The Proper Treatment of Quantification in Ordinary English, Part 1:
The Fragment of English

We will now explore the analysis of English that Montague puts forth in his seminal paper, PTQ. As we’ve already seen, there are three principle parts to the analysis:

- The syntactic fragment of English (easy)
- The logical language, Intensional Logic (moderate)
- The translation from English to IL (difficult; after all, this is the actual semantic analysis)

1. The Syntactic Categories of English

1.1 The Categories

- In PTQ, Montague employs a system of syntactic categories (i.e., category labels) for English that are similar in structure to the types (i.e., a so-called categorical grammar)

- This allows for an elegant statement of the category-to-type correspondence between English and IL.

\( Cat \) is the smallest set such that:

a. \( e, t \in Cat \)

b. If \( A, B \in Cat \), then \( A/B, A//B \in Cat \)

1.2 On Reading the Category Labels

From the point of view of our type notation, the category notation is ‘flipped backwards’

\( A/B \) and \( A//B \) = Expressions that when ‘combined’ with an expression of category \( B \) yield an expression of category \( A \).

Illustration:
\( t/e, t//e \): Combines with an expression of category \( e \) to yield one of category \( t \)
\( \approx <e, t> \)
(3) a. **Question:** What is the difference between $A/B$ and $A//B$?

b. **Answer:** Nothing substantial. As we’ll see in a second, it allows Montague to distinguish between CNs and IVs.

c. **Quote:**
“We shall regard the categories $A/B$ and $A//B$ as playing the same semantical but different syntactical roles.”
(i.e., CN and IV have translations/meanings of the same type, but are different syntactic categories in English.)

(4) **Abbreviations for the Syntactic Categories**

- Up until now, we’ve been treating the labels below as the actual category labels of our language.

- In PTQ, though, Montague introduces them as *meta-linguistic abbreviations* for the more complex category labels defined in (1).

a. **Some New Definitions for Old Friends**

(i) $IV$ abbreviates $t/e$

(ii) $T$ abbreviates $t/IV$ ($= t/(t/e)$)

(iii) $CN$ abbreviates $t/e$

(iv) $TV$ abbreviates $IV/T$ ($= (t/e)/(t/(t/e))$)

**Note:** TVs are expressions that combine with *Terms* $(t/(t/e))$ to yield IVs $(t/e)$.

*Thus, TVs will directly combine syntactically with quantificational terms...*

b. **Some New Friends**

<table>
<thead>
<tr>
<th>Category</th>
<th>Informal Name</th>
<th>Abbreviates</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) IAV</td>
<td>‘IV-modifying adverb’</td>
<td>IV/IV</td>
<td>slowly, allegedly</td>
</tr>
<tr>
<td>(ii) t/t</td>
<td>‘Sentence-modifying adverb’</td>
<td></td>
<td>necessarily</td>
</tr>
<tr>
<td>(iii) IAV/T</td>
<td>‘IAV-making preposition’</td>
<td></td>
<td>in, about</td>
</tr>
<tr>
<td>(iv) IV/t</td>
<td>‘Sentence-taking verb phrase’</td>
<td></td>
<td>believe that</td>
</tr>
<tr>
<td>(v) IV//IV</td>
<td>‘IV-taking verb phrase’</td>
<td></td>
<td>try to, wish to</td>
</tr>
</tbody>
</table>

**Notes:**
- Montague uses the term ‘verb phrase’ in (iv) and (v) a bit differently from syntacticians.
- Just like TVs, prepositions in PTQ directly combine with quantificational terms.
- The only uses of the $A/B$ and $A//B$ distinction are:
  - $IV$ vs. $CN$  
    - *(run vs. man)*
  - IAV vs. IV//IV  
    - *(slowly vs. try to)*
(5) **Remark**  
In PTQ, there are an infinite number of category labels for English, some of which don’t seem to apply to any actual expression of English (e.g. e/t)

- Again, what this buys us is mainly just an elegant statement of the type-category correspondence in the translation system…

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2. **The Basic Expressions of English**

*The following are the basic expressions of the English fragment, exactly as written out by Montague in PTQ.*

(6) **Basic Expressions of the English Fragment**

- $B_{IV} = \{\text{run, walk, talk, rise, change}\}$
- $B_T = \{\text{John, Mary, Bill, ninety, he}_0, \text{he}_1, \text{he}_2, \ldots\}$
- $B_{TV} = \{\text{find, lose, eat, love, date, be, seek, conceive}\}$
- $B_{IAV} = \{\text{rapidly, slowly, voluntarily, allegedly}\}$
- $B_{CN} = \{\text{man, woman, park, fish, pen, unicorn, price, temperature}\}$
- $B_{t/t} = \{\text{necessarily}\}$
- $B_{IAV/T} = \{\text{in, about}\}$
- $B_{IV/A} = \{\text{believe that, assert that}\}$
- $B_{IV/IV} = \{\text{try to, wish to}\}$
- $B_A = \emptyset$ if A is any category other than those mentioned above. (In particular, the sets $B_e$ of basic entity expressions and $B_t$ of basic declarative sentences are empty.)

(7) **Remark**  
In the PTQ system, pronouns are all terms. They are also all masculine. Finally, note that the pronouns and indices are not syntactically separate expressions (unlike in UG).

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1 Just to forestall any confusion, in the PTQ paper itself, Montague uses ‘$\Lambda$’ to denote the null set.
(8) **Key Observation, Previewing Some Fun to Come…**

- Note that the following pairs of expressions are all members of the same category:

a. eat, seek [both TVs]
b. rapidly, allegedly [both IAVs]
c. in, about [both IAV/Ts]

- In each of these pairs, the second expression creates an ‘opaque’ environment, whereas the first one does not (i.e., the first creates a ‘transparent’ environment)

a. (i) John ate a unicorn. (entails ‘there is a unicorn’)
   (ii) John seeks a unicorn. (doesn’t entail ‘there is a unicorn’)
b. (i) John rapidly danced. (entails ‘John danced’)
   (ii) John allegedly danced. (doesn’t entail ‘John danced’)
c. (i) John talked in a house. (entails ‘there is a house’)
   (ii) John talked about a unicorn. (doesn’t entail ‘there is a unicorn’)

- Given that *seek*, *allegedly*, and *about* create ‘opaque’ contexts, we’d ideally want them to take *intensions* as arguments (LING 620).
  - *Seek* takes as argument the intension of *a unicorn*
  - *Allegedly* takes as argument the intension of *danced*.
  - *About* takes as argument the intension of *a unicorn*.

- **Recall, however, Montague’s requirement that expressions of the same category map to translations/meanings of the same type.**

- Consequently, since *seek*, *allegedly*, *about* have a meaning that takes intensions as arguments, we’ll also need for *eat*, *rapidly*, and *in* to have such meanings…

3. **The Syntactic Rules**

Given the basic expressions in (6), the syntactic rules outlined in this section will simultaneously define the set of meaningful expressions of English, $\bigcup_{A \in \text{Cat}} P_A$

(9) **Rule S1** $B_A \subseteq P_A$, for every category A.

(10) **Rule S2 (Forming Quantificational Terms)**

If $\zeta \in P_{CN}$, then $F_0(\zeta), F_1(\zeta), F_2(\zeta) \in P_T$,
where $F_0(\zeta) = \textit{every } \zeta$
$F_1(\zeta) = \textit{the } \zeta$
$F_2(\zeta)$ is a $\zeta$ or an $\zeta$ according as the first word in $\zeta$ takes a or an
(11) **Remarks on Rule S2**

a. Via the addition of \( F_1 \), we are adding definite determiners to our fragment.

Montague will provide a ‘Russellian’ analysis of definite terms: ²

\[
\text{The man smokes is true iff } \exists x_0 \forall x_1 (((\text{man’ } x_1) \leftrightarrow x_0 = x_1) \& (\text{smokes’ } x_0))
\]

*There is exactly one man, and he smokes.*

b. The definition of \( F_2 \) in (10) appeals to a notion that Montague does not ever explicitly define in the paper: whether a given word takes a or an

- In this case, though, we could easily revise (10) to explicitly contain the generalization ‘an before a vowel’.

(12) **Rule S3 (Relativization)**

If \( \zeta \in P_{\CN} \) and \( \phi \in P_t \), then \( F_{3,n}(\zeta,\phi) \in P_{\CN} \), where \( F_{3,n}(\zeta,\phi) = \zeta \text{ such that } \phi' \), and \( \phi' \) comes from \( \phi \) by replacing each occurrence of \( \text{he}_n \) or \( \text{him}_n \) by \{he, she, it\} or \{him, her, it\}, respectively, according as the first \( B_{\CN} \) in \( \zeta \) is of \{masc., fem., neuter\} gender.

**Illustration:**

(i) \( \text{woman} \in P_{\CN}, \ \text{John loves him}_5 \in P_t \)  \text{(Rules)}

(ii) \( F_{3,5}(\text{woman }, \text{John loves him}_5) \in P_{\CN} \)  \text{(Rule S3)}

(iii) \( \text{woman such that John loves her} \in P_{\CN} \)  \text{(def. of } F_{3,5} \text{)}

(13) **Remarks on Rule S3**

a. Rule S3 is the rule for forming relative clauses in PTQ. Note that it only forms ‘such that’ relatives; there are no mechanisms in PTQ for filler-gap dependencies.

b. Like \( F_{10,n} \) from the last handout, \( F_{3,n} \) is an infinite family of operations.

c. Also like \( F_{10,n} \), the translation operation corresponding to \( F_{3,n} \) will lambda abstract over the variable with index \( n \).

**Rough, Simplified Illustration:**

\( F_{3,5}(\text{woman }, \text{John loves him}_5) \) translates to: \((\lambda x_5 ((\text{woman’ } x_5) \& ((\text{loves } x_5) \text{ john’})))\)

\( \text{woman such that John loves her} \) translates to: \((\lambda x_5 ((\text{woman’ } x_5) \& ((\text{loves } x_5) \text{ john’})))\)

² Note that, since Intensional Logic is being used as the translation language, it would not be possible in the PTQ system to provide a Fregean/presuppositional analysis of definite terms.
(14) **Rule S4 (Subject-Predicate Rule)**

If $\alpha \in P_{\text{IV}}$ and $\delta \in P_{\text{IV}}$, then $F_4(\alpha, \delta) \in P_{\text{t}}$, where $F_4(\alpha, \delta) = \alpha \delta'$ and $\delta'$ is the result of replacing the first *verb* (i.e., member of $B_{\text{IV}}, B_{\text{TV}}, B_{\text{IV/t}},$ or $B_{\text{IV//IV}}$) in $\delta$ by its third person singular present.

(15) **Remarks on Rule S4**

a. In PTQ, the operation $F_4$ doesn’t just combine a subject with a predicate, it is also responsible for adding the tense and agreement morphology to the verb.

b. Due to its additional morpho-syntactic role, the definition of $F_4$ appeals to two important, fundamental concepts: (a) the notion of a ‘verb’, and (b) the notion of a verb’s ‘third person singular present’.

c. The notion ‘verb’ is defined as part of the definition of $F_4$. *Notice, though, that ‘verb’ is not actually a syntactic category in the system.*

- In fact, *verb can’t* be a syntactic category in MG, given the need for category-to-type correspondence.

d. The notion ‘third person singular present’ is nowhere defined in the paper. To properly implement this, though, we could imagine defining a function ‘3sgPRES’ which maps an English verb root to its 3rd singular present form:

\[
3\text{sgPRES}(\text{root}) = \begin{cases} 
\text{does,} & \text{if } \text{root} = \text{do} \\
\text{is,} & \text{if } \text{root} = \text{be} \\
\text{has,} & \text{if } \text{root} = \text{have} \\
\text{...} & \\
\text{root+s otherwise} 
\end{cases}
\]

- With this function, we’d simply say that $\delta'$ is obtained from $\delta$ by replacing the first verb $v$ in $\delta$ with $3\text{sgPRES}(v)$.

(15) **Rule S5 (Direct Object Rule)**

If $\delta \in P_{\text{IV/T}}$ and $\beta \in P_{\text{T}}$, then $F_5(\delta, \beta) \in P_{\text{IV}}$, where $F_5(\delta, \beta) = \delta \beta$ if $\beta$ does not have the form $\text{he}_n$, and $F_5(\delta, \text{he}_n) = \delta \text{ him}_n$.

*Remark:*

Again, the definition of $F_5$ captures the behavior of objective case on English pronouns.
(16) **Rule S6 (Prepositional Phrase Rule)**

If $\delta \in P_{IAV/T}$ and $\beta \in P_T$, then $F_5(\delta, \beta) \in P_{IAV}$

**Illustration:**

(i) $in \in P_{IAV/T}$, $he_2 \in P_T$ (Rule S1)
(ii) $F_5(in, he_2) \in P_{IAV}$ (Rule S6)
(iii) $in him_2 \in P_{IAV}$ (def. of $F_5$)

**Remark:**

S6 forms prepositional phrases in PTQ. Note its use of the operation $F_5$.
- This captures the presence of ACC on pronominal complements of Ps.
- This also nicely illustrates the difference between syntactic operations and syntactic rules in PTQ.

(17) **Rule S7 (Finite Complement Clause Rule)**

If $\delta \in P_{IV/t}$ and $\beta \in P_T$, then $F_6(\delta, \beta) \in P_{IV}$, where $F_6(\delta, \beta) = \delta \beta$

**Illustration:**

(i) $believe that \in P_{IV/t}$, $John runs \in P_T$ (Rule S1, S4)
(ii) $F_6(believe that, John runs) \in P_{IV}$ (Rule S7)
(iii) $believe that John runs \in P_{IV}$ (def. of $F_6$)

**Question:**

Did Montague really need to introduce a new operation $F_6$ here? Couldn’t he have simply continued to use $F_5$?
(After all, nothing in $P_T$ will be of the form $he_n$)

(18) **Rule S8 (Infinitival Control Rule)**

If $\delta \in P_{IV//IV}$ and $\beta \in P_{IV}$, then $F_6(\delta, \beta) \in P_{IV}$

**Illustration:**

(i) $try to \in P_{IV//IV}$, $find a unicorn \in P_{IV}$ (Rule S1, S5)
(ii) $F_6(try to, find a unicorn) \in P_{IV}$ (Rule S8)
(iii) $try to find a unicorn \in P_{IV}$ (def. of $F_6$)

(19) **Rule S9 (Sentential Adverbs Rule)**

If $\delta \in P_{vt}$ and $\beta \in P_t$, then $F_6(\delta, \beta) \in P_t$

**Illustration:**

(i) $necessarily \in P_{vt}$, $John runs \in P_t$ (Rule S1, S4)
(ii) $F_6(necessarily, John runs) \in P_t$ (Rule S9)
(iii) $necessarily John runs \in P_t$ (def. of $F_6$)

(20) **Rule S10 (Adverbs Rule)**

If $\delta \in P_{IV/IV}$ and $\beta \in P_{IV}$, then $F_7(\delta, \beta) \in P_{IV}$, where $F_7(\delta, \beta) = \beta \delta$

**Illustration:**

(i) $about a unicorn \in P_{IV/IV}$, $talk \in P_{IV}$ (Rule S1, S6)
(ii) $F_7(about a unicorn, talk) \in P_{IV}$ (Rule S10)
(iii) $talk about a unicorn \in P_{IV}$ (def. of $F_7$)
(21) **Rule S11 (Sentential Conjunction/Disjunction Rule)**
If $\phi, \psi \in P_t$, then $F_8(\phi, \psi), F_9(\phi, \psi) \in P_t$, where $F_8(\phi, \psi) = \phi$ and $F_9(\phi, \psi) = \phi$ or $\psi$.

(22) **Rule S12 (IV Conjunction/Disjunction Rule)**
If $\phi, \psi \in P_{IV}$, then $F_8(\phi, \psi), F_9(\phi, \psi) \in P_{IV}$

**Illustration:**
(i) *runs, loves Mary* $\in P_{IV}$  
   (Rule S1, S5)
(ii) $F_8(\text{runs, loves Mary}), F_9(\text{runs, loves Mary}) \in P_{IV}$  
   (Rule S12)
(iii) *runs and loves Mary, runs or loves Mary* $\in P_{IV}$  
   (def. of $F_8$ and $F_9$)

(23) **Remark**

- Montague uses the *same* syntactic operations, $F_8$ and $F_9$, to do conjunction/disjunction of sentences and conjunction/disjunction of IVs.

- As we’ll see later, this will cause some difficulty converting the PTQ system into the algebraic UG format
  - In the putative translation base, we’d need a *single* polynomial operation over IL to correspond to $F_8$ ($F_9$)
  - But a single such operation won’t give us the right translations for both sentential conjunction (disjunction) and IV conjunction (disjunction)

(24) **Rule S13 (Term Disjunction)**
If $\alpha, \beta \in P_T$, then $F_9(\alpha, \beta) \in P_T$

**Illustration:**
(i) *every man, John* $\in P_T$  
   (Rule S1, S2)
(ii) $F_9(\text{every man, John}) \in P_T$  
   (Rule S13)
(iii) *every man or John* $\in P_T$  
   (def. of $F_9$)

(25) a. **Question:** Why didn’t Montague also include a rule for *conjoining* terms?

b. **Educated Guess:** He didn’t want to have to deal with the collective reading of sentences like *John and Mary ate a unicorn*.

*And, at last we come to one of the centerpieces of the paper... the rules for 'Quantifying In'...*
(26) **Rule S14 (Quantifying In to Sentences)**
If $\alpha \in P_T$ and $\varphi \in P_t$, then $F_{10,n}(\alpha, \varphi) \in P_t$, where either:

(i) $\alpha$ does not have the form $he_k$ and $F_{10,n}(\alpha, \varphi)$ comes from $\varphi$ by replacing the first occurrence of $he_n$ or $him_n$ by $\alpha$, and all other occurrences of $he_n$ or $him_n$ by \{he, she, it\} or \{him, her, it\} respectively, according as the gender of the first $B_{CN}$ or $B_T$ in $\alpha$ is \{masculine, feminine, neuter\}, or

(ii) $\alpha = he_k$ and $F_{10,n}(\alpha, \varphi)$ comes from $\varphi$ by replacing all occurrences of $he_n$ or $him_n$ by $he_k$ or $him_k$ respectively.

(27) **Remark 1**
The definition of $F_{10,n}(\alpha, \varphi)$ is disjunctive; its value depends upon whether the ‘term argument’ $\alpha$ is a pronoun or not.

a. If $\alpha$ is not a pronoun, then we do the following:
   (i) Replace the first instance of $he_n$ or $him_n$ with $\alpha$, and
   (ii) All subsequent instances of $he_n$ or $him_n$ with a pronoun agreeing in gender with $\alpha$

   **Illustration:**
   (i) a woman $\in P_T$, he$_2$ runs and John likes him$_2$ $\in P_t$ (Rules)
   (ii) $F_{10,2}$ (a woman, he$_2$ runs and John likes him$_2$) $\in P_t$ (Rule S14)
   (iii) a woman runs and John likes her $\in P_t$ (def. of $F_{10,n}$)

b. If $\alpha$ is a pronoun, then we do the following:
   Replace all instances of $he_n$ or $him_n$ with $\alpha$ or its accusative variant, respectively

   **Illustration:**
   (i) he$_3$ $\in P_T$, he$_2$ runs and John likes him$_2$ $\in P_t$ (Rules)
   (ii) $F_{10,2}$ (he$_3$, he$_2$ runs and John likes him$_2$) $\in P_t$ (Rule S14)
   (iii) he$_3$ runs and John likes him$_3$ $\in P_t$ (def. of $F_{10,n}$)

(28) a. **Question:** Why did Montague use this disjunctive definition of $F_{10,n}$?
   After all, if we simply extended the first condition in (26ai) to pronouns, we’d end up getting almost the same strings:

   **Illustration:**
   (ii) $F_{10,2}$ (he$_3$, he$_2$ runs and John likes him$_2$) $\in P_t$
   (iii) he$_3$ runs and John likes him $\in P_t$

b. **Answer:** For whatever reason, condition (26ai) doesn’t copy the index onto the subsequent pronouns. Consequently, if we quantified-in a woman on (28aiii), we wouldn’t get gender agreement on the following pronouns.
The definition of $F_{10,a}(\alpha,\varphi)$ again appeals to a notion that Montague doesn’t ever define in the paper PTQ itself: the gender of a CN or T.

a. **A Possible Implementation:**

We could imagine defining a function $GEN$ whose domain is $B_{CN}$ and $B_T$ and whose range is \{MASC, FEM, NEUT\}.

\[
\begin{align*}
GEN(John) &= \text{MASC} \\
GEN(Mary) &= \text{FEM} \\
GEN(ninety) &= \text{NEUT} \\
\vdots \\
GEN(man) &= \text{MASC} \\
GEN(woman) &= \text{FEM} \\
GEN(fish) &= \text{NEUT} \\
\vdots
\end{align*}
\]

We could then reformulate (26) so that it makes reference to this $GEN$ function.

**Rule S15 (Quantifying In to NPs)**

If $\alpha \in P_T$ and $\varphi \in P_{CN}$, then $F_{10,a}(\alpha,\varphi) \in P_{CN}$

**Illustration:**

(i) every man $\in P_T$, woman such that he$_2$ likes her $\in P_{CN}$  \quad (Rules)
(ii) $F_{10,2}(\text{every man, woman such that he}_2 \text{ likes her}) \in P_{CN}$ \quad (Rule S15)
(iii) woman such that every man likes her $\in P_{CN}$ \quad (def. of $F_{10,a}$)

**Remark:** It still isn’t clear to me what use this rule is. Neither Montague himself nor Dowty et al. (1981) discuss it in detail.

**Rule S16 (Quantifying In to VPs)**

If $\alpha \in P_T$ and $\varphi \in P_{IV}$, then $F_{10,a}(\alpha,\varphi) \in P_{IV}$

**Illustration:**

(i) a unicorn $\in P_T$, find him$_2$ and eat him$_2$ $\in P_{IV}$ \quad (Rules)
(ii) $F_{10,2}(\text{a unicorn, find him}_2 \text{ and eat him}_2) \in P_{IV}$ \quad (Rule S16)
(iii) find a unicorn and eat it $\in P_{IV}$ \quad (def. of $F_{10,a}$)

**Remark:** As we’ll see, this allows us to capture the opaque/bound reading of “John wants to find a unicorn and eat it.”

Finally, to wrap things up, Montague introduces rules for adding negation and tense morphology...
Rule S17 (Rules for Tense and Negation)

If $\alpha \in P_T$ and $\delta \in P_{IV}$, then $F_{11}(\alpha, \delta), F_{12}(\alpha, \delta), F_{13}(\alpha, \delta), F_{14}(\alpha, \delta), F_{15}(\alpha, \delta) \in P_t$, where:

(i) $F_{11}(\alpha, \delta) = \alpha \delta'$ and $\delta'$ is the result of replacing the first verb in $\delta$ by its **negative third person singular present**.

**Illustration:**

(i) $\text{John} \in P_T, \text{run} \in P_{IV}$ (Rule S1)
(ii) $F_{11}(\text{John, run}) \in P_t$ (Rule S17)
(iii) $\text{John doesn’t run} \in P_t$ (def. of $F_{11}$)

(ii) $F_{12}(\alpha, \delta) = \alpha \delta'$ and $\delta'$ is the result of replacing the first verb in $\delta$ by its **third person singular future**.

**Illustration:**

(i) $\text{John} \in P_T, \text{run} \in P_{IV}$ (Rule S1)
(ii) $F_{12}(\text{John, run}) \in P_t$ (Rule S17)
(iii) $\text{John will run} \in P_t$ (def. of $F_{12}$)

(iii) $F_{13}(\alpha, \delta) = \alpha \delta'$ and $\delta'$ is the result of replacing the first verb in $\delta$ by its **negative third person singular future**.

**Illustration:**

(i) $\text{John} \in P_T, \text{run} \in P_{IV}$ (Rule S1)
(ii) $F_{13}(\text{John, run}) \in P_t$ (Rule S17)
(iii) $\text{John won’t run} \in P_t$ (def. of $F_{13}$)

(iv) $F_{14}(\alpha, \delta) = \alpha \delta'$ and $\delta'$ is the result of replacing the first verb in $\delta$ by its **third person singular present perfect**.

**Illustration:**

(i) $\text{John} \in P_T, \text{run} \in P_{IV}$ (Rule S1)
(ii) $F_{14}(\text{John, run}) \in P_t$ (Rule S17)
(iii) $\text{John has run} \in P_t$ (def. of $F_{14}$)

(v) $F_{15}(\alpha, \delta) = \alpha \delta'$ and $\delta'$ is the result of replacing the first verb in $\delta$ by its **negative third person singular present perfect**.

**Illustration:**

(i) $\text{John} \in P_T, \text{run} \in P_{IV}$ (Rule S1)
(ii) $F_{15}(\text{John, run}) \in P_t$ (Rule S17)
(iii) $\text{John hasn’t run} \in P_t$ (def. of $F_{15}$)
(33) **Remarks**

a. Again, Montague doesn’t ever properly define the terms negative...present, future, negative...future, present perfect, or negative...present perfect.

b. But, again, we could imagine defining an array of morpho-syntactic functions mapping every basic ‘verb’ to these values.

(i) \( \text{Neg3sgPRES}(\text{root}) = \begin{cases} 
  \text{isn’t,} & \text{if } \text{root} = \text{be} \\
  \text{doesn’t } \text{root} & \text{otherwise}
\end{cases} \)

(ii) \( \text{3sgFUT}(\text{root}) = \text{will } \text{root} \)

(iii) \( \text{Neg3sgFUT}(\text{root}) = \text{won’t } \text{root} \)

(iv) \( \text{3sgPERF}(\text{root}) = \text{has } \text{root}^{+\text{ed}} \)

(v) \( \text{NEG3sgPERF}(\text{root}) = \text{hasn’t } \text{root}^{+\text{ed}} \)

c. Montague doesn’t ‘decompose’ these morphological forms in the way we’ve come to expect from transformational morpho-syntactic analyses of English.

   o In PTQ, **negative 3rd singular future** and **3rd singular future** are just two different primitives; the ‘negative’ and ‘future’ isn’t separately factored out

d. Also, these features are introduced by the same rule that adds the subject with the predicate.

   o Thus, **doesn’t run** and **won’t run** aren’t themselves meaningful expressions.

   o Thus, we can’t get conjunctions like **John doesn’t run and won’t run**.

(34) **Snarky Side Remark**

Despite how little Montague regards the syntactic research ‘emanating from MIT’, he could have paid a bit closer attention to *Syntactic Structures* ;)

Those are all the rules! We can now use them to properly define the set of meaningful expressions for the English fragment.

(35) **The Phrases of the English Fragment**

\( \{ \text{P}_A \}_{A \in \text{Cat}} \) is the smallest family of sets indexed by \( \text{Cat} \) such that S1-S17 are true.

(36) **The Meaningful Expressions of the English Fragment**

\( \varphi \) is a meaningful expression of the fragment if there is an \( A \in \text{Cat} \) such that \( \varphi \in \text{P}_A \).

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3 Actually, what we really need here is a separate function PST-PARTICIPLE, mapping every verbal root to its past participle.
4. Some Illustrations of the Fragment

As the calculations below illustrate, the following are members of \( P_1 \):

\[(37) \quad \text{every man loves a woman such that she loves him} \]

(i) \( \text{man, woman} \in B_{\text{CN}}, \text{love} \in B_{\text{TV}}, \, \text{he}_0, \, \text{he}_1 \in B_T \) (by (6))

(ii) \( \text{man, woman} \in P_{\text{CN}}, \text{love} \in P_{\text{TV}}, \, \text{he}_0, \, \text{he}_1 \in P_T \) (by S1)

(iii) \( \text{love} \, \text{him}_0 \in P_{\text{IV}} \) (by S5)

(iv) \( \text{he}_1 \, \text{loves} \, \text{him}_0 \in P_1 \) (by S4)

(v) \( \text{woman such that she loves} \, \text{him}_0 \in P_{\text{CN}} \) (by S3)

(vi) \( \text{a woman such that she loves} \, \text{him}_0 \in P_T \) (by S2)

(vii) \( \text{love a woman such that she loves} \, \text{him}_0 \in P_{\text{IV}} \) (by S5)

(viii) \( \text{he}_0 \, \text{loves a woman such that she loves} \, \text{him}_0 \in P_1 \) (by S4)

(ix) \( \text{every man} \in P_T \) (by S2)

(x) \( \text{every man loves a woman such that she loves him} \in P_T \) (by S14)

Analysis Tree, Illustrating the Derivation

```
       every man loves a woman such that she loves him, 10,0
         /               \
      every man, 0    he_0 loves a woman such that she loves him_0, 4
         |                     \
    man       he_0       love a woman such that she loves him_0, 5
       |                     \
    love       a woman such that she loves him_0, 2
          |                         \
      woman       he_1 loves him_0, 4
             \
      he_1       love him_0, 5
            \
      love     he_0
```
(38) **John seeks a unicorn.**

   a. **Derivation / Analysis One**
      (i) \( \text{unicorn} \in B_{CN}, \text{seek} \in B_{TV}, \text{John} \in B_T \) (by (6))
      (ii) \( \text{unicorn} \in P_{CN}, \text{seek} \in P_{TV}, \text{John} \in P_T \) (by S1)
      (iii) \( \text{a unicorn} \in P_T \) (by S2)
      (iv) \( \text{seek a unicorn} \in P_{IV} \) (by S5)
      (v) \( \text{John seeks a unicorn} \in P_t \) (by S4)

\[\text{John seeks a unicorn, 4} \]

\[\text{John} \quad \text{seek a unicorn, 5} \]

\[\text{seek} \quad \text{a unicorn, 2} \]

\[\text{unicorn} \]

   b. **Derivation / Analysis Two**
      (i) \( \text{unicorn} \in B_{CN}, \text{seek} \in B_{TV}, \text{John, he}_0 \in B_T \) (by (6))
      (ii) \( \text{unicorn} \in P_{CN}, \text{seek} \in P_{TV}, \text{John, he}_0 \in P_T \) (by S1)
      (iii) \( \text{a unicorn} \in P_T \) (by S2)
      (iv) \( \text{seek him}_0 \in P_{IV} \) (by S5)
      (v) \( \text{John seeks him}_0 \in P_t \) (by S4)
      (vi) \( \text{John seeks a unicorn} \in P_t \) (by S14)

\[\text{John seeks a unicorn, 10,0} \]

\[\text{a unicorn, 2} \quad \text{John seeks him}_0, 4 \]

\[\text{unicorn} \quad \text{John} \quad \text{seek him}_0, 5 \]

\[\text{seek} \quad \text{he}_0 \]

(39) **Remarks**

- Again, we see that our English fragment admits of syntactically (and semantically) ambiguous expressions.
- Again, this will present no problems for PTQ, where the translation relation needn’t be a function.
- In addition, we’ll see that under the derivation in (38a), sentence (38) receives the *de dicto* reading, while under the derivation in (38b), it receives *de re* reading.
(40) **Some Optional Exercises for Students**

Show how our English fragment predicts that each of the following are members of $P_1$

a. **John doesn’t love the man such that he runs**

($\approx$ *John doesn’t love the man who runs.*)

b. **every man will talk about Mary**

c. **Bill has thought that a man loves him**

d. **every woman runs or loves Bill**