Montague’s Theory of Translation: Translation Bases and Indirect Interpretations 1

1. From Translation Functions to Interpretations

1. What We Currently Have

   a. Mini-English (and Disambiguated-Mini-English):
      Mini-English is $< E, K, X, S > \gamma \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}, \delta \in \Delta, R >$
      (i) $< E, K, X, S > \gamma \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}, \delta \in \Delta$ is Disambiguated
          Mini-English
      (ii) $R$ maps trees in $E$ to the first member of their root node.

   b. Politics-NoQ
      Politics-NoQ is the language $< A, F, X, S, t > \gamma \in \{ \text{Concat, Not, And} \}, \tau \in T, \text{Id} >$
      (i) $< A, F, X, S, t > \gamma \in \{ \text{Concat, Not, And} \}, \tau \in T$ is the disambiguated language
          Politics-NoQ.
      (ii) $\text{Id}$ is the identity function.

   c. Interpretation for Politics-NoQ
      Let the set $S = \{ \text{Michelle, Barack, Mitt} \}$. Let $\mathbf{B} = < B, G, f > \gamma \in \{ \text{Concat, Not, And} \}$ be the Fregean interpretation based on $S$, such that $f$ is as defined before.

   d. Translation Base from Mini-English to Politics-NoQ
      Let be $\mathbf{T}$ is the translation base $< g, H, j > \gamma \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$, where $g$, $H$, and $j$ are as defined before.

Where $k$ is the translation function determined by $\mathbf{T}$ and $h$ is the meaning assignment determined by $\mathbf{B}$, we have the following homomorphisms.

\[
\begin{align*}
< E, K > \gamma & \in \{ \text{Merge-S, Merge-IV, Not, And, If} \} & < A, F > \gamma & \in \{ \text{Concat, Not, And} \} \\
\downarrow k & & \downarrow h \\
< A, H > \gamma & \in \{ \text{Merge-S, Merge-IV, Not, And, If} \} & < B, G > \gamma & \in \{ \text{Concat, Not, And} \}
\end{align*}
\]

(2) Key Issue
   • What we want is a homomorphism that maps $E$ to $B$.
   • But since the operation indices are different in $< A, H > \gamma \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$
     and $< A, F > \gamma \in \{ \text{Concat, Not, And} \}$ we don’t have such a homomorphism yet…

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1 These notes are based upon material in the following readings: Halvorsen & Ladusaw (1979), Dowty et al. (1981) Chapter 8, and Thomason (1974) Chapter 7 (Montague’s “Universal Grammar”).
(3) **The Key Theorem Relating Translation to Interpretation**
If \( <A, F_\gamma> \in \Gamma \) is an algebra, \( h \) is a homomorphism from \( <A, F_\gamma> \in \Gamma \) to some algebra \( <B, G_\gamma> \in \Gamma \), and for each \( \gamma \in \Pi \), \( H_\gamma \) is a polynomial operation over \( <A, F_\gamma> \), then there is exactly one algebra \( <B, G_\gamma> \in \Pi \) such that \( h \) is a homomorphism from \( <A, H_\gamma> \in \Pi \) to \( <B, G_\gamma> \in \Pi \) (Montague 1974: 225)

(4) **Restatement of the Key Theorem**
Suppose the following conditions all hold:

a. There are two algebras \( A = <A, F_\gamma> \in \Gamma \) and \( B = <B, G_\gamma> \in \Gamma \) and a homomorphism \( h \) from \( A \) to \( B \).

b. There is an algebra \( A' = <A, H_\gamma> \in \Pi \) where \( \gamma \in \Pi \) are all polynomial operations over \( <A, F_\gamma> \), and \( \gamma \in \Gamma \).

c. There is therefore one (exactly one) algebra \( B' = <B, G_\gamma> \in \Pi \) such that \( h \) is also a homomorphism from \( A' \) to \( B' \).

(5) **The Importance of the Key Theorem**

- Conditions (4a-c) are exactly what we have in (1)!

  a. \( <A, F_\gamma> \in \{\text{Concat, Not, And}\} \) and \( <B, G_\gamma> \in \{\text{Concat, Not, And}\} \) are algebras.

  b. Meaning assignment \( h \) is a homomorphism from \( \gamma \in \Pi \) to \( \gamma \in \Gamma \).

  c. \( <A, H_\gamma> \in \{\text{Merge-S, Merge-IV, Not, And, If}\} \) is an algebra where the operations are all polynomial operations over \( <A, F_\gamma> \).

- Therefore, the key theorem in (3)/(4) guarantees us the following:

  There is an algebra \( <B, G_\gamma> \in \{\text{Merge-S, Merge-IV, Not, And, If}\} \) such that \( h \) is also a homomorphism to it from \( <A, H_\gamma> \in \{\text{Merge-S, Merge-IV, Not, And, If}\} \)

\[
\begin{align*}
&<E, K_\gamma> \in \{\text{Merge-S, Merge-IV, Not, And, If}\} \\
&\downarrow \quad k \\
&<A, H_\gamma> \in \{\text{Merge-S, Merge-IV, Not, And, If}\} \\
&\downarrow \quad h \\
&<B, G_\gamma> \in \{\text{Merge-S, Merge-IV, Not, And, If}\}
\end{align*}
\]

\[
\begin{align*}
&<A, F_\gamma> \in \{\text{Concat, Not, And}\} \\
&\downarrow \quad h \\
&<B, G_\gamma> \in \{\text{Concat, Not, And}\}
\end{align*}
\]
Remarks

- Note that in the algebra $< \mathcal{B}, \gamma \mathcal{G}_\gamma > \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$ the set $\mathcal{B}$ is the set of meanings in our interpretation of Politics-NoQ.

- Although Montague doesn’t say it, the operations $\{ \mathcal{G}_\gamma \} \in \{ \text{Concat, Not, And} \}$ whose definitions mirror those of the operations in our translation base $\{ \mathcal{H}_\gamma \} \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$ are all polynomial operations over $< \mathcal{B}, \gamma \mathcal{G}_\gamma > \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$

- As desired, $h \circ k$ is a homomorphism from $< \mathcal{E}, \gamma \mathcal{K}_\gamma > \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$ (Disambiguated-Mini-English) to $< \mathcal{B}, \gamma \mathcal{G}_\gamma > \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$ (our derived semantic algebra).

- Thus, thanks to our translation base $T$, we now have an interpretation for Mini-English.

The Interpretation for Mini-English: $< \mathcal{B}, \gamma \mathcal{G}_\gamma \cdot h \circ j > \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$

(i) $< \mathcal{B}, \gamma \mathcal{G}_\gamma > \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$ is the derived algebra guaranteed by (3)/(4),

(ii) $h$ is the meaning assignment determined by $< \mathcal{B}, \gamma \mathcal{G}_\gamma > \in \{ \text{Concat, Not, And} \}$

(iii) $j$ is the lexical translation function in our translation base $T$.

- Note that this structure will satisfy our general definition of an interpretation:
  
  o Because $h \circ k$ is a homomorphism, we know that for all $\gamma \in \{ \text{Merge-S, Merge-IV, Not, And, If} \}$, $K_\gamma$ and $G_\gamma$ are of the same arity.

  o $h \circ j$ is a function from the basic expressions of Mini-English into $\mathcal{B}$.

Clearly, this result in (5)/(6) generalizes to all languages, allowing us to state the following general theorem...

(7) **General Theorem on Indirect Interpretation**

Let $L$ and $L'$ be languages such that there is an interpretation $\mathcal{B}$ for $L'$ and a translation base $T$ from $L$ to $L'$. There is an interpretation $\mathcal{B}'$ for $L$.

You can no doubt already see how (7) follows from what we’ve seen so far...

For those who are interested, we can give a more explicit proof of it as in (8).
Proof of the General Theorem on Indirect Interpretation

- Let $h$ be the meaning assignment determined by $B$. Let $k$ be the translation function determined by $T$. Let $j$ be the lexical translation function in $T$.

- By definition, $h$ is a homomorphism from the syntactic algebra of $L' <A, F_{\gamma}, \gamma \in \Gamma>$ to the semantic algebra of $B <B, G_{\gamma}, \gamma \in \Gamma>$.

- By definition, $k$ is a homomorphism from the syntactic algebra of $L <E, K_{\gamma}, \gamma \in \Pi>$ to the algebra $<A, H_{\gamma}, \gamma \in \Pi>$ where $\{H_{\gamma}\}_{\gamma \in \Pi}$ are all polynomial operations over $<A, F_{\gamma}, \gamma \in \Gamma>$.

- Therefore, by the theorem in (3)/(4), there is an algebra $<B, G_{\gamma}, \gamma \in \Pi>$ such that $h$ is also a homomorphism from $<A, H_{\gamma}, \gamma \in \Pi>$ to $<B, G_{\gamma}, \gamma \in \Pi>$.

- Consequently, $h^o k$ is a homomorphism from $<E, K_{\gamma}, \gamma \in \Pi>$ to $<B, G_{\gamma}, \gamma \in \Pi>$.

- Therefore, we know that for all $\gamma \in \Pi$, $K_{\gamma}$ and $G_{\gamma}$ have the same arity.

- Moreover, $h^o j$ is a function from the basic categories in $L$ to $B$.

- Therefore, the structure $<B, G_{\gamma}, h^o j, \gamma \in \Pi>$ is an interpretation for $L$.

Remark
Furthermore, the meaning assignment determined by the interpretation $<B, G_{\gamma}, h^o j, \gamma \in \Pi>$ will be $h^o k$ (proof left as exercise for the student).

The Big Upshot

a. Our Initial Question:
Given our background theory of language and meaning, under what conditions can we guarantee that translating from one language $L$ into another language $L'$ gives us a compositional semantics for $L$.

b. Answer:
If we can provide an interpretation for $L'$ and the translation from $L$ to $L'$ satisfy the conditions of a translation base, then we are guaranteed a compositional semantics for $L$.

Thus, a new viable path to providing a semantics for a (natural) language is to provide a translation base from that language to a logical language whose semantics is already defined.
2. Illustration: Mini-English and Politics-NoQ

(11) First Observation
When we compose together $k$ and $h$ in (1), we get a mapping from sentences of Disambiguated-Mini-English to meanings in B.

**Illustration:** Let $T$ be the tree such that $R(T) = Barack$ loves Michelle.

a. $h^o k(T) = (by$ definition of function composition)
b. $h(k(T)) = (by$ definition of Mini-English)
c. $h(k(K_{\text{Merge-S}}(<Barack, \emptyset>, K_{\text{Merge-IV}}(<\text{loves } \emptyset>, <\text{Michelle, } \emptyset>) ))) = (by$ homomorphism prop.)
d. $h(H_{\text{Merge-S}}(k(<Barack, \emptyset>), H_{\text{Merge-IV}}(k(<\text{loves } \emptyset>), k(<\text{Michelle, } \emptyset>)))) = (by$ definition of $k$)
e. $h(H_{\text{Merge-S}}(\text{barack'}, H_{\text{Merge-IV}}(\text{loves'}, \text{michelle'}))) = (by$ def. of $H_{\text{Merge-S}}$)
f. $h(H_{\text{Merge-S}}(\text{barack'}, \text{(loves' michelle')})) = (by$ definition of $H_{\text{Merge-S}}$)
g. $h((\text{loves' michelle'}) \text{ barack'}) = (by$ definition of Politics-NoQ)
h. $h(F_{\text{Concat}}(F_{\text{Concat}}(\text{loves'}, \text{michelle'}), \text{barack'})) = (by$ homomorphism prop.)
i. $G_{\text{Concat}}(G_{\text{Concat}}(h(\text{loves'}), h(\text{michelle'})), h(\text{barack'})) = (by$ def. of $h$

j. $G_{\text{Concat}}(G_{\text{Concat}}(j, \text{Michelle }, \text{Barack })) = (by$ def. of $G_{\text{Concat}}$
k. $G_{\text{Concat}}(j(\text{Michelle } ), \text{Barack })) = (by$ def. of $G_{\text{Concat}}$
l. $j(\text{Michelle})(\text{Barack}) = (by$ def. of $j$
m. $1$

(12) Second Observation
Observing the behavior of $h^o k$ over a range of examples, we can directly construct an interpretation for Disambiguated-Mini-English, which will mirror the behavior of $h^o k$
The Interpretation of Disambiguated Mini-English

Let \(\langle B, G, l \rangle \in \{\text{Merge-S}, \text{Merge-IV}, \text{Not}, \text{And}, \text{If}\}\) be the structure defined as follows:

a. The Definition of the Set \(B\)
   The set \(B\) is the same set as the set \(B\) in \(\langle B, G, f \rangle \in \{\text{Concat}, \text{Not}, \text{And}\}\), the interpretation of Politics-NoQ we had defined previously.
   - That is \(B = \bigcup_{\tau \in T} D_{\tau}\) where \(\{\text{Michelle, Barack, Mitt}\}\).

b. The Definition of the Semantic Operations
   The operations \(\{G_\gamma\} \in \{\text{Merge-S}, \text{Merge-IV}, \text{Not}, \text{And}, \text{If}\}\) are defined as follows:
   (i) \(G_{\text{Not}} = G_{\text{Not}}\) (defined previously)
   (ii) \(G_{\text{And}} = G_{\text{And}}\) (defined previously)
   (iii) \(G_{\text{If}} = G_{\text{Not}} < G_{\text{And}} < \text{Id}_{1,2}, G_{\text{Not}} < \text{Id}_{2,2} \rangle \rangle \)
   (iv) \(G_{\text{Merge-IV}} = G_{\text{Concat}}\) (defined previously)
   (v) \(G_{\text{Merge-S}} = G_{\text{Concat}} < \text{Id}_{2,2}, \text{Id}_{1,2} \rangle\)

Note:
- The definitions of the operations \(\{G_\gamma\} \in \{\text{Merge-S}, \text{Merge-IV}, \text{Not}, \text{And}, \text{If}\}\) mirror the definitions of the operations \(\{H_\gamma\} \in \{\text{Merge-S}, \text{Merge-IV}, \text{Not}, \text{And}, \text{If}\}\) in our translation base.
- Moreover, \(\{G_\gamma\} \in \{\text{Merge-S}, \text{Merge-IV}, \text{Not}, \text{And}, \text{If}\}\) are all polynomial operations over the semantic algebra \(\langle B, G_\gamma \rangle \in \{\text{Concat}, \text{Not}, \text{And}\}\).
- Consequently, \(\langle B, G_\gamma \rangle \in \{\text{Merge-S}, \text{Merge-IV}, \text{Not}, \text{And}, \text{If}\}\) is an algebra.

c. The Definition of the Lexical Interpretation Function
   The lexical interpretation function \(l\) is defined as follows:
   (i) \(l(\langle \text{Barack}, \emptyset \rangle) = \text{Barack}\)
   (ii) \(l(\langle \text{Michelle}, \emptyset \rangle) = \text{Michelle}\)
   (iii) \(l(\langle \text{Mitt}, \emptyset \rangle) = \text{Mitt}\)
   (iv) \(l(\langle \text{smokes}, \emptyset \rangle) = \text{the function } h \text{ equal to } f(\text{smokes}^*)\)
   (v) \(l(\langle \text{loves}, \emptyset \rangle) = \text{the function } j \text{ equal to } f(\text{loves}^*)\)

Note: Where \(g\) is the meaning assignment determined by \(B\), \(l = g^0 j\)

Remark \(\langle B, G_\gamma, l \rangle \in \{\text{Merge-S}, \text{Merge-IV}, \text{Not}, \text{And}, \text{If}\}\) is an interpretation of Mini-English
Remark
The meaning assignment $g$ determined by $<B, G, I> \gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}$ is equal to $h^\gamma k$

Illustration
Let $g$ be the meaning assignment determined by $<B, G, I> \gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}$. Let $T$ be the tree such that $R(T) = \text{Barack loves Michelle}.$

(a) $g(T) = (\text{by definition of Mini-English})$

(c) $g(\text{Merge-S}(<\text{Barack}, \emptyset>, \text{Merge-IV}(<\text{loves}, \emptyset>, <\text{Michelle}, \emptyset>))) = (\text{by homomorphism prop.})$

(d) $G_{\text{Merge-S}}(g(<\text{Barack}, \emptyset>), G_{\text{Merge-IV}}(g(<\text{loves}, \emptyset>), g(<\text{Michelle}, \emptyset>))) = (\text{by definition of } g)$

(e) $G_{\text{Merge-S}}(\text{Barack}, G_{\text{Merge-IV}}(j, \text{Michelle})) = (\text{by def. of } G_{\text{Merge-IV}})$

(f) $G_{\text{Merge-S}}(\text{Barack}, j(\text{Michelle})) = (\text{by def. of } G_{\text{Merge-S}})$

(g) $j(\text{Michelle})(\text{Barack}) = (\text{by def. of } j)$

3. Indirect Interpretation: A Summary

Direct Interpretation
Let $L$ be a language. **Direct interpretation of $L$** is the specification of an interpretation $B$ of $L$.

Indirect Interpretation
Let $L$ be a language. **Indirect interpretation of $L$** is the specification of a language $L'$, an interpretation $B'$ of $L'$ and a translation base $T$ from $L$ to $L'$.

Indirect Interpretation Always Yields Direct Interpretation
- Suppose that we have indirectly interpreted the language $L$.
  - That is, we have defined a language $L'$ an interpretation $B'$ of $L'$ and a translation base $T$ from $L$ to $L'$

  - Given the result in (7)/(8) it is thus trivial to construct an interpretation $B$ of $L$.  

Some Choice Quotes

• From Halvorsen & Ladusaw (1979), p. 210:
  “An understanding of [the eliminability of indirect interpretation] is necessary to understand the use of...logics of PTQ and most other analyses within Montague grammar. As has been stated elsewhere, the [logical language] is an expository device and is in no way a necessary part of an analysis of any language offered within this theory. By using the easily interpreted [logical language] as a mediator, natural languages can be analyzed syntactically and then provided with a translation...from them to [the logical language] to induce their interpretation....This method of analysis amounts to direct interpretation of natural language.” (emphasis mine)

• From Dowty et al. (1981), p. 263:
  “Translating English into [a logical language] was therefore not essential to interpreting the English phrases we generated; it was simply a convenient intermediate step in assigning them meanings. This step could have been eliminated had we chosen to describe the interpretation of English directly... This point is important, because anyone who does not appreciate it may misunderstand the role of [logical languages] in applications of Montague’s descriptive framework to natural languages.” (emphasis mine)

Why Do Indirect Interpretation?

It’s Just Sometimes Conceptually ‘Easier’

If the logical language is well-designed and familiar to readers, then it can provide a more ‘perspicuous’ representation (statement / name) of the meanings that we wish to be assigned to the (natural) language expressions.

Illustration (From 610):

‘[\lambda x : x smokes ]’ vs. ‘The function f from De to Dt such that for all x \in De, f(x) = 1 iff x smokes.’

• From Halvorsen & Ladusaw (1979), p. 216:
  “Since the translation process is less involved than interpretation, presentation of fragments of the languages becomes clearer.”

• From Dowty et al. (1981), p. 264:
  “[The purpose of indirect interpretation] was to have a convenient, compact notation for giving a briefer statement of semantic rules than we were able to give in earlier chapters of this book, where semantic rules were formulated rather long-windedly in English... [The logical language] could provide us with names for meanings...”
Road Map of Where We’ve Been and Where We’re Going

- We’ve now covered the conceptual core of the Montague Grammar architecture, as well as the key sections of Montague’s “Universal Grammar”.
  - Sections of UG Covered: 1, 2, 3, 5

- What we haven’t covered from UG is:
  - The general theory of Fregean interpretations
  - Montague’s presentation of the syntax/semantics of his Intensional Logic
  - Montague’s presentation of the syntax/translation of a fragment of English

- However, those sections of UG are largely superceded by Montague’s paper “The Proper Treatment of Quantification in Ordinary English” (PTQ)
  - If we had all the time in the world, we would cover both systems and compare them (hint for final paper).
  - Given our limited time, however, I’d like to now move us from UG to PTQ.

- What’s Next on the Agenda:
  - Extending this system to handle quantification in FOL and English.
    - An algebraic treatment of FOL (with quantification)
    - An translation base from a fragment of English to FOL.
  
  Following that, we’ll examine Montague’s Intensional Logic and its applications to English (and various puzzles therein).