N-flationary magnetic fields

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Looking for inflation in String Theory...
...as in Field Theory...

**radiative stability**
of the inflaton potential is required!

As usual, symmetries help:
a *shift symmetry* can keep the inflaton potential flat
natural inflaton
= pseudo Nambu-Goldstone boson (pNBG)

\[ V(\phi) = \Lambda^4 [1 + \cos(\phi/f)] \]

WMAP requires \( f > 3 \, M_P \)...

Freese and Kinney 2004

...while String Theory seems to tolerate only \( f < M_P \)!

Banks, Dine, Fox and Gorbatov 2003
A way out:

more than one $pNGB$

- Mixture of a $pNGB$ and a modulus
  - Blanco-Pillado et al 2004

- Two $pNGB$s
  - Kim, Nilles and Peloso 2004

- A lot of $pNGB$s
  - Dimopoulos, Kachru, McGreevy and Wacker 2005

N-flation
How does N-flation work?

Start from N pNGBs:

\[
\mathcal{L} = -\sqrt{-g} \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + \Lambda_i^4 \left[ 1 + \cos(\phi_i/f_i) \right] \right\}
\]

Assume that all the $\phi_i$, all the $f_i$ and all the $\Lambda_i$ are equal:

\[
\mathcal{L} = -\sqrt{-g} \left\{ \frac{N}{2} (\partial \phi)^2 + N \Lambda^4 \left[ 1 + \cos(\phi/f) \right] \right\}
\]

Canonically normalized field \( \Phi = \sqrt{N} \phi \)

\[
\mathcal{L} = -\sqrt{-g} \left\{ \frac{1}{2} (\partial \Phi)^2 + N \Lambda^4 \left[ 1 + \cos\left( \frac{\Phi}{\sqrt{N} f} \right) \right] \right\}
\]

Can be $> M_p$

N>600 required by WMAP  
Kim and Liddle 2006
pNGBs are coupled to the electromagnetic field!

\[ \mathcal{L} \supset \sum_{i=1}^{N} \alpha \frac{\phi_i}{4 \, M_P} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

\[ (\alpha = \mathcal{O}(1)) \]

Magnetic fields can be produced by the rolling pNGBs at inflation

M. Anber, LS
Cosmological magnetic fields

- Observed with intensities of order $\mu$Gauss
- Coherence lengths of 10s of kpcs
- Unknown origin

Can be amplified by a dynamo mechanism

Seed field required $\sim 10^{-30}$ G

Davis, Lilley and Tornkvist 1999
back to our model...

Electromagnetic field coupled to the *sum* of the pNGBs

the *direction* of rolling of the pNGBs matters:

\[
\mathcal{L} \supset \sum_{i=1}^{N} \frac{\alpha}{4 M_P} \phi_i F_{\mu\nu} \tilde{F}^{\mu\nu}
\]

define \( \gamma = (N_+ - N_-)/N \) where

\( N_+ = \# \text{ of pNGBs with } \dot{\phi} > 0 \)

\( N_- = \# \text{ of pNGBs with } \dot{\phi} < 0 \)

\([-1 < \gamma < 1]\)
The main equation

\[
\frac{\partial^2 F_\pm}{\partial \tau^2} + \left( k^2 \pm \frac{\alpha \gamma \sqrt{N}}{M_P} \frac{d\Phi}{d\tau} k \right) F_\pm = 0
\]

\((F_\pm = >\text{ve and } <\text{ve helicity modes of the magnetic field})\)

The result depends only on one combination of parameters

\[
\xi \equiv |\alpha \gamma| \sqrt{N \epsilon / 2}
\]

where \(\epsilon\) is the slow-roll parameter
The main result

\[ F(\tau, \vec{k}) \simeq \sqrt{\frac{k}{2}} \left( \frac{k}{2\xi aH} \right)^{1/4} e^{-2\sqrt{2\xi k/aH}} e^{\pi \xi} \]
A Constraint...

The energy in the magnetic field should not exceed the energy in the inflaton condensate!

If insist on COBE normalization (H\sim10^{13}\text{GeV}),

\[ \xi < 7 \]

If require just H>10^{-3}\text{eV},

\[ \xi < 25 \]

Cf. Garretson, Field and Carroll 1992
Evolving the field in the cosmic plasma

The magnetic field produced has *maximal helicity*

Magnetohydrodynamic processes (*inverse cascade*) transfer power to large scales

From Jedamzik and Banerjee 2004
Final value of the magnetic field (before the dynamo)

\[ B \simeq 10^{-33} \frac{e^\pi \xi}{\xi^{17/12}} \left( \frac{T_{RH}}{10^9 \text{GeV}} \right)^{11/36} \left( \frac{l_{\text{phys}}}{10 \text{ kpc}} \right)^{-9/4} \text{ G} \]

\( \xi \geq 2 \)

is sufficient to initiate the dynamo
In terms of the original parameters

\[ \alpha \gamma \sqrt{N} \geq 10 \]

Enough magnetic field for \( \alpha \) and/or \( \gamma \sqrt{N} \) of \( O(\text{few}) \)!
Discussion...

- One obvious possibility: $N=\text{few}$, $\alpha \sim 10$
- More difficult: insist on $\alpha = 1$

E.g. for $N=600$, need $N_+ \sim 420$ and $N_- \sim 180$...

...rather improbable, if the theory is exactly symmetric wrt $\phi_i \rightarrow -\phi_i$

...but an asymmetry can exist:

(Blanco-Pillado et al 2004)

$$V(\phi) = \Lambda_1^4 \cos a\phi + \Lambda_2^4 \cos b\phi + \Lambda_3^4 \cos (a - b) \phi$$
An asymmetric axion potential...
Conclusions

• Models of inflation in string theory might naturally lead to the observed magnetic fields

• Overproduction of magnetic fields could kill some of these models

• $O(1)$ coefficients matter!