Practice Test III  
Unit 6 – Estimation  
Unit 7 – Hypothesis Testing  

SOLUTIONS

1. 

a. True or False.  
A hypothesis test for which the type I error occurs with probability $\alpha$ has probability of type II error equal to $(1 - \alpha)$.

Answer: FALSE

Solution:  
A type I error can only occur when the null hypothesis is true and a type II error can only occur when the alternative hypothesis is true.

b. True or False.  
If a one sided test indicates that the null hypothesis can be rejected at the 5% level, then a two sided test performed on the same set of data is necessarily significant at the 5% level.

Answer: FALSE

Solution: 
The significance level for a two sided test is the sum of two areas under the curve whereas it is the sum of one area under the curve when the test is one sided. Thus, for a given test statistic value, the two sided p-value will always be larger than the one sided p-value. Accordingly, if a one sided p-value is .05 or close to it, then the two sided p-value cannot necessarily be less than or equal to .05.
c. True or False.
For a given sample variance $s^2$ and sample mean $\bar{X}$, a 90% confidence interval for an unknown mean $\mu$ is narrower than a 99% confidence interval.

Answer: TRUE

Solution:
All other things constant, the confidence coefficient multiplier for a 90% CI is smaller than the confidence coefficient multiplier for a 99% CI. As a result the width of the CI, being \[ \text{upper limit} - \text{lower limit} \] will be narrower. For example, 90% and 99% confidence intervals based on the Normal distribution have multipliers 1.645 and 2.576, respectively.

d. True or False.
An investigator is performing a t-test for which the assumptions are satisfied could, in the absence of a student’s t-distribution tables, use a Normal(0,1) probability table provided the degrees of freedom is sufficiently large.

Answer: TRUE

Solution: Consider

<table>
<thead>
<tr>
<th>Type I Error (2 sided)</th>
<th>Test Statistic</th>
<th>Critical Value of Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>.10</td>
<td>t df=30</td>
<td>1.697</td>
</tr>
<tr>
<td></td>
<td>t df=50</td>
<td>1.676</td>
</tr>
<tr>
<td></td>
<td>t df=100</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>Normal (0,1)</td>
<td>1.645</td>
</tr>
<tr>
<td>.01</td>
<td>t df=30</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td>t df=50</td>
<td>2.678</td>
</tr>
<tr>
<td></td>
<td>t df=100</td>
<td>2.626</td>
</tr>
<tr>
<td></td>
<td>Normal (0,1)</td>
<td>2.576</td>
</tr>
</tbody>
</table>
e. Choose ONE.

The meaning of a p-value is
___ i.  the power of the test
___ ii. the probability of getting a result as extreme or more extreme than
      the one observed if the null hypothesis is false.
___ iii. the probability that the null hypothesis is true
___ iv. the probability of making a type II error
___ v.  the probability of getting a result as extreme or more extreme than
       the one observed if the null hypothesis is true.

Answer: v.

Solution: By definition, the p-value is the probability of obtaining a result as extreme or
more extreme as the one observed under the assumption of the null hypothesis.

2.

In September 1991, the Vermont Yankee Rowe nuclear power plant was closed by the
Nuclear Regulatory Commission. New England Electric System now buys electricity
from Canada to meet the energy needs of residents of Vermont.

It is known that the per customer yearly energy demand follows a normal distribution
with mean $\mu = 2,383$ kilowatt hours and variance $\sigma^2 = 1,050,625$ kilowatt hours squared.

Quebec Power and Light have hired you as a consultant. Their first request is to compute
an interval such that 95% of the per customer yearly consumption of energy are contained
in this interval. Find the lower and upper limits of the required interval.
Answer: \((374, 4392)\)

Solution:

For \(Z \sim \text{Normal}(0,1)\) the required 95\% coverage interval is \(-1.96 < Z < +1.96\)

Use the standardization formula backwards to obtain the corresponding coverage interval for a random variable \(X \sim \text{Normal}(\mu = 2383, \sigma^2 = 1,050,625)\)

Thus, for \(X \sim \text{Normal}(\mu = 2383, \sigma^2 = 1,050,625)\), the required coverage interval is

\[
(-1.96)[\sigma] + \mu \leq X \leq (+1.96)[\sigma] + \mu
\]

\[
= (-1.96)[1025] + 2383 \leq X \leq (+1.96)[1025] + 2383
\]

\[
= 374, 4392
\]

3.
A report from a clinical trial concludes that “\textit{difference between treatments is not statistically significant, }p < 0.05\textit{}”. What caveat/s must accompany this conclusion? Why is it potentially wrong to interpret this as meaning that there is no clinically important difference between treatments?

Answer:

The caveats that must accompany this conclusion pertain to the possibility that the alternative is true and that the statistical procedure performed missed its detection. There are several ways in which this might have occurred, including but not limited to the following:

1. the alternative is true and an event of low probability occurred and resulted in an incorrect retention of the null hypothesis
2. the alternative is true and the sample size was too small so that the power of the statistical test performed was low
3. the alternative is true but the choice of indicator was a very imprecise one so that the power of the statistical test was low
4.

Consider testing the null hypothesis that the mean of a normal probability distribution is \( \mu = 98.6 \). The population standard deviation is known and is equal to \( \sigma = 1.5 \). You draw a sample of size 25 from this distribution and calculate a sample mean. If it is of interest to perform a two sided test of the null hypothesis with a pre-specified type I error equal to 0.02, what values must the sample mean lie between in order for the null hypothesis to be not rejected?

**Answer:** Between 97.9 and 99.29

**Solution:**

1. For \( Z \sim \text{Normal}(0,1) \) and desired two sided type I error = .02, obtain the critical region for the Z test. To do this, obtain from a Normal (0,1) calculator that
   
   \[ \Pr \left[ -2.326 < Z < +2.326 \right] = .98 \]

2. Identify the null hypothesis distribution of \( \bar{X}_{n=25} \)
   
   \( \bar{X}_{n=25} \sim \text{Normal} \left( \mu_{\text{null}} = 98.6 \text{ and } \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{1.5^2}{25} \right) \) under null.

3. Solve for critical region values of \( \bar{X}_{n=25} \) by using the standardization formula backwards. Here, it takes the form
   
   \[ (-2.326) \left[ \sigma_{\bar{X}} \right] + \mu_{\text{null}} \leq \bar{X}_{n=25} \leq (+2.326) \left[ \sigma_{\bar{X}} \right] + \mu_{\text{null}} \]
   
   \[ = (-2.326) \left[ 1.5/\sqrt{25} \right] + 98.6 \leq \bar{X}_{n=25} \leq (+2.326) \left[ 1.5/\sqrt{25} \right] + 98.6 \]
   
   \[ = 97.9, 99.29 \]
5.

A randomized trial of “diet continuation” versus “diet termination” was performed to evaluate whether diet termination is associated with a reduction in normal development among children with PKU. The outcome measured was Bitnet IQ, a measure of mental retardation. The following summary statistics were obtained from complete data on 22 children:

<table>
<thead>
<tr>
<th>Group</th>
<th>Diet Continuation</th>
<th>Diet Termination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size, n</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Sample mean IQ, X</td>
<td>103.6</td>
<td>99.8</td>
</tr>
</tbody>
</table>

Carry out the appropriate statistical hypothesis test to evaluate the study data. In developing your answer you may assume that the observations are distributed normal in both groups. You may also assume that the population standard deviations of a Bitnet IQ are $\sigma = 13$ in the diet continuation population and $\sigma = 17$ in the diet termination population.

**Answer:**

- $H_{null}$: $\mu_C = \mu_T$
- $H_{alternative}$: $\mu_T < \mu_C$, one sided

- $Z$ statistic $= \frac{(\bar{X}_T - \bar{X}_C) - [0]}{SE(\bar{X}_T - \bar{X}_C)}$; where $SE(\bar{X}_T - \bar{X}_C) = \sqrt{\frac{\sigma_T^2}{n_T} + \frac{\sigma_C^2}{n_C}} \sim \text{Normal}(0,1)$ under null

- $Z = \frac{(99.8 - 103.6)}{6.56} = -0.5793$; where $6.56 = \sqrt{\frac{17^2}{10} + \frac{13^2}{12}}$

- $p$-value $= 0.28$

- Do not reject. Conclude these data are insufficient evidence of an effect of diet continuation on Bitnet IQ in children with PKU
6.

It has been reported anecdotally that the onsets of menses for women living together (for example, a college dormitory) are more synchronized than those of women living separately. An investigator for the *Journal of Irreproducible Results* decides to investigate this hypothesis.

During the month of December 2008, the dates of menses onset were recorded for two samples of women. The data for the first sample consisted of menses onset histories for 10 women who had lived together continuously in a single dormitory at Smith College in Northampton, MA during 2008. The sample variance was calculated to be $s^2 = 30.3$. The data for the second sample included menses onset histories for 25 women of comparable ages attending the University of Massachusetts/Amherst, all of whom lived in separate dwellings. The sample variance for this sample was $s^2 = 69.7$.

Using the appropriate statistical hypothesis test, do these data suggest that menses onset dates are more synchronized for women living together relative to women living separately?

**Answer:**

- $H_{null}: \sigma^2_{separately} = \sigma^2_{together}$
- $H_{alternative}: \sigma^2_{separately} > \sigma^2_{together}$, one sided

- $F$ statistic $= \frac{S^2_{separately}}{S^2_{together}} \sim F_{24,9}$ under null

- $F = \frac{69.7}{30.3} = 2.3$
- $pvalue = 0.09726$

- Do not reject. Conclude these data are insufficient evidence of a synchrony of menses among women who live together, relative to women who do not live together.
7.

Consider the setting of a single sample of $n=16$ data values that are a random sample from a normal distribution. Suppose it is of interest to perform a type I error $\alpha = 0.01$ statistical hypothesis test of $H_0: \mu \geq 100$ versus $H_A: \mu < 100$, $\alpha = 0.01$. Suppose further that $\sigma$ is unknown. State the appropriate test statistic (2 points) and determine the critical region for values of the sample mean $\bar{X}$ (8 points).

**Answer:**

**Appropriate test statistic is student’s t with df=15**

**Critical region is** $\bar{X}_{n=16} \leq (-0.65)[S] + 100$

**Solution:**

(1) For a one sided test with type I error $= 0.01$, critical region for t-statistic is $t\text{-statistic} \leq t_{df=15; .01}$ or $t\text{-statistic} \leq -2.60$

(2) Substitution of t-statistic $= \frac{\bar{X}_{n=16} - \mu_{NULL}}{SE(\bar{X}_{n=16})} = \frac{\bar{X}_{n=16} - 100}{S/\sqrt{16}}$ yields critical region

$$\frac{\bar{X}_{n=16} - 100}{S/\sqrt{16}} \leq -2.60 \Rightarrow \bar{X}_{n=16} \leq -2.60S/\sqrt{16} + 100 \Rightarrow$$

$$\bar{X}_{n=16} \leq -0.65(S) + 100$$