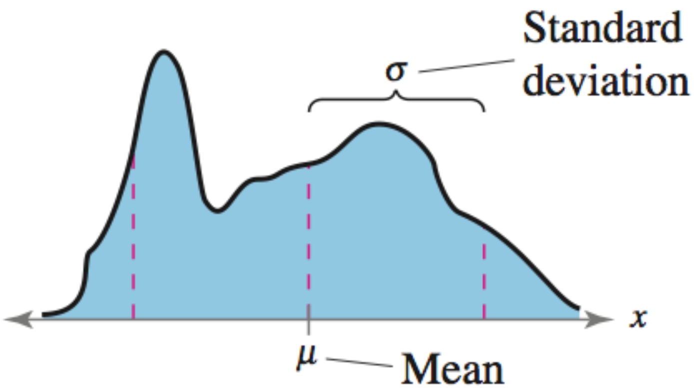
$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \qquad \text{Variance}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

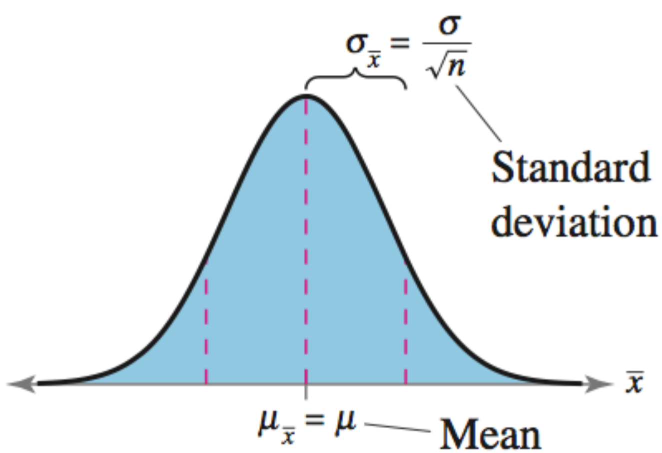
Standard deviation (**standard error of the mean)**

The Central Limit Theorem

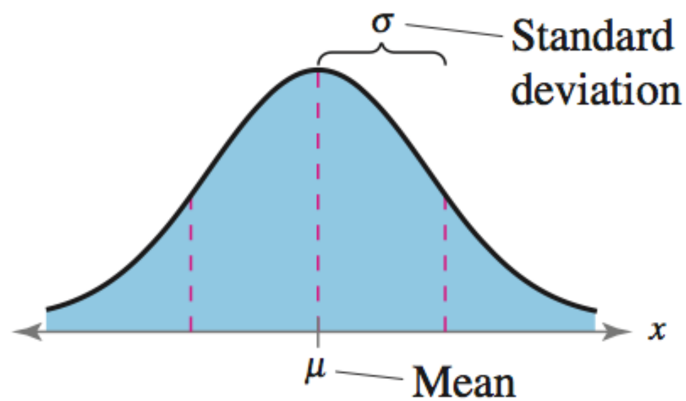
1. Any Population Distribution



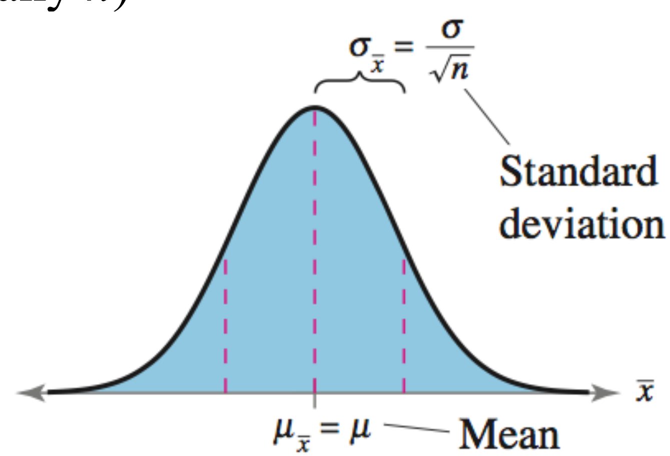
Distribution of Sample Means, $n \geq 30$



2. Normal Population Distribution

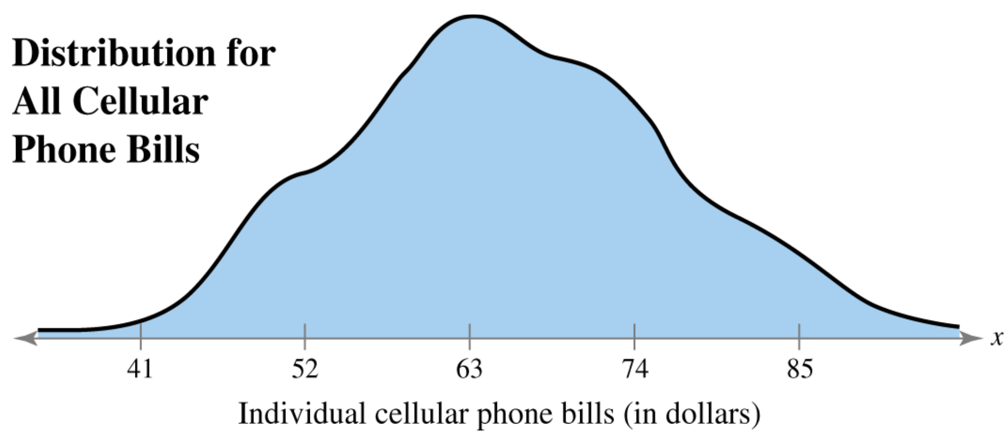


Distribution of Sample Means, (any n)



Example: Interpreting the Central Limit Theorem

Cellular phone bills for residents of a city have a mean of \$63 and a standard deviation of \$11. Random samples of 100 cellular phone bills are drawn from this population and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling distribution of sample means.



Solution: Interpreting the Central Limit Theorem

- The mean of the sampling distribution is equal to the population mean

$$\mu_{\bar{x}} = \mu = 63$$

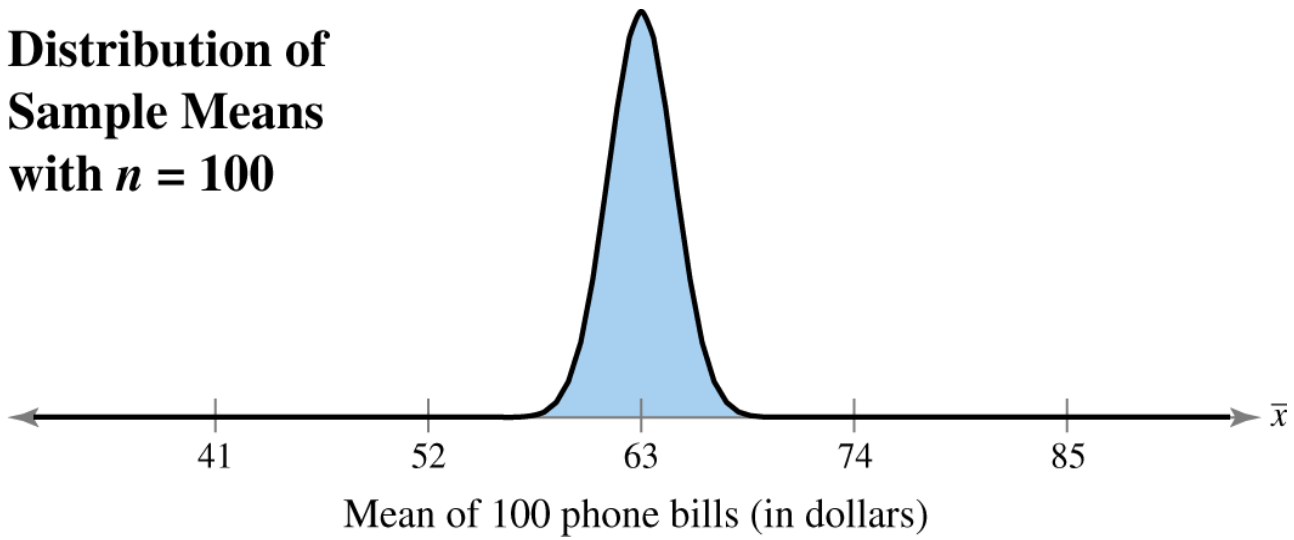
- The standard error of the mean is equal to the population standard deviation divided by the square root of n.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{11}{\sqrt{100}} = 1.1$$

Solution: Interpreting the Central Limit Theorem

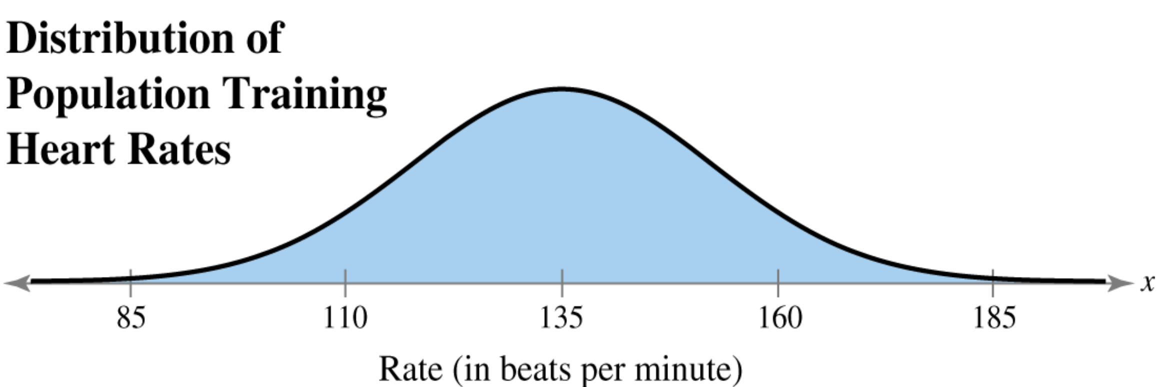
- Since the sample size is greater than 30, the sampling distribution can be approximated by a normal distribution with

$$\mu_{\bar{x}} = \$63 \qquad \sigma_{\bar{x}} = \$1.10$$



Example: Interpreting the Central Limit Theorem

Suppose the training heart rates of all 20-year-old athletes are normally distributed, with a mean of 135 beats per minute and standard deviation of 18 beats per minute. Random samples of size 4 are drawn from this population, and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling distribution of sample means.



Solution: Interpreting the Central Limit Theorem

- The mean of the sampling distribution is equal to the population mean

$$\mu_{\bar{x}} = \mu = 135$$

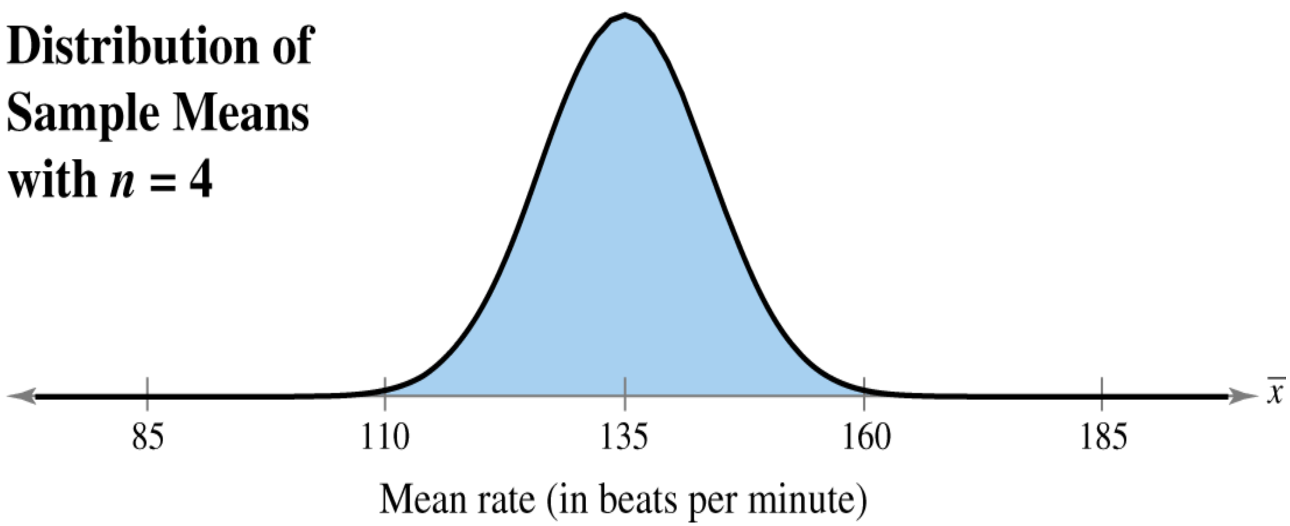
- The standard error of the mean is equal to the population standard deviation divided by the square root of n.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{4}} = 9$$

Solution: Interpreting the Central Limit Theorem

- Since the population is normally distributed, the sampling distribution of the sample means is also normally distributed.

$$\mu_{\bar{x}} = 135 \qquad \sigma_{\bar{x}} = 9$$



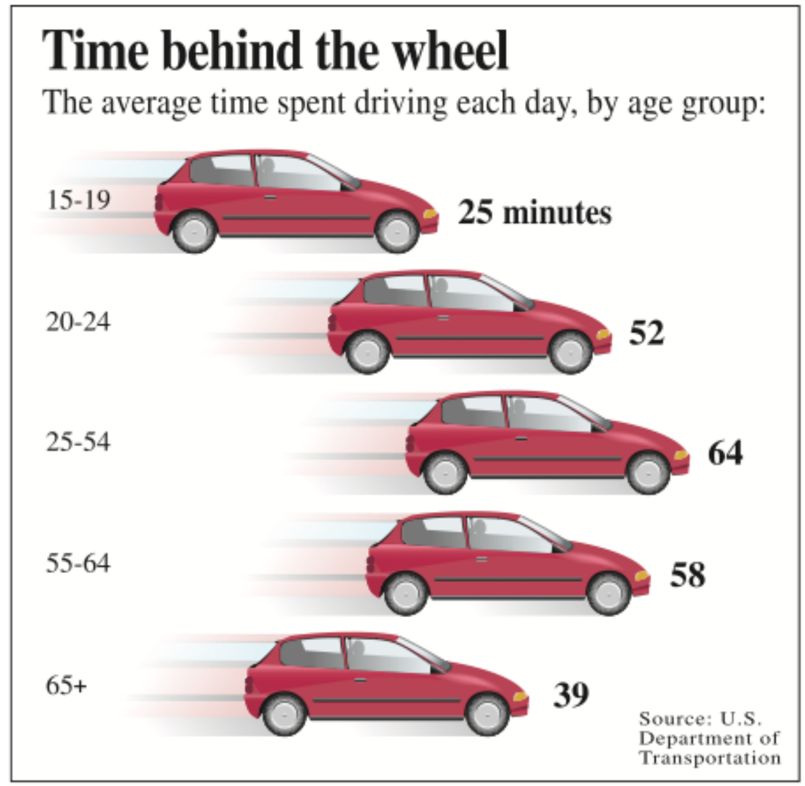
Probability and the Central Limit Theorem

- To transform $\bar{x}$ to a z-score

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard error}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Example: Probabilities for Sampling Distributions

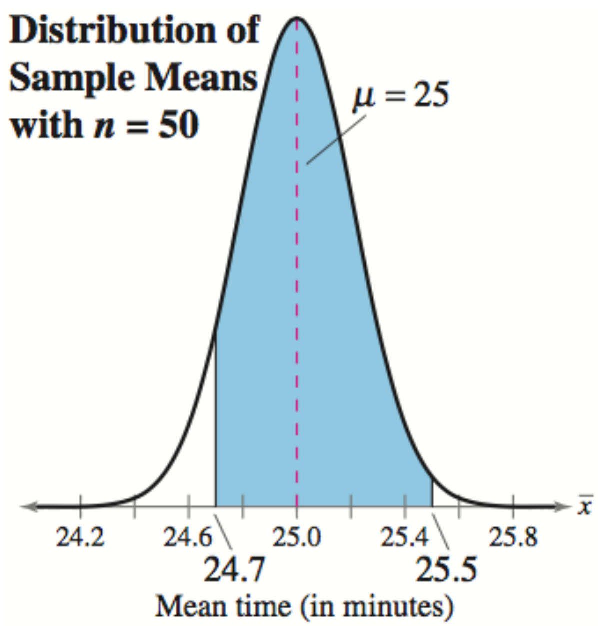
The graph shows the length of time people spend driving each day. You randomly select 50 drivers ages 15 to 19. What is the probability that the mean time they spend driving each day is between 24.7 and 25.5 minutes? Assume that $\sigma = 1.5$ minutes.



Solution: Probabilities for Sampling Distributions

From the Central Limit Theorem (sample size is greater than 30), the sampling distribution of sample means is approximately normal with

$$\mu_{\bar{x}} = \mu = 25 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{50}} \approx 0.21213$$



Solution: Probabilities for Sampling Distributions

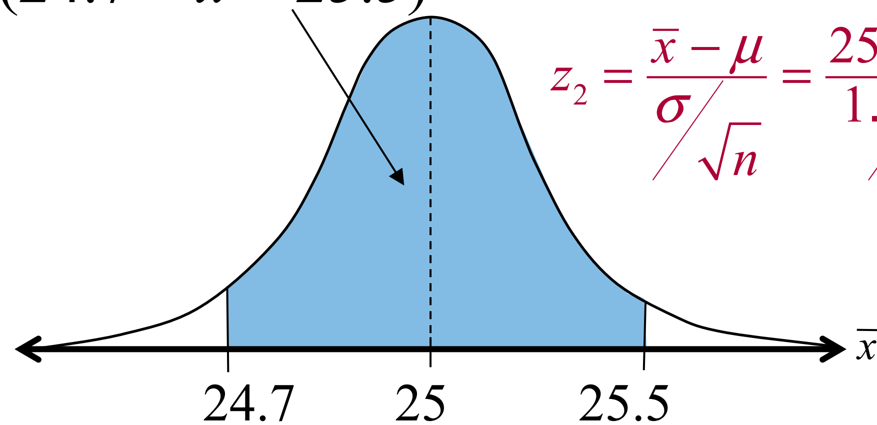
Normal Distribution
$\mu = 25 \quad \sigma = 0.21213$

Standard Normal Distribution
$\mu = 0 \quad \sigma = 1$

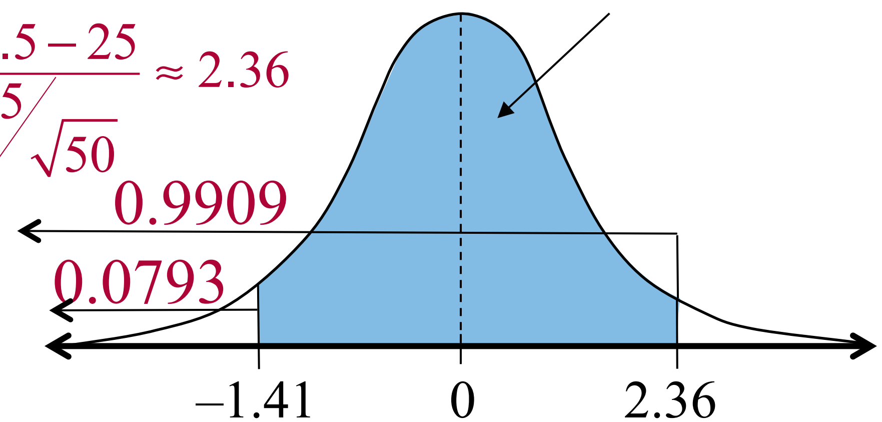
$$z_1 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{24.7 - 25}{1.5/\sqrt{50}} \approx -1.41$$

$$z_2 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{25.5 - 25}{1.5/\sqrt{50}} \approx 2.36$$

$P(24.7 < \bar{x} < 25.5)$



$P(-1.41 < z < 2.36)$

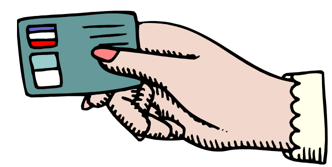


$$P(24 < \bar{x} < 54) = P(-1.41 < z < 2.36)$$
$$= 0.9909 - 0.0793 = \mathbf{0.9116}$$

Example: Probabilities for *x* and $\bar{x}$

An education finance corporation claims that the average credit card debts carried by undergraduates are normally distributed, with a mean of \$3173 and a standard deviation of \$1120. *(Adapted from Sallie Mae)*

1. What is the probability that a randomly selected undergraduate, who is a credit card holder, has a credit card balance less than \$2700?



Solution:

You are asked to find the probability associated with a certain value of the random variable *x*.

Solution: Probabilities for *x* and $\bar{x}$

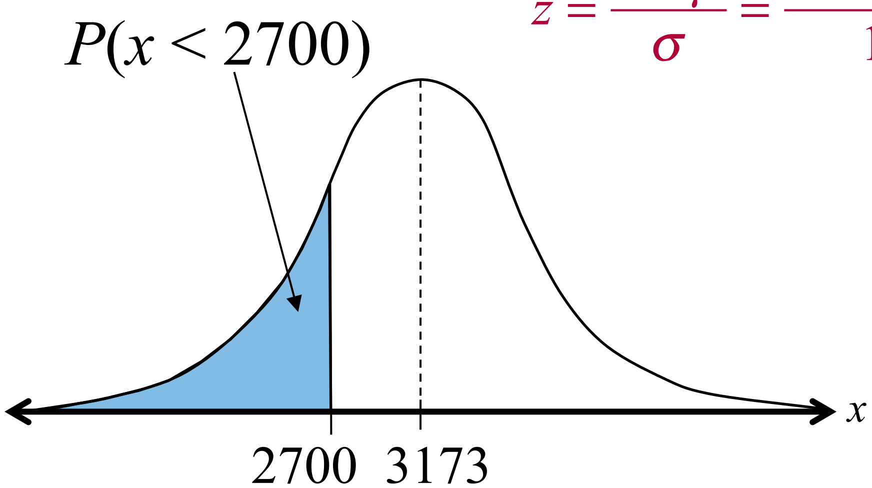
Normal Distribution

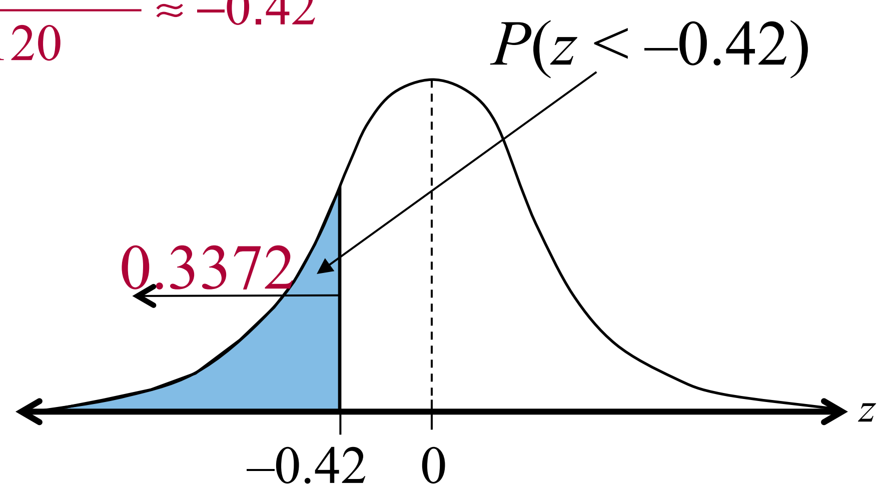
$\mu = 3173 \quad \sigma = 1120$

Standard Normal Distribution

$\mu = 0 \quad \sigma = 1$

$$z = \frac{x - \mu}{\sigma} = \frac{2700 - 3173}{1120} \approx -0.42$$

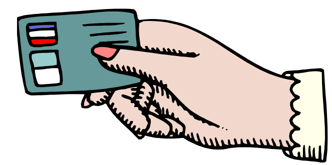




$$P(x < 2700) = P(z < -0.42) = \mathbf{0.3372}$$

Example: Probabilities for x and $\bar{x}$

2. You randomly select 25 undergraduates who are credit card holders. What is the probability that their mean credit card balance is less than \$2700?



Solution:

You are asked to find the probability associated with a sample mean $\bar{x}$.

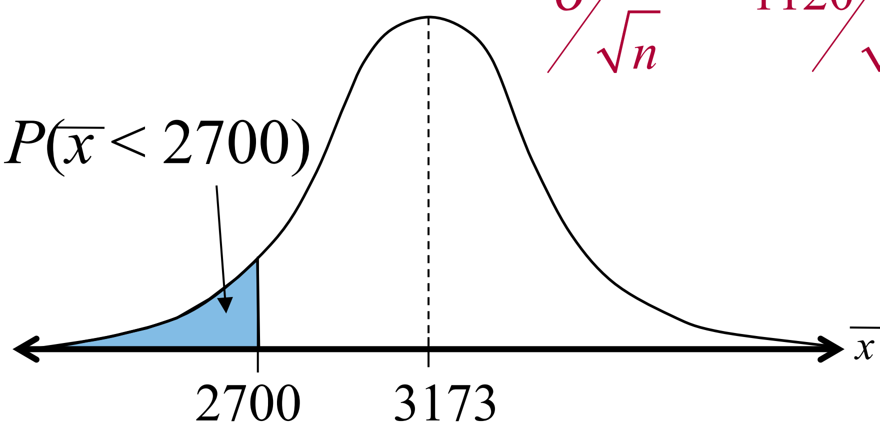
$$\mu_{\bar{x}} = \mu = 3173 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1120}{\sqrt{25}} = 224$$

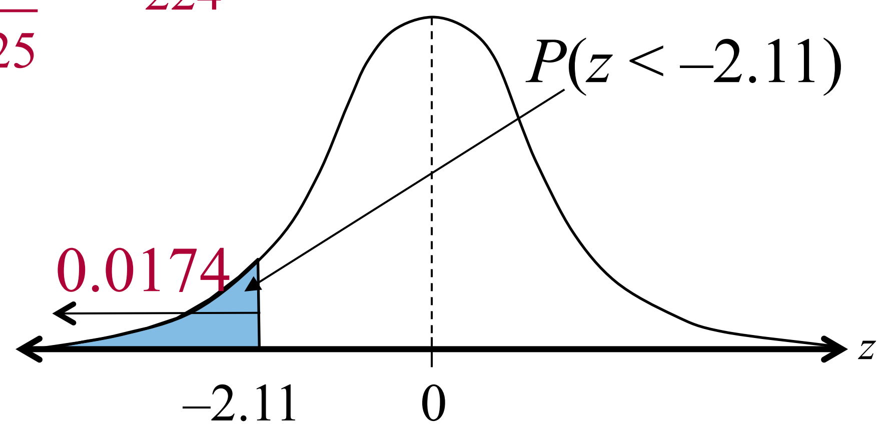
Solution: Probabilities for *x* and $\overline{x}$

Normal Distribution
$\mu = 3173 \quad \sigma = 1120$

Standard Normal Distribution
$\mu = 0 \quad \sigma = 1$

$$z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} = \frac{2700 - 3173}{1120 / \sqrt{25}} = \frac{-473}{224} \approx -2.11$$





$$P(\overline{x} < 2700) = P(z < -2.11) = \mathbf{0.0174}$$

Solution: Probabilities for *x* and $\bar{x}$

- There is about a 34% chance that an undergraduate will have a balance less than $2700.
- There is only about a 2% chance that the mean of a sample of 25 will have a balance less than $2700 (unusual event).
- It is possible that the sample is unusual or it is possible that the corporation's claim that the mean is $3173 is incorrect.

Section 5.4 Summary

- Found sampling distributions and verified their properties
- Interpreted the Central Limit Theorem
- Applied the Central Limit Theorem to find the probability of a sample mean

Section 5.5

Normal Approximations to Binomial Distributions

Section 5.5 Objectives

- Determine when the normal distribution can approximate the binomial distribution
- Find the continuity correction
- Use the normal distribution to approximate binomial probabilities

Normal Approximation to a Binomial

- The normal distribution is used to approximate the binomial distribution when it would be impractical to use the binomial distribution to find a probability.

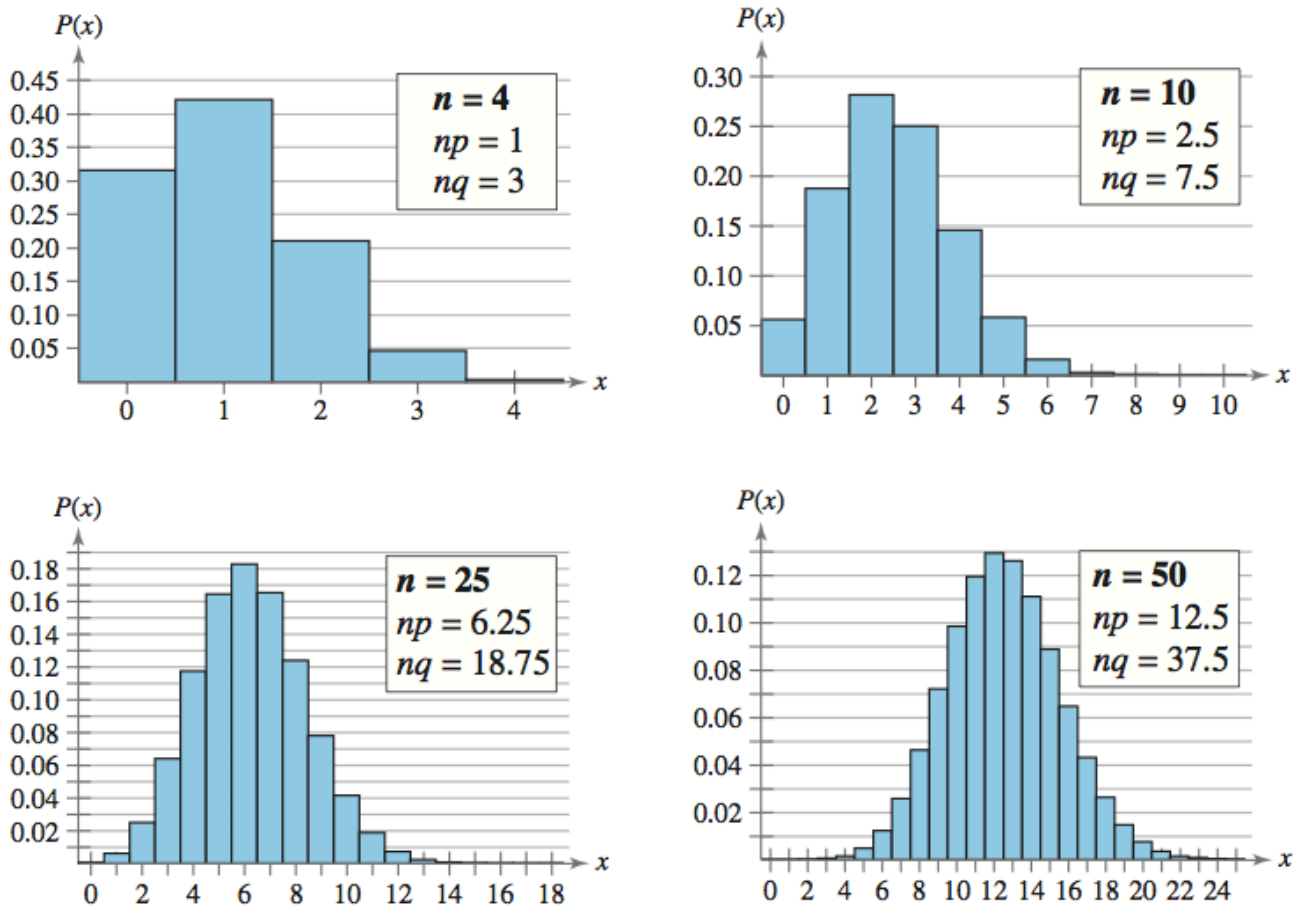
Normal Approximation to a Binomial Distribution

- If $np \geq 5$ and $nq \geq 5$, then the binomial random variable x is approximately normally distributed with
  - mean $\mu = np$
  - standard deviation $\sigma = \sqrt{npq}$

  where n is the number of independent trials, p is the probability of success in a single trial, and q is the probability of failure in a single trial.

Normal Approximation to a Binomial

- Binomial distribution: $p = 0.25$



- As n increases the histogram approaches a normal curve.

Example: Approximating the Binomial

Decide whether you can use the normal distribution to approximate x, the number of people who reply yes. If you can, find the mean and standard deviation.

1. Sixty-two percent of adults in the U.S. have an HDTV in their home. You randomly select 45 adults in the U.S. and ask them if they have an HDTV in their home.

Solution: Approximating the Binomial

- You can use the normal approximation

$n = 45, \quad p = 0.62, \quad q = 0.38$

$$np = (45)(0.62) = 27.9$$

$$nq = (45)(0.38) = 17.1$$

- Mean: $\mu = np = 27.9$
- Standard Deviation: $\sigma = \sqrt{npq} = \sqrt{45 \cdot 0.62 \cdot 0.38} \approx 3.26$

Example: Approximating the Binomial

Decide whether you can use the normal distribution to approximate x, the number of people who reply yes. If you can, find the mean and standard deviation.

2. Twelve percent of adults in the U.S. who do not have an HDTV in their home are planning to purchase one in the next two years. You randomly select 30 adults in the U.S. who do not have an HDTV and ask them if they are planning to purchase one in the next two years.

Solution: Approximating the Binomial

- You cannot use the normal approximation

  $n = 30, \quad p = 0.12, \quad q = 0.88$

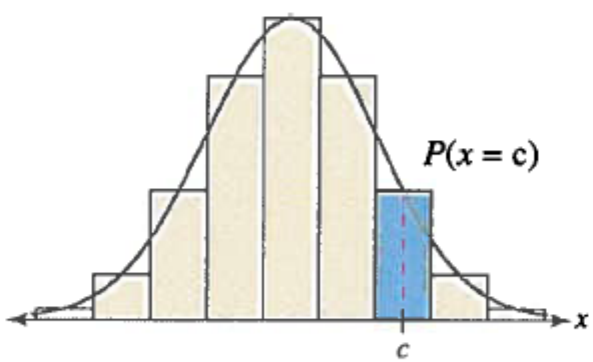
  $np = (30)(0.12) = 3.6$

  $nq = (30)(0.88) = 26.4$

- Because $np < 5$, you cannot use the normal distribution to approximate the distribution of x.

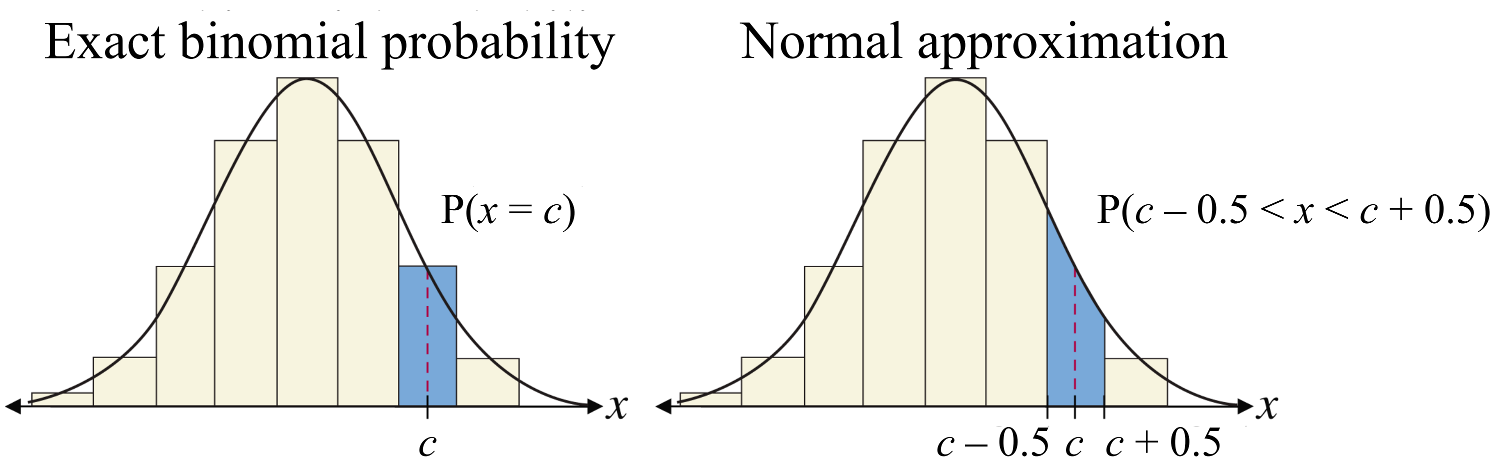
Correction for Continuity

- The binomial distribution is discrete and can be represented by a probability histogram.
- To calculate *exact* binomial probabilities, the binomial formula is used for each value of x and the results are added.
- Geometrically this corresponds to adding the areas of bars in the probability histogram.



Correction for Continuity

- When you use a *continuous* normal distribution to approximate a binomial probability, you need to move 0.5 unit to the left and right of the midpoint to include all possible x-values in the interval (**continuity correction**).



Example: Using a Correction for Continuity

Use a continuity correction to convert the binomial interval to a normal distribution interval.

1. The probability of getting between 270 and 310 successes, inclusive.

Solution:

- The discrete midpoint values are 270, 271, …, 310.
- The corresponding interval for the continuous normal distribution is

$$269.5 < x < 310.5$$

Example: Using a Correction for Continuity

Use a continuity correction to convert the binomial interval to a normal distribution interval.

2. The probability of getting at least 158 successes.

Solution:

- The discrete midpoint values are 158, 159, 160, ….
- The corresponding interval for the continuous normal distribution is

$$x > 157.5$$

Example: Using a Correction for Continuity

Use a continuity correction to convert the binomial interval to a normal distribution interval.

3. The probability of getting fewer than 63 successes.

Solution:

- The discrete midpoint values are …, 60, 61, 62.
- The corresponding interval for the continuous normal distribution is

$$x < 62.5$$

Using the Normal Distribution to Approximate Binomial Probabilities

In Words	*In Symbols*
1. Verify that the binomial distribution applies.	Specify n, p, and q.
2. Determine if you can use the normal distribution to approximate x, the binomial variable.	Is $np \geq 5$? Is $nq \geq 5$?
3. Find the mean μ and standard deviation σ for the distribution.	$\mu = np$ $\sigma = \sqrt{npq}$

Using the Normal Distribution to Approximate Binomial Probabilities

In Words	*In Symbols*
4. Apply the appropriate continuity correction. Shade the corresponding area under the normal curve.	Add or subtract 0.5 from endpoints.
5. Find the corresponding z-score(s).	$z = \dfrac{x - \mu}{\sigma}$
6. Find the probability.	Use the Standard Normal Table.

Example: Approximating a Binomial Probability

Sixty-two percent of adults in the U.S. have an HDTV in their home. You randomly select 45 adults in the U.S. and ask them if they have an HDTV in their home. What is the probability that fewer than 20 of them respond yes? *(Source: Opinion Research Corporation)*

Solution:

- Can use the normal approximation (see slide 91)

$$\mu = 64\,(0.62) = 27.9 \qquad \sigma = \sqrt{45 \cdot 0.62 \cdot 0.38} \approx 3.26$$

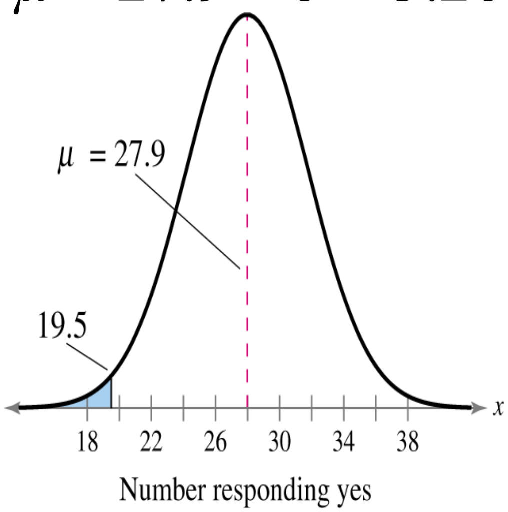
Solution: Approximating a Binomial Probability

- Apply the continuity correction:

  Fewer than 20 (…17, 18, 19) corresponds to the continuous normal distribution interval $\boldsymbol{x < 19.5}$.

Normal Distribution

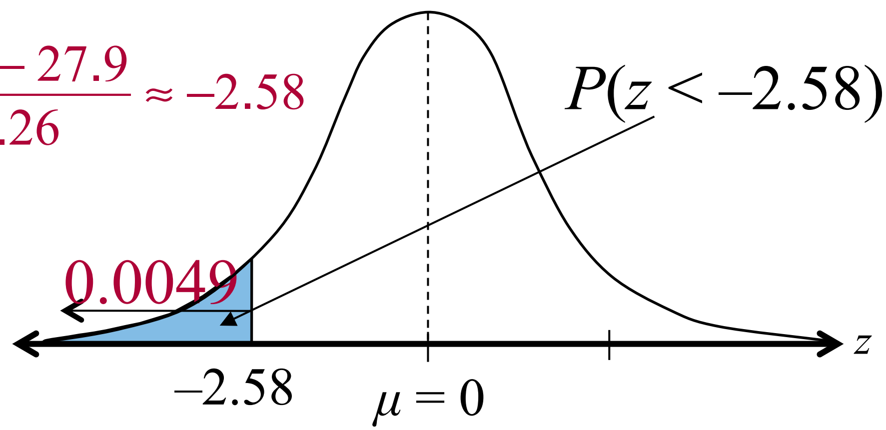
$\mu = 27.9 \quad \sigma \approx 3.26$



$$z = \frac{x - \mu}{\sigma} = \frac{19.5 - 27.9}{3.26} \approx -2.58$$

Standard Normal

$\mu = 0 \quad \sigma = 1$



$P(z < -2.58) = \mathbf{0.0049}$

Example: Approximating a Binomial Probability

A survey reports that 62% of Internet users use Windows® Internet Explorer® as their browser. You randomly select 150 Internet users and ask them whether they use Internet Explorer® as their browser. What is the probability that exactly 96 will say yes? *(Source: Net Applications)*

Solution:

- Can use the normal approximation

$$np = 150 \cdot 0.62 = 93 \geq 5 \quad nq = 150 \cdot 0.38 = 57 \geq 5$$

$$\mu = 150 \cdot 0.62 = 93 \qquad \sigma = \sqrt{150 \cdot 0.62 \cdot 0.38} \approx 5.94$$

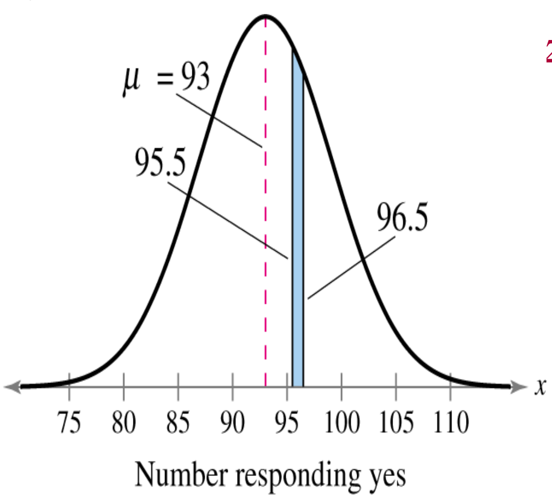
Solution: Approximating a Binomial Probability

- Apply the continuity correction:

  Rewrite the discrete probability P(x=96) as the continuous probability **P(95.5 < *x* < 96.5)**.

Normal Distribution

$\mu = 27.9 \quad \sigma = 3.26$

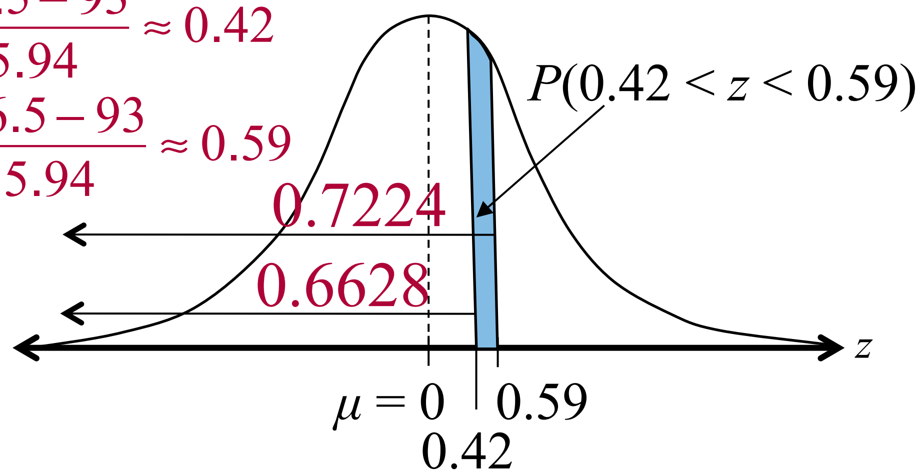


$$z_1 = \frac{x-\mu}{\sigma} = \frac{95.5-93}{5.94} \approx 0.42$$

$$z_2 = \frac{x-\mu}{\sigma} = \frac{96.5-93}{5.94} \approx 0.59$$

Standard Normal

$\mu = 0 \quad \sigma = 1$



$$P(0.42 < z < 0.59) = 0.7224 - 0.6628 = \mathbf{0.0596}$$

Section 5.5 Summary

- Determined when the normal distribution can approximate the binomial distribution
- Found the continuity correction
- Used the normal distribution to approximate binomial probabilities