The Role of Imagistic Simulation in Scientific Thought Experiments

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Abstract

Interest in thought experiments (TEs) derives from the paradox: “How can findings that carry conviction result from a new experiment conducted entirely within the head?” Historical studies have established the importance of TEs in science but have proposed disparate hypotheses concerning the source of knowledge in TEs, ranging from empiricist to rationalist accounts. This article analyzes TEs in think-aloud protocols of scientifically trained experts to examine more fine-grained information about their use. Some TEs appear powerful enough to discredit an existing theory—a disconfirmatory purpose. In addition, confirmatory and generative purposes were identified for other TEs. One can also use details in transcript data, including imagery reports and gestures, to provide evidence for a central role played by imagistic simulations in many TEs, and to suggest that these simulations can generate new knowledge using several sources, including the “extended application” of perceptual motor schemas, implicit prior knowledge, and spatial reasoning operations, in contrast to formal arguments. These sources suggest what it means for TEs to be grounded in embodied processes that can begin to explain the paradox above. This leads to a rationalistic view of TEs as using productive internal reasoning, but the view also acknowledges the historical role that experience with the world can play in forming certain schemas used in TEs. Understanding such processes could help provide a foundation for developing a larger model of scientific investigation processes grounded on imagistic simulation (Clement, 2008).

Keywords: Scientific thinking; Imagery; Mental simulation; Embodied cognition; Reasoning; Creativity

1. Introduction

This article focuses on understanding what is happening when an expert makes new or novel predictions in a scientific thought experiment. Scientists such as Galileo, Newton,
Einstein, and Maxwell have included powerful thought experiments (TEs) in their published works. History of science investigations of TEs include that of Nersessian (2002) who has proposed, based on her reading of historical records of Maxwell’s work on electromagnetic field theory, that TEs can play a role in the development of scientific theories, not just in their exposition. Related processes that can be interpreted as utilizing TEs have been documented using the laboratory notes of Michael Faraday by Gooding (1992) and Ippolito and Tweney (1995). Thought experiments are intriguing because (a) they appear to play a powerful role in science; and (b) the subject seems to gain what feels like new empirical information without making any new observations. This raises what I call the fundamental paradox of thought experiments, expressed as: “How can findings that carry conviction result from a new experiment conducted entirely within the head?” The idea of an experiment (involving observation) being conducted in the head (without observations) is paradoxical in that it seems self-contradictory. Interestingly, previous attempts to deal with the fundamental paradox have ranged widely on a spectrum from empiricist to rationalist. For example, Norton (1996) claims that TEs are simply disguised arguments using the rules of inductive or deductive reasoning and that these start from tacit premises that can be suppressed empirical knowledge; he supports this position by displaying his own logical reconstructions of arguments he discerns in famous TEs. On the rationalist side, Gendler (1998) argues that the power of certain TEs used by scientists such as Galileo cannot be assigned to an underlying formal argument. And Brown (1991, 2004) believes that certain TEs utilize Platonistic apperceptions and nonformal reasoning to arrive at new conclusions that transcend experience, rather than starting from empirical propositions and extending them by formal arguments. (Unless qualified, in this paper I will use the term “reasoning” in a broad sense that can include nonformal heuristic reasoning that may not be rule based.) For example, he describes a method Galileo used to disconfirm Aristotle’s theory that heavy objects will fall faster than light objects. Galileo imagined a light object joined to a heavy object while falling, reasoning that under Aristotle’s assumptions the light object would slow the heavy object down; yet under those same assumptions, the total mass of the conjoined objects should fall faster than the heavy object alone, leading to a contradiction. Also on the rationalist side is Shepard (2008), who hypothesized that TEs may draw on imagined transformations depending on innate, presymbolic knowledge of three-dimensional space and symmetry.

Nersessian (1992) hypothesized that “simulative reasoning” using analog mental models played a role in Maxwell’s TEs on electromagnetic effects. Miščević (2007) also focused on mental models, hypothesizing on theoretical grounds that the use of visualization could tap implicit spatial-geometric knowledge as a source. However, Gooding (1992), through his analysis of Faraday’s laboratory notebooks, emphasized the similarities between real experiments and TEs, arguing against a purely rationalist description. Although he recognizes the role of reasoning, he argues that TEs also involve distillations of practice, including material world experience. Thus, there are some disparate but interesting views on the nature of TEs. However, because of the lack of fine-grained information in historical data, we lack empirical information on whether something like mental simulation is actually used, and if so what its components are and what the sources of knowledge are within it.
My own interest in this area began during a study of spontaneous analogies in think-aloud studies of expert reasoning in which we noticed that, in contrast to the usual view of an analogous case as already residing in memory, several of the cases were quite novel, indicating that they had been invented rather than retrieved (Clement, 1988). The papers described this finding from protocols as documenting the spontaneous generation of Gedanken experiments by experts, since that is a term often used in the history of physics for invented TEs. Subsequently, Clement (1994) analyzed data from expert protocols to propose mechanisms for what was termed the imagistic simulation process, with the goal of providing a foundation for understanding thought experiment processes. Eventually this was expanded into a larger theory of sources of knowledge in TEs (Clement, 2002, 2008). The present paper elaborates this theory on the basis of think-aloud case studies and places it in the context of other work on the problem of TEs.

With respect to the question of purposes for TEs, Kuhn (1977) theorized that the role of TEs was to disclose a conflict between one’s existing concepts and nature. On the other hand, Brown (1991) identified several purposes that included constructive as well as destructive (conflict generating) TEs. He also theorized that a few special TEs could serve both functions. Nersessian’s (2002) analysis of Maxwell’s work hypothesized that a TE could expose conflicts in an existing theory but that it could also point to new constraints that help guide positive modifications of the theory, thus playing both a destructive and constructive role.

Thus, a disparate variety of interesting theories about thought experimenting have been proposed in historical and philosophical studies. However, in order to seek more direct and detailed evidence than can be provided by historical studies, there is a need to examine real-time evidence on purposes and mechanisms for TEs as they are being used for the first time by a reasoner.

2. Purpose and method

In this paper I will analyze examples of qualitative thought experiment episodes from think-aloud case studies in an attempt to provide some evidence of this kind. Unfortunately, there is not even a consensus in the literature on a definition for “thought experiment.” The goal of this study is to provide some initial documentation of the use of spontaneous TEs and to use these to suggest a modeling framework for a mechanism that can begin to address the paradox. In doing so, I follow Anzai and Simon (1979) in considering think-aloud case studies to be an important strategy for constraining initial modeling in an underdeveloped area. Partly because the TEs discussed here appear to involve dynamic imagery and internal simulations, I do not intend to develop a computational model, but to concentrate on the prior task of trying to sort out a consistent way to describe relationships between concepts like “simulation,” “perceptual motor schema,” “imagery,” and “mental model” during the running of a thought experiment. To do this I will focus most on a single expert subject in an attempt to account for a wide range of detailed phenomena visible in a videotaped case study (including imagery reports and depictive gestures), supplemented by episodes from other subjects and history of science.
3. Case study

This study derives from a set of protocols from subjects who were professors and advanced graduate students in scientific fields, including physics, mathematics, and computer science. The primary database for this paper comes from an expert in mathematics thinking aloud about the following ‘‘spring problem’’:

A weight is hung on a spring (Fig. 1). The original spring is replaced with a spring made of the same kind of wire, with the same number of coils, but with coils that are twice as wide in diameter. Will the spring stretch from its natural length more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible.) Why do you think so?

The subject had a working knowledge of mechanics but the fact that he did not have direct expertise in mechanical engineering or springs means that his thinking occurs on the frontier of his personal knowledge. Thus, it is plausible that his thinking may share some characteristics with processes used on the frontiers of science. I will present some examples of TEs from this subject and then give a definition of ‘‘thought experiment,’’ followed by more examples from this subject and others. In the following subject S2 reports simplest example, ‘‘imagining what would happen’’ to the wide and narrow springs:

Protocol Episode 1: S: ‘‘I’m going to try to visualize it to imagine what would happen—my guess would be that it [wider spring] would stretch more—this is a kind of kinesthetics sense that somehow a bigger spring is looser—Umm, that’s high uncertainty.’’

I will use italicized type to identify transcript segments that have potential as evidence for imagery use (both kinesthetic and visual). This appears to be a TE in the sense that he makes a prediction for a concrete situation, and it appears to be one he has not previously observed. In this case his visualization, as he puts it, gives him the correct answer, but he is not very confident in the result, so it is a high-uncertainty TE.

![Fig. 1. Spring problem.](image-url)
S2 also generated an analogy in which he predicted that a long horizontal rod fixed at one end would bend more than a short one (with the same sized weight attached to the other end of each rod), inferring that segments of the wider spring would bend more and therefore stretch more. However, he was concerned about the appropriateness of this analogy at a deeper level because of the apparent lack of a match between: (a) bending producing an increasing slope in the rod; and (b) a lack of increasing slope in the wire in a stretched spring. One can visualize this discrepancy by thinking of the increasing slope a bug would experience walking down a bending rod and the constant slope the bug would experience walking down the helix of a stretched spring. (The latter is my own descriptive analogy for purposes of clarity—not the subject’s.) This discrepancy led him to question whether the bending rod was an adequate model for the spring.

Episode 2: ‘‘But then it occurs to me that there’s something clearly wrong with that [bending rod] metaphor, because it would (raises hands together in front of face) droop (moves r. hand to the right in a downward curve) like that, its slope (retraces curved path in air with l. hand) would steadily increase, whereas in a [real] spring, the slope of the [stretched] spiral is constant…. You get a spring which stretches more and more at the bottom. The loops are wider apart there. But that is not the case...they’re uniform all the way around.’’

Imagining the spring with an increasing slope appears to be a thought experiment in which he ‘‘runs’’ the idea of bending taking place in the spring as it stretches, as shown in Fig. 2. This can be seen as a novel thought experiment in which he examines the consequences of running the ‘‘bending model’’ in consecutive segments of the spring. This anomaly of a spring with an increasing slope produces a mismatch with his prior knowledge or intuition that spring coils should not become wider apart at one end. This anomaly appears to bother him considerably and drives further work on the problem.

3.1. First definition of ‘‘thought experiment’’

These two examples motivate the following definition. Performing an (untested) thought experiment (TE in the broad sense) is the act of considering an untested, concrete system
(the ‘‘experiment’’ or case) and attempting to predict aspects of its behavior. Those aspects of behavior must be new and untested in the sense that the subject has not observed them before nor been informed about them. I use the phrase ‘‘untested thought experiment’’ to emphasize that this does not include cases where the subject simply replays a previously observed event. For example, when the subject compared the displacement of long and short bending rods after Episode 1 above, I did not count this as a TE since that situation was deemed sufficiently common to indicate that his knowledge of it would probably be from direct prior experience. Still, the above definition is intentionally quite broad. Later I will give a second definition for a more specialized type of thought experiment. However, one useful feature of the broad definition above is that it appears to designate cases that raise the fundamental paradox.

After spending nearly 30 min considering the bending rod and other analogies and becoming frustrated by not being able to resolve the "increasing slope" anomaly, S2 generates the polygonal coil cases in Figs. 3 and 4. While analyzing the hexagon in terms of bending effects, below, it occurs to him in an Aha episode that there will also be twisting effects in the segments. Each episode below is considered a TE under the definition given.

Episode 3: ‘‘Just looking at this it occurs to me that when force is applied here [From weight in Fig. 3], you not only get a bend on this segment, but because there’s a pivot here [point x], you get a torsion [strain produced by twisting the wire in segment b] effect. Aha!! Maybe the behavior of the spring has something to do with twist (makes twisting motion with right hand) forces as well as bend forces. That’s a real interesting idea.’’

![Fig. 3. Hexagonal coil case.](image)

![Fig. 4. Square coil case.](image)
Twisting of the wire and the resulting torsional strain is in fact the most important source of stretching and restoring force, in the analysis of spring behavior as understood by engineers. Its discovery here represents a scientific insight in finding a new causal mechanism. The act of predicting the effects of stretching a hexagonal coil fits the given definition of an untested thought experiment since it is certainly unlikely to have been tried before by the subject. (The generation of the polygonal coils and the torsion discovery and Aha! phenomenon, above, are interesting processes in themselves and are discussed in Clement [1989, 2008]. However, in this paper I will focus on describing processes for performing and making inferences from such TEs rather than for how they are generated). The plausibility of the wire twisting is clarified as the subject runs a TE on the square coil in Fig. 4:

Episode 4: ‘‘Let me accentuate the torsion force by making a square where there’s a right angle. I like that, a right angle. That unmixes the bend from the torsion. Now I have two forces introducing a stretch. I have the force that bends this segment a [Fig. 4] and in addition I have a torsion (makes twisting motion with right hand) force which twists [rod b] at vertex, um, x.’’

(It is as if side a were a wrench acting at x to twist the end of side b through an angle, while c keeps the other end of b from turning, resulting in a twisting deformation of the metal in b. The same would be true for segment c.)

Episode 5: ‘‘Now let’s assume that torsion and bend (motions with hands together as if bending something) do not interact…does this (points to square) gain in slope—toward the bottom? Indeed, we have a structure here which does not have this increasing slope as you get to the bottom. It’s only if one looks at the fine structure, the rod between the Y and the X, that one sees the flop (moves left hand horizontally in a downward curve) effect…. Now I feel I have a good model of sp—one of a spring.’’

Here S2 implies that a spring made of square coils will stretch with an equal distance between the coils, unlike the problematic and inaccurate situation he imagined in Fig. 2, a spring with increasing slope and an increasing distance between the coils toward the bottom. That is, there is bending in each side, but because bending and slope start over from zero at each corner; the slope from the bends does not accumulate by adding. The square coil is a new case in which the increasing slope difficulty does not occur, suggesting it may be a way to resolve his previous anomaly. He then goes on to ask about the effect of coil width for the square coil case.

Episode 6: ‘‘Now making the sides longer certainly would make the [square] spring stretch more… The longer the segment (holds hands up in front as if holding something between them), the more (makes bending motion with right hand) the bendability.’’

Episode 7: ‘‘Now the same thing would happen to the torsion I think, because if I have a longer rod (moves hands apart), and I put a twist on it (moves hands as if twisting a rod),
it seems to me—again, physical intuition—that it will twist more, uhhhhh (looks to side and pauses 4 s.) I’m—I think I trust that intuition… I’m (raises hands in same position as before) imagining holding something that has a certain twistiness to it, a—and twisting it.…”

Later the subject distinguishes between confidence in the answer to the spring problem and confidence in his understanding of it, and indicates that the torsion analysis has increased his understanding of the system from “way, way down” up to “like, 80%.” At this point S2 appears to have a mental model of the spring as working like a square coil that contains elements that both bend and twist. Thus, S2 uses TEs to predict correctly that the wider spring will stretch more and that twisting of the wire is an important source of stretching. These also suggest that the slope of the stretched spring will be constant throughout (also correct), resolving S2’s previous anomaly about increasing slope.

4. Initial analysis

Under the definition proposed, each of the seven numbered episodes above can be considered to contain an untested thought experiment. In the present case there appears to be evidence that (a) TEs have been part of the subject’s generation of important new hypotheses (e.g., protocol Episodes 1 and 3); (b) a TE (Episode 2) has raised serious doubts about one hypothesis; (c) TEs have boosted his confidence in other hypotheses about certain aspects of system behavior and have increased his feeling of understanding how the system works (Episodes 5, 6, 7). Thus, TEs appeared to play a central, not just peripheral, role within the thinking of this subject in helping to generate, cast serious doubt on, or support hypotheses.

4.1. Relationship between TEs and mental models

In reviewing usage for the term “mental model,” Nersessian (2002) notes that authors’ definitions have varied widely on different dimensions such as whether their format is propositional or imagistic. My use of the term “mental model” in the broad sense focuses on a simplified, somewhat general, internal representation of a system that can predict or account for its structure or behavior. The degree of generality can vary. This usage of “mental model” does not specify an imagistic format; this is desirable because I do not want to postulate by definition that mental models are imagistic; rather I am interested in using case study data to speak to the question of whether the format of a prediction generated by a mental model in a TE can be imagistic. Using a mental model to make a prediction for an untested question about a system then qualifies as a TE. (Conversely, using a model within its normal domain of application to make a prediction for a familiar question about a system would not be a TE.) These definitions are quite broad, reflecting a history of broad usage, and more substance needs to be added by proposing more detailed mechanisms for how TEs work, with initial support from case study data.
4.2. Imagistic simulation mechanism underlying TEs

4.2.1. Evidence for imagery

Italicized type in the protocol episodes above identifies examples of several imagery-related observation categories, listed here in the order they occur in Episode 7: personal action projections (spontaneously redescribing a system action in terms of a human action) consistent with the use of kinesthetic imagery, depictive gestures\(^1\) (gestures that depict objects, forces, locations, or movements of entities), and imagery reports. The latter occurs when a subject spontaneously uses terms like “imagining,” “picturing” a situation, or “feeling what it’s like to manipulate” a situation. In this last case it is a dynamic imagery report (involving movement or forces). None of these observations are infallible indicators on their own but are used here as evidence for imagery, and this is reinforced when more than one appear together. All indicators above except object or location gestures or reports of static imagery are also evidence for dynamic imagery of the kind that could be used in a simulation. Such indicators appear alongside new thought experiment predictions in the protocol segments, supporting the hypothesis that some type of internal imagistic simulation is occurring.

4.2.2. Schema-driven simulations

One can also draw on the historical precedent of motor schema theory (Schmidt, 1982) in hypothesizing that analog perceptual motor knowledge structures that can control real actions over time (e.g., a schema for “twisting” objects) are involved here. The observations in Episode 7, for example, can be explained via what I have called a schema-driven imagistic simulation, as follows: (a) The subject has activated a somewhat general and permanent perceptual motor schema that can control the action of twisting real objects; the schema is capable of coordinating real actions and perceptions over time and does this partly by generating action command trajectories and perceptual expectations. This capability allows it to generate imagery of anticipated actions and perceptual expectations in the absence of real objects and actions, presumably by driving presymbolic activity top down in some of the upper layers of the perceptual and motor systems; (b) The schema assimilates images of two rods of different lengths that are more specific and temporary than the schema itself; and (c) The schema “runs through an action” of twisting vicariously over time without touching real objects, generating an imagistic simulation of twisting each rod, and the subject compares the anticipated effort required for each. I follow Kosslyn’s (1980) theory of static imagery in referring to this last step as image interrogation and inspection, here extended to images of events and actions. Such a simulation may draw out implicit knowledge in the schema—knowledge the subject has not attended to and/or not described linguistically before. For example, the simulation may draw out presymbolic knowledge embedded in analog tuning parameters of a motor schema to anticipate differences in the effort required to twist a long and short rod. In other words, a hypothesis can be made, with initial grounding in data such as that in Episode 7, that the subject is going through a process wherein a general action schema assimilates the image of a
particular object and produces expectations about its behavior in a subsequent dynamic image (simulation) (Clement, 1994). The knowledge being used there is ‘embodied’ in this sense. In the present cases, the action schema is equivalent to what, in natural language, one might call a ‘physical intuition.’

4.3. Sources of new knowledge and conviction in TEs

In attempting to identify sources of new knowledge and conviction in TEs that can address the paradox cited earlier of doing experiments in one’s head, one can start with the two possible sources proposed above. One begins from the general hypothesis that the subject can run an imagistic simulation via a perceptual motor schema that generates a prediction by generating dynamic imagery. In order to go beyond cases where such a simulation is simply a routine ‘playing back’ of a familiar everyday action (which would not count as a TE), one can specify the following four additional features, any one of which could allow it to work for an unfamiliar (‘untested’) or even novel question in a thought experiment.

(a) Flexible perceptual motor schemas that can generate (run) imagistic simulations via the extended application of the schema, extended outside of its normal domain of application. (‘Outside’ means that the schema is being applied to either an unfamiliar situation or a familiar situation along with a question that has never been asked before about its operation.) For example, the projection of a human action into the problem and the accompanying depictive gestures in Episode 7 suggest the use of a perceptual motor schema for twisting that is being ‘run’ on an unfamiliar situation to produce a simulation.

(b) Converting implicit into explicit knowledge: As described above, this aspect also explains and is supported by Episode 7, as well as Episode 1 where imagining stretching the wide and narrow springs directly can be explained by the idea that the subject’s schema for handling springs or metal is being tapped for implicit knowledge about stretchability. (The subject confirmed that he had not investigated this question about springs before). These first two possible sources of conviction are shown as (a) and (b) in Fig. 5, where an imagistic simulation is shown within the dotted box.

Each of the seven episodes above contains at least one kind of evidence for the involvement of imagery (shown in italics). The presence of depictive gestures or imagery reports that in most episodes (1, 3, 4, 6, 7) represent dynamic events via a human action is taken as evidence for the involvement of a perceptual motor schema that is generating visual and/or kinesthetic imagery, as opposed to the subject working directly from memorized factual knowledge or equations. In addition, the wording in most episodes conveys the sense that the subject’s conclusions are self-evaluated inferences rather than facts derived from an external authority.

With respect to possible sources of conviction in TEs beyond (a) and (b) above, other evidence indicates that schema-driven simulations can work in concert with more general spatial reasoning skills. These general skills are reasoning operations...
that reflect spatio-temporal constraints on any system of objects, such as the constraint that solid objects may not occupy the same space, or that two sequential displacements of the same object will add together, or that the face of an object turning on an axis will disappear and reappear (Shepard & Cooper, 1982). Elementary spatial reasoning operations are processed automatically without noticeable effort. One can add to the list of possible subprocesses in an imagistic simulation:

(c) **Spatial reasoning** can contribute inferences, such as were apparently needed in Episode 2 where the bending going on in successive sections of the spring is imagined to add together and produce an increasing slope in the wire. A similar process is hypothesized to occur in Episodes 4 and 5 when the subject imagines whether the contributions to stretching and slope in each side of the square coil accumulate. (He predicts correctly that stretch accumulates, but slope does not since any bending in each side would ‘‘start over’’ from zero slope at each joint.) These two conclusions are viewed as emergent features (marked [2] in Fig. 5) that are predicted by running schemas plus spatial reasoning on the imagery of the square coil.

(d) Episodes 4 and 5 also appear to involve a *compound simulation* wherein either multiple schemas (e.g., for bending or twisting in each side) or multiple applications of the same schema are used on the same image to generate a simulation and the results added together imagistically. Compound simulation encompasses the ability to make predictions from chains of actions or parallel coordination of simultaneous actions, but only at a level of complexity that is within the limits of the human imaging
system. Presumably it can use neural capabilities related to those used for primitive cases where a child coordinates two actions being used on the same object (e.g., using a knife and fork on a piece of food).

I refer to the schema and (b) in Fig. 5 as prior knowledge, and running the simulation along with aspects (a, c, d) as reasoning. One can point to the above sources (a through d) as potential sources of knowledge and conviction during the imagistic simulation of a thought experiment, as shown in Fig. 5. In this framework then, there are multiple sources that contribute to addressing the fundamental paradox of thought experiments.²

5. Imagery enhancement episodes

Further support for the imagistic simulation framework above comes from its ability to explain ‘‘imagery enhancement’’ episodes. The simplest example comes from another subject, S7, comparing the narrow and wide springs:

Episode 8: ‘‘If we had a case where the second one went—had huge diameters compared to the first, it would appear to sag a lot more. It just feels like it would be a lot more spongy.’’

By changing the problem into an extreme case comparison, the subject is able to predict a result, apparently via kinesthetic imagery. He subsequently appears to enhance the imagery still more by imagining placing a very heavy weight on each spring:

‘‘Imagine putting a very heavy weight on it so it disturbs it a lot…that [very wide spring] would seem a lot easier…it would stretch more.’’

The size of the equal weights in the problem is not specified and indeed changing this parameter for both springs should be completely irrelevant to the answer to the problem. So it is puzzling as to why the subject changes it. But one can hypothesize that it acts to enhance the imagery in the simulation so that the effects in the system are larger and the imagined result becomes clearer, especially if the initially imagined weights and displacements were small. I call this imagery enhancement.

(A) (B) (C)

Fig. 6. (A–C) Extreme case comparison gesture for twisting a rod.
A second example is an extreme case of twisting a very short rod that comes immediately after S2 says, “I’m imagining holding something that has a certain twistiness to it, a—and twisting it (with hands held in air as if holding a rod 2 ft. long)…” in Episode 7 above:

Episode 9: (see Fig. 6) “Now I’m confirming (moves clenched right hand toward clenched left hand) that, by using this method of limits. As (moves right hand to left hand until they almost touch at the first word ‘closer’) I bring my hand up closer and closer (keeps holding clenched right hand next to left hand, making slight vertical punctuating motions at the words ‘hold,’ ‘clearly,’ ‘harder,’ and ‘harder’) to the original place where I hold it, I realize very clearly that it will get harder and harder to twist. So that confirms my intuition so I’m quite confident of that.”

(The reader may wish to try this thought experiment with images of thin coat hanger wire, bent to have 1 inch “handles” at each end of the wire.) This extreme case poses an interesting challenge for theory because it simply seems to repeat the exact same reasoning and result as the previous Episode 7, but yields a much higher level of confidence. Weld (1990) proposed that one mechanism for the effectiveness of an extreme case is to allow access to the second of two data points (pairs) for the values of two related variables. If one assumes a monotonic relationship, one can predict an increasing or decreasing function from knowing two data points. But how can considering the extra extreme case above add so much confidence since S2 has already just consulted about his knowledge on this issue in Episode 7 and already has determined the answer from the equivalent of at least two “data points”? Is it a logical inference? There appears to be no basis for a new rule-based inference that could produce considerably greater conviction. And his saying “I realize very clearly that it will get harder” indicates there is something special about the extreme case that makes it count more than simply adding a third data point of equal weight from which to induce a pattern.

Given the above observations it is more plausible to explain this as “imagery enhancement” (or “simulation enhancement”)—from increasing the difference between the two dynamic images being compared. In this view the main source of conviction in the simulations is the tapping of implicit knowledge embedded in motor schemas and its conversion into explicit knowledge. The extreme case makes differences in implicit expectations larger and more “perceivable” in this case. A second problem that questions the adequacy of describing this as “accessing a stored data point” is the difficulty this presents in explaining the following imagery indicators: a personal action projection, depictive gestures, and kinaesthetic imagery reports. Why did the subject bother to run through an imagistic simulation for the extreme case? The evidence that he did so suggests he was accessing implicit perceptual-motor knowledge that was not stored as a linguistic description. For if it were already explicitly described and stored, why form an image of the situation and make the effort to run through a simulation? Why not just report it as a fact? The subject’s saying, along with hand motions, that as he moves his right hand very close to his left hand (for a very short rod), that he realizes very clearly that it will get harder and harder to twist, provides interesting evidence that the subject is actually imagining a special case and his actions upon it,
focusing on the amount of effort required, and gaining a higher level of confidence from simulated comparison, as opposed to remembering a fact.\textsuperscript{5}

The novelty of both this extreme case and the very wide spring argues that the origin of new confidence is the schema-driven imagistic simulation itself being applied to the extreme case, rather than another fact or episodic memory being activated. The most parsimonious interpretation is that the extreme case enhances the subject’s ability to anticipate and compare contrasting kinesthetic images, and that the main source of conviction in the simulation is the tapping of implicit knowledge and its conversion into explicit knowledge (Clement, 1994, 2008).

A case has now been built from protocol evidence that the models used by S2 in his protocol were analog, runnable, and dynamically imageable. I use the term ‘‘runnable model’’ for this kind of mental model.

Episode 10. In a third example of enhancement, another subject, S6, attempted to imagine the direction in which the wire in a spring coil would twist when it was stretched, but he found it difficult. After adding to his image of thick spring wire ‘‘little paint dots on it all along its length....and saying ... would I see a torsional displacement,’’ he was eventually able to do this. When asked whether he was thinking of an equation, he said, ‘‘Oh no...I’m going pull and release, pull and release, and so I’m constantly putting it through its paces. And asking, you know, how would I see the dots move?’’

This ‘‘markers’’ strategy appears to be another kind of imagery enhancement strategy designed directly to aid or enhance visualization and spatial reasoning during an imagistic simulation. Again, it is difficult to explain with the use solely of a rule-based, propositional account why the subject would do this. A traditional propositional account would have inferences being made on symbolically expressed relationships between variables; for example, of the form ‘‘an increase in variable A causes variable B to (increase or decrease).’’ However, adding dots to the system plays no causal role in the system, and therefore, in the propositional account, should be irrelevant to inferencing. In comparison, the imagistic simulation framework developed so far appears to offer a more viable way to explain ‘‘imagery enhancement’’ episodes. And the subjects’ efforts to produce them also argues that they valued imagery-based methods as important.

5.1. Imagery enhancement in a historical TE

Enhancement may also have played a role in historical TEs such as those Einstein created for general relativity. His goal was to show ‘‘how the problem of the general relativity theory (for accelerated motion) is closely connected with that of gravitation’’ (Einstein & Infeld, 1938, p. 222). He describes his first version of a TE in this area coming to him in the patent office as the case of a person falling (e.g., from a tall building). He concluded, ‘‘If a person falls freely he will not feel his own weight.’’ That this was an important insight for him is supported by his referring to it as ‘‘the happiest thought in my life’’ (letter by Einstein quoted in Isaacson, 2007, p. 145). He subsequently enhanced the experiment by
placing the person in a closed, windowless container such as an elevator. He invites us to imagine that we could not tell the difference between being in an elevator floating in space and one in free fall dropping toward the earth. Nor could we distinguish between an elevator accelerating “upward” in outer space and one at rest in a gravitational field, since objects would appear to drop to the floor in the same way. This particular TE plays a generative rather than an evaluative role in building up the connection between relativity and gravity (c.f., Brown, 1991). (Similarly, in several of the present protocol episodes [3, 7, 10], TEs played a theory-generating role, complementing their evaluative role in other cases.)

It would be possible to pose this experiment using abstract descriptions of reference frames. Instead, Einstein enhanced the concrete imagery and meaning of the experiment by modifying the original “falling man” experiment and tapping into the elevator schema with its familiar kinesthetic imagery of the sensation of acceleration or deceleration, as well as visual imagery of a box we cannot see outside of to detect its movement. In the present view, this constitutes enhancing the imagery of the original thought experiment to make it easier to run comparative imagistic simulations with it and to detect findings more unambiguously.

5.2. Summary for basic untested TEs

In summary, the presence of TEs can be documented within think-aloud protocols, and there is evidence that those documented here involved the use of imagery. The successful
uses of TEs in the above episodes can be explained by hypothesizing that they involve an imagistic simulation process wherein a perceptual motor schema generates dynamic imagery that can be interrogated. Four possible sources of new knowledge and conviction in TEs that can begin to explain the TE paradox were proposed in Fig. 5, based on TE transcript episodes. This framework obtains additional initial support from its ability to explain imagery enhancement episodes (Clement, 2008).

6. Second, more specialized TE definition: Evaluative Gedanken experiments

A more specialized type of thought experiment is what I call an evaluative Gedanken experiment. Performing an evaluative Gedanken experiment is the act of considering an untested, concrete system designed to help evaluate a scientific concept, model, or theory—and attempting to predict aspects of the system’s behavior. An example from subject S2 is the case of a spring made of a vertically oriented band of material, depicted at the bottom of Fig. 7A. (The reader might imagine the thin metal strip unwound from a coffee can, reshaped to make a spring, say, 3” wide.) He described this as follows:

Episode 11: Band spring. “How about a spring made of something that cannot bend. And if you showed that it still behaved like a spring you would be showing that the bend is not the most important part. Or is not particularly relevant at all maybe somehow... How could I imagine such a structure?... I’m thinking of something that’s made of a band... we’re trying to imagine configurations that would not bend. Since its cross section is like that (see bottom of Fig. 7A)... it can not bend in the up-down (indicates up/down directions with hands) direction like that because it’s too tall. But it can easily twist (motions as if twisting an object).

Given the imagery indicators here, I interpret this to mean that the subject imagined that such a spring would still be quite stretchable even though the band “cannot bend in the up-down direction,” which challenges the mechanism of bending as necessary for springs to behave as they do. In this evaluatory Gedanken experiment, he designs a special case for which the bending model yields a prediction (predicts “negligible stretch” either by deduction or by simulation—the choice is not clear from the transcript), but he also has another independent source of information that can be used to evaluate that prediction (in this instance, a physical intuition, which yields a different prediction) as shown in Fig. 7A. This is an evaluative Gedanken experiment because it is designed to help evaluate an explanatory model. Each side of the “V” in Fig. 7A can be viewed as a simpler untested thought experiment (previously unobserved prediction); this illustrates how the evaluative Gedanken can be a more complex reasoning pattern than the simple TEs discussed earlier. In this particular kind of Gedanken experiment, the subject considers a new system for which the present model is predictive but for which another source of knowledge is also predictive, giving the potential for conflict or coherence between the two prediction methods. (Other examples are given in Clement, 2008.)
6.1. Galileo's cannonball through a hole in the Earth experiment

A similar pattern can be seen in the beautiful experiment shown in Fig. 7B, used by Galileo (2001) in his *Dialogues on the Two Chief World Systems*. (Versions can also be traced back at least as far as his predecessor Tartaglia.) The larger issue at hand is whether Aristotle’s distinction of “Natural” and “Violent” motion as two different types of motion is valid. Here, “Natural” motion is motion toward the center of the Earth, whereas upward movement, such as throwing a ball up, is “Violent” (preternatural or “constrained”) motion. Drilling a hole through the Earth and dropping a cannonball into it appears to produce, at the center of the Earth, a smooth but very sudden transition from natural to violent motion. And yet the cause of the ensuing upward (“violent”) motion seems eminently “natural,” challenging Aristotle’s distinction:

Salviati: But [the cannonball] having arrived at the center is it your belief that it would pass on beyond.....?

Simplicio: I think it would keep on going a long way.

Salviati: Now would not this motion beyond the center be upward, and according to what you have said preternatural and constrained? Let me see you find an external thrower who shall overtake it...to throw it upward. (Galileo, 2001, p. 236).

The inference that the cannonball appears to pass through the center absolutely smoothly without any discontinuous change or violent source of motion argues that it is undergoing one and the same kind of motion on each side of the center. This attacks Aristotle’s central distinction between Natural and Violent motion and helps discredit that explanatory model. Again, as shown in Fig. 7B, the Gedanken case evokes one prediction from the old model and a conflicting prediction from another source, here an intuition about the strong momentum of a speeding cannonball. Thus, the pattern of reasoning in the band spring experiment from the expert protocol appears to have the same form as an example from Galileo’s dialogues, suggesting that this is one general method for conducting evaluative Gedanken experiments.

7. Discussion

The fundamental paradox of TEs, “‘How can findings that carry conviction result from a new experiment conducted entirely within the head?’” is a long-standing problem. Historical studies reviewed in the introduction have established the importance of TEs in science but have proposed some disparate hypotheses concerning the source of knowledge in TEs. In this paper I have attempted to seek more direct and detailed evidence on the purposes and mechanism for TEs from think-aloud protocols. An initial step was to promote clarification by providing both broad and narrow definitions of “thought experiment.” The paradox
appears to apply surprisingly widely under the broader definition to all “untested thought experiments”: the act of predicting the behavior of an untested, concrete system.

To address the paradox, previous authors have taken different positions, ranging from more empiricist to more rationalist, on the question of the source of knowledge in TEs. Hypothesized sources have ranged from hidden premises from prior experiential knowledge (Norton, 1996) to Platonic intuitions (Brown, 1991, 2004), fostering a spirited and thought-provoking debate.

What the present data indicate is that when experts are studied under conditions that preclude any new empirical input, they are still able to make confident predictions about aspects of systems they have never observed, so this study can be seen initially as weighing in against an empiricist view. Instead, subjects appeared to generate new simulations to make new predictions. Evidence for the use of imagistic simulations was provided from the co-occurrence of predictions with imagery indicators such as dynamic imagery reports, personal action projections, and depictive gestures. This evidence was reinforced by cases of imagery enhancement—via the use of extreme cases or adding “markers” to the imagery to make comparisons easier—arguing that imagistic simulations can be important to the subject and centrally involved, not just a decorative side effect.

However, using protocols to unpack a number of sources of knowledge within the process of imagistic simulation, depicted in Fig. 5, led to a somewhat more nuanced view—still strongly rationalistic, as it stresses nonformal reasoning, but also recognizing the possible historical role of empirical sources as follows. Source (a) in Fig. 5 starts from a generalized perceptual motor schema, and these will most often have been built up at least partly during experiences with the world historically. However, to count as a thought experiment, subjects must use such an old schema in a new situation (or for a new question), extending it outside its normal domain of application. This means that there is some uncertainty in the application of the schema, even though there is considerable conviction in some cases. The same can be said for source (b) where one uses implicit knowledge, probably built up from prior empirical experiences, that is embodied in a schema in analog form and needs to be made explicit. In both (a) and (b) one must make an applicability judgment as to whether the experimentally developed knowledge applies to the unfamiliar TE case.

Sources (c) and (d) in Fig. 5 utilize additional reasoning capabilities to make further extensions of thought. In (c), spatial reasoning—for example, about perceptual transformations or the way object movements add together—is a type of plausible reasoning that is presumably automatic and ubiquitous. And in (d), compound simulation, the subject runs two or more schemas on the same case in a way that involves sequencing or coordinating and predicting the effects of a new combination of actions.

In this framework, TEs can utilize old knowledge schemas developed from past experiences in interacting with the world. This is reminiscent of Norton’s (1996) focus on hidden empirical premises extended by deductive or inductive arguments, mentioned in the introduction. It therefore gives partial support to the view that empirical knowledge, in some sense, can be involved. However, such empirical aspects are historical, not part of the current thinking. And such knowledge was only one element in the creative construction of new structures and the nonformal reasoning involved in the imagistic simulations run on
those structures. Brown (1991, 2004) has argued via examples of famous TEs, such as the one for heavy and light objects mentioned in the introduction, that not all TEs need to start from empirical propositions and that the ensuing reasoning need not always be a formal argument. Evidence in the present study suggests that some TEs can begin from implicit physical intuitions apprehended via imagistic simulations, rather than explicit linguistic propositions or axioms; and that subsequent reasoning can also operate on imagistic representations (Clement, 1994). Reasoning via imagistic simulation, as represented in Fig. 5, appears more like the playing out of an imagined experience than the articulation of an argument. It differs from rule-based argument by utilizing creative, extended applications of schemas, involving applicability judgments; emergent properties from compound simulations; and spatial reasoning, which in elementary cases is applied “automatically” rather than in an explicit argument. The process in the dotted box in Fig. 5, faced with an unfamiliar context, is more like one or more motor schemas extending creatively to engage in an exploratory action than it is a rule-based argument. This view is consistent with other authors such as Hegarty (2004) who distinguish between analog reasoning via mental simulation and rule-based reasoning.

For precedents on the rationalist side, the use of (c), spatial reasoning processes, is the source in Fig. 5 that comes closest to Brown’s (2004) hypothesized Platonic “intuitions of the laws of nature” since spatial reasoning may be so deeply embedded or embodied in the system. Shepard (2008) hypothesized that many of these spatial reasoning processes are innate, but specifying the extent to which some could be innate is a current developmental research issue.

Nersessian (1992) provided evidence for the role of TEs within Maxwell’s work and hypothesized that they involve constructing and simulating a mental model of a situation. Her concluding hypothesis about the source of knowledge in these TEs was, “The constructed situation inherits empirical force by being abstracted from both our experiences and activities in, and our knowledge, conceptualizations, and assumptions of, the world” (p. 297). She has framed similar views along with other examples and careful reviews of previous literature on mental modeling and simulation (Nersessian, 2002, 2008). I take her view to be an intermediate one, referring to a mixture of empirical and rationalist sources.

Building on Clement (1994), the present study attempted to address these questions by using protocol evidence to document the use of imagistic simulations, and to help unpack the process and specify multiple sources of conviction within it, thereby attempting to outline a set of mechanisms underneath the term “simulation” as used by others. To address the TE paradox, the idea of perceptual motor schemas running imagistic simulations, and the four more specific sources of conviction within imagistic simulations identified above and in Fig. 5, can account for ways that a TE can feel empirical (via the inspection of imagery) or necessary (via confident schema extension or spatial reasoning). Yet these processes actually involve a considerable amount of nonformal reasoning and inference that goes beyond prior observations. No one source in Fig. 5 needs to be involved to the same degree in every TE. This means that TEs can vary as to where they are on an empiricist-rationalist continuum. In cases such as the twisting rod in Episode 7, extensions to an unfamiliar case by
an existing knowledge schema with experiential roots may play a primary role. In other cases such as the square coil, a subject can put several imagistic elements together and “run” this new assembly in the mind—not just by using a perceptual motor schema as above but also by using spatial reasoning and compound simulations to decide whether contributions from each side add or cancel. Here, rationalistic reasoning is more predominant. Thus, a combination of several mechanisms is needed in order to handle the variety of TEs observed in the protocols. This leads to the view that the processes depicted within the dotted box in Fig. 5 constitute a flexible set of resources for reasoning via imagistic simulation that can begin to explain the TE paradox.

7.1.1. Gedanken experiments

Building on Kuhn’s (1977) ideas, Gooding (1992) observed that the successful TE author “selects and isolates just those features of a phenomenon, the environmental framework, and the conceptual scheme that are mutually problematic” (p. 71). Here it appears he is describing some general features of one design for what I have termed an evaluative Gedanken experiment. In doing so, he also took a position that is intermediate between the empiricist and rationalist ends of the spectrum, concluding with elegant simplicity, “What is needed is a combination of empirical knowledge and the ability to reason with it” (Gooding, 1994, p. 1041).

Gedanken experiments are designed to help evaluate a theory, model, or concept. Examples like the band spring in Fig. 7 are more similar to real experiments in design and can include something like the control of variables, which may make them “feel” even more empirical. Galileo’s dropping balls experiment, discussed by Brown and mentioned in the introduction, has a similar Gedanken form to that of Fig. 7B except that the sources of conflict are both internal to the theory being attacked. The conclusions about nonformal mechanisms for untested TEs, reached in the discussion section above, can also apply to major subprocesses of many evaluatory Gedanken experiments because Gedankens can be based on (contain) one or more simple untested TEs, as was seen in Galileo’s Hole through the Earth TE. The high-level structure, however, of the Gedankens examined here, took the form of a designed argument leading to a contradiction. This allows one to say that although an elemental TE itself may reach its conclusion via something very different from a rule-based argument, the result of the TE may be used as part of a larger formal argument, such as an evaluative Gedanken experiment.

7.1.2. Purposes of TEs

Kuhn (1964) emphasized a disconfirmatory role for TEs, and while some of the present episodes support this, other cases indicate that TEs can play a role that is confirmatory (lending support; e.g., Episodes 5, 8, 9). TEs can also play a theory-generating role (e.g., Episode 3). Brown (1991) developed a related distinction between destructive and constructive thought experiments. I am able to cover the episodes in this paper using a consolidating typology of evaluative (either disconfirmatory [destructive] or confirmatory), or generative (constructive) purposes of TEs.
8. Limitations and extensions

8.1. Limitations

One limitation of this article is that simulation is not the only mechanism used for operating with TEs; some TEs can use qualitative or mathematical deduction from principles, and I do not have space here to deal with those cases. However, simulations appear to be a very powerful mechanism. Additionally, as mentioned, imagistic simulation does not explain how the experiments were generated (see Clement, 1988, 2008). In this paper I have focused only on mechanisms for running and drawing conclusions from TEs. More work is needed to evaluate, augment, and refine our understanding of these mechanisms as well as generation mechanisms.

8.2. Flexibility

Interest has increased in the role perceptual processing plays in grounding the meaning of cognitive representations (e.g., Barsalou, 1999); one can also ask related questions such as: Is there any role for thinking at a perceptual motor level that connects it in an important way with creative scientific reasoning and theory formation at higher levels? Piaget (1955) emphasized that the natural extendibility and flexibility of perceptual motor schemas are extremely valuable properties, allowing them to be used in new circumstances. It can be argued that each of the four sources of new knowledge and conviction described earlier exemplifies how the natural flexibility of perceptual motor schemas can be utilized to advantage during scientific thinking through imagistic simulation. For example, in the square coil in Episodes 4 and 5, one sees how the flexible perceptual motor schemas for bending and twisting can be combined and run in a new context in order to produce emergent predictions such as the effect of greater coil width and the lack of accumulating slope. The flexibility of a system that can produce compound simulations and that can be run in a representation that incorporates physical and spatial constraints through spatial reasoning pays off in being able to generate novel predictions from a new runnable model. Ippolito and Tweney (1995) discuss how abilities related to these may also guide the interactions of scientists with real experiments.

8.3. Imagistic simulation can ground higher levels of processing

An extension of this theme is to suggest that imagistic simulations provide a foundation for more sophisticated types of reasoning. Clement (1989) discusses a subtype of mental model, explanatory models; these hypothesize a hidden theoretical mechanism operating within a system to explain the behavior of the system. In this paper I have cited protocol examples of TEs run via schema-driven imagistic simulations where the simulations are not only run directly on the target (Episode 1), but are run within an explanatory model for the target (Episode 2), within an analogy (Episodes 3, 4, 5, 9, cf. Clement, 2004), and within an evaluative Gedanken experiment (Episode 11). Thus, TEs using imagistic simulation may
be an important mechanism operating within several types of nonformal reasoning. This theme and additional episodes of TEs conducted on more sophisticated qualitative and mathematical models of the spring are analyzed in Clement (2008); evidence is presented that argues for imagistic simulations as the grounding level that influences three higher levels of processing during creative theory formation. These levels are, from next higher to highest: nonformal reasoning processes (including analogies and Gedanken experiments); cycles of generation and evaluation of runnable explanatory models; and higher-level investigation and application processes.

8.4. Implications

A relatively small number of specific evaluative Gedanken experiments in the history of science have been recognized for their pedagogical value in convincing students of the validity or nonvalidity of a theory (Reiner & Gilbert, 2000). Occasionally, useful and interesting Gedanken experiments are generated by students themselves (Gilbert & Reiner, 2004). But with regard to untested TEs in the broad sense, in the present view, they are likely to be more ubiquitous in classrooms than previously thought because they appear within many types of reasoning (Stephens & Clement, in press). This suggests that they—along with imagistic simulation as a major mechanism—deserve additional study.

8.5. Summary

Both the broad and specialized concepts of “thought experiment” proposed here appear to be useful, and both can be documented in think-aloud protocols. The broad concept of an untested thought experiment is appropriate for expressing the fundamental paradox. The narrow concept of an evaluative Gedanken experiment encompasses cases such as “ Dropping a Cannonball Through a Hole in the Earth” used by Galileo, impressive in its role of disconfirming an established theory. In addition, confirmatory and generative roles were identified for other TEs. One can also use details in transcript data to provide evidence that, rather than using only formal arguments, expert TEs can make use of nonformal inferencing via embodied perceptual-motor schemas and imagistic simulation; this evidence was reinforced by cases where experts appear to modify a case in order to enhance the imagery being attempted. Unpacking these processes further led to hypothesizing the role of extended schema application, implicit knowledge, compound simulations, and spatial reasoning. All of these processes were proposed as mechanisms that can speak to the paradox of appearing to make experimental observations within one’s head. This emphasizes a rationalistic view of TEs in terms of the creative internal reasoning used to generate new beliefs that go beyond experience, but it also acknowledges the historical role of experience in forming certain schemas employed by TEs.

It is hoped that the descriptors generated here will contribute to future work needed to develop a fuller theory of TEs based on evidence from naturalistic observations of behaviors, and that this can interact fruitfully with both educational applications and the analysis of historical TEs.
Notes

1. There is not space here, but Clement (2008) discusses more than a dozen other indicators associated with imagery and reviews an increasing variety of studies of depictive gestures that suggest they are concurrent expressions of core meanings or reasoning strategies and not simply delayed translations of speech. Other studies indicate that the same brain areas are active during real actions and corresponding imagined actions (Decety & Ingvar, 1990; Roland, Larsen, Lassen, & Skinhoj, 1980). This suggests that depictive gestures are natural, abbreviated outputs reflecting internal imagery.

2. With regard to the relationship between schemas, implicit knowledge, and mental models, by considering the example of using a twisting schema to examine whether longer objects are easier to twist than shorter ones, one can ask whether “twisting the image of a rod” should be considered the use of a mental model. If no, then some TEs can be performed without models. If yes, then one is stretching the use of the term “model” to include the action of a basic perceptual motor schema on an imaged system. The latter is not as inappropriate as it may seem when one accepts Schmidt’s (1982) idea of general action schemas that can adapt to apply to different situations via tuning parameters, and when one provides evidence that they can extend to produce predictions via imagery for new unfamiliar cases or questions. And one can imagine that more complex models involve assemblies of multiple perceptual motor action schemas, such as bending and twisting in the square coil example. So in the present usage, a simple perceptual motor schema operating on the image of a generic system can be a primitive type of mental model for a range of situations. Another interesting question is whether it is desirable to draw a lower-level “line of demarcation” that prevents the concept of mental model from including implicit aspects of its constituent perceptual motor schemas. In my present view, a primitive model, T, of twisting a rod, was assembled and run in Episode 7 to answer a new question by simulating and comparing specific cases. This led to uncovering some implicit knowledge that then became an explicit part of a somewhat richer model, T’. In this view then, one can refer to implicit knowledge in a model T that is converted to explicit knowledge in the refined model T’.

3. I interpret his statement to mean that as the hand gets extremely close to the end of the rod, it becomes very hard to turn. Because of his way of expressing this, it could be objected that this is rather something like an empirical induction (by enumeration) from considering many positions A to G for the end of the rod. I submit that it is implausible that the subject could have stored data on each comparison in such a sequence, A to B, B to C…etc., and that it is the contrast between imagining twisting a 20-inch rod (Fig. 6A) and an extremely short rod (Fig. 6C) that yields new confidence, supported by his repeated punctuating gestures with his hands in the extreme position. I interpret his statement as also expressing the resulting conclusion of a monotonic increase.
References


