1. Let $\vec{r}(t) = \langle t^2, t^2, t^3 \rangle$ represent the position of an object.

   (a) Find the acceleration of the object at the point $(1, 1, -1)$.

$\vec{v}(t) = \vec{r}'(t) = \langle 2t, 2t, 3t^2 \rangle$

$\vec{a}(t) = \vec{v}'(t) = \langle 2, 2, 6t \rangle$

$\vec{r}(t_0) = \begin{cases} t_0^2 = 1 \\ t_0^2 = 1 \\ t_0^3 = -1 \end{cases} \Rightarrow t_0 = -1$

$\vec{a}(-1) = \langle 2, 2, 6(-1) \rangle = \langle 2, 2, -6 \rangle$

[Ans: $\langle 2, 2, -6 \rangle$]

   (b) Find the unit tangent vector at the point $(1, 1, -1)$.

$t_0 = -1$ still

$$\frac{\vec{v}(-1)}{|\vec{v}(-1)|} = \frac{\langle 2(-1), 2(-1), 3(-1)^2 \rangle}{\sqrt{(-2)^2 + (-2)^2 + (3)^2}} = \frac{1}{\sqrt{17}} \langle -2, -2, 3 \rangle$$

[Ans: $\left\langle -\frac{2}{\sqrt{17}}, -\frac{2}{\sqrt{17}}, \frac{3}{\sqrt{17}} \right\rangle$]

2. Suppose the acceleration vector of a particle is given by $\vec{a}(t) = \langle t, 3\cos t, 3\sin t \rangle$. Find the velocity and position vector if $\vec{v}(0) = \langle 0, 1, 0 \rangle$, $\vec{r}(0) = \langle 1, 0, 0 \rangle$. What is the speed at $t = 1$?

$\int \vec{a}(t)\, dt = \vec{v}(t) + \vec{v}_c$ & $\int \vec{v}(t)\, dt = \vec{r}(t) + \vec{r}_c$

$\int \langle t, 3\cos(t), 3\sin(t) \rangle\, dt = \langle \frac{1}{2}t^2, 3\sin(t), -3\cos(t) \rangle + \vec{v}_c$

at $t=0$ $\rightarrow \langle 0, 0, -3 \rangle + \vec{v}_c = \langle 0, 1, 0 \rangle$

so $\vec{v}_c = \langle 0, 1, 3 \rangle$

$\vec{v}(t) = \langle \frac{1}{2}t^2, 3\sin(t) + 1, -3\cos(t) + 3 \rangle$

$\int \langle \frac{1}{2}t^2, 3\sin(t) + 1, -3\cos(t) + 3 \rangle\, dt$

$= \langle \frac{1}{6}t^3, -3\cos(t) + t, -3\sin(t) + 3t \rangle + \vec{r}_c$

at $t=0$: $\langle 0, -3, 0 \rangle + \vec{r}_c = \langle 1, 0, 0 \rangle \Rightarrow \vec{r}_c = \langle 1, 3, 0 \rangle$

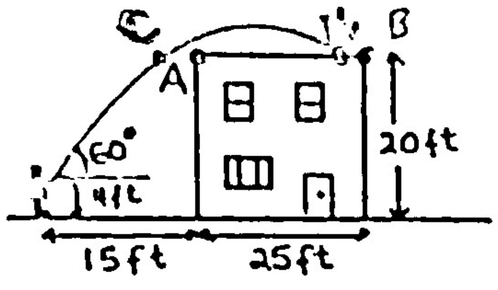
$\Downarrow$

$\vec{r}(t) = \langle \frac{1}{6}t^3 + 1, -3\cos(t) + t + 3, -3\sin(t) + 3t \rangle$

[Ans: $\sqrt{\frac{1}{4} + (3\sin(1) + 1)^2 + (-3\cos(1) + 3)^2}$]

speed at $t=1$ is $|\vec{v}(1)|$

3. A fire hose sprays water with an initial velocity of 40 ft/s at an angle of 60° with the horizontal.



$g \approx -32\ ft/s^2$

(a) Will the water clear corner point A? Justify your answer.

$\vec{a}(t) = \langle 0, -32 \rangle$ $\quad \vec{v}(t) = \langle 40 \cdot \overset{\cos(60°)}{\left(\frac{1}{2}\right)}, 40 \cdot \overset{\sin(60°)}{\left(\frac{\sqrt{3}}{2}\right)} - 32t \rangle$

$\vec{r}(t) = \langle 20t, 4 + 20\sqrt{3}t - 16t^2 \rangle$ so $x = 15ft$ at $t = \frac{15}{20} = \frac{3}{4}$ s

and y at $t = \frac{3}{4}$ is $4 + 20\sqrt{3}\left(\frac{3}{4}\right) - 16\left(\frac{3}{4}\right)^2$

$= 15\sqrt{3} - 5 \approx 20.98$ ft

[Ans: Yes, since its height at that point will be ft.]

(b) Will the water hit the roof? Justify your answer.

(i.e. check if water clears point B or not)

so $x = 15 + 20$ ft $= 40$ ft at $t = 2$ s.

then $y = 4 + 20\sqrt{3}(2) - 16(2)^2 = 40\sqrt{3} - 60$

≈ 9.28 ft

[Ans: Yes. The height will be which is less than 20 ft.]

(c) How far from the corner point A will the water hit the roof?

want to find when $\underline{y = 20ft}$ i.e. $4 + 20\sqrt{3}t - 16t^2 = 20$

$\Downarrow$

$16t^2 - 20\sqrt{3}t + 16 = 0$

$4t^2 - 5\sqrt{3}t + 4 = 0$

quadratic formula

$t = \frac{5\sqrt{3} \pm \sqrt{75 - 64}}{8}$

$= \frac{5\sqrt{3} \pm \sqrt{11}}{8}$.

Take the larger value of t → since smaller value of t corresponds to point C in diagram

[Ans: ≈ 14.942 ft]

x at $t = \frac{5\sqrt{3} + \sqrt{11}}{8}$: $\frac{5}{2}(5\sqrt{3} + \sqrt{11}) \approx \underline{29.942 \text{ ft}}$