1. Introduction

(1) Our Main Outstanding Problem: Cumulative Readings

How do we augment our semantics of plural so that a sentence like (a) is true in a situation like (b)?

a. Sentence: Three TAs graded thirty exams.

b. Situation: Dave graded 10 exams. Mary graded 5. Sue graded 15.

(2) Some Related Problems

a. Cover Readings

How is it that a sentence like (i) is true in a situation like (ii)?

(i) Sentence: Six boys met.

(ii) Situation: Dave and Bill met. Tom and Frank met. John and Joe met.

b. Co-Distributive Readings

How is that a sentence like (i) is true in a situation like (ii)?

(i) Sentence: The sides of square 1 are parallel to the sides of square 2.

(ii) Situation: 

(3) A Desideratum

It would be ideal if one mechanism could capture all the facts in (1)-(2).

(4) Observation

- We’ve already seen that our null ‘*’-operator provides a solution to (2a).
- Is there, then, a way that the ‘*’-operator could provide a solution to (1) and (2b)??

(5) How Krifka (1999) Fits In

- Krifka’s main target here is really the semantics of modifiers like at most/least, and how their meaning differs from what ‘classic’ GQ theory would predict.
- However, along the way, he lays out a treatment of cumulative readings that has since become rather popular (see also Krifka 1989, Krifka 1992)
2. Krifka’s Proposal: The Universal Cumulativity of VPs

(6) A Quick Comment on My Exposition

- Krifka’s approach to cumulative readings is given a rather compact presentation by Krifka (1989, 1992, 1999)
- For this reason, I’m going to present his main proposal in a series of steps, each of which should not actually be attributed to Krifka himself.

2.1 Step One: Generalizing our ‘**’-Operator

Thus far, our meta-language ‘**’-operator can only apply to one-place predicates

- This is because only one-place predicates are equivalent to sets of entities, and our ‘**’-operator can really only apply to sets of entities.
  - This, in turn, is because our ‘**’-operator is defined as in (7), as the closure of a set under our ‘plurality-forming’ operator ‘+’, and ‘+’ itself is defined only to take entities as arguments.

(7) Definition of our Meta-Language ‘**’-Operator

\[ *S = \begin{align*}
  \text{The smallest set such that} & \quad a. \quad D \subseteq *D \\
  & \quad b. \quad \text{If } x, y \in *D, \text{ then } x+y \in *D
\end{align*} \]

(8) Illustration

Let \( S = \{ \langle \text{Dave, Sue} \rangle, \langle \text{Bill, Frank} \rangle \} \). \( *S \) is not defined, because it is not possible for ‘+’ to join together the pairs \( \langle \text{Dave, Sue} \rangle \) and \( \langle \text{Bill, Frank} \rangle \)

(9) Observation

If the operator ‘+’ were defined for arbitrary tuples of entities, then our ‘**’-operator would be able to apply to predicates of any arity!

So let’s extend our definition of ‘+’ so that it can apply to tuples

(10) The Plurality Forming Operator ‘+’, New Definition

a. Base Case: Let \( x \) and \( y \) be two entities. \( x+y \) = the plurality formed from \( x \) and \( y \)

b. Induction Case:
   - Let \( \langle x_1, ..., x_n \rangle \) and \( \langle y_1, ..., y_n \rangle \) be two \( n \)-tuples of entities.
   - \( \langle x_1, ..., x_n \rangle + \langle y_1, ..., y_n \rangle = \langle x_1 + y_1, ..., x_n + y_n \rangle \)
(11) **Illustration**

a. \( \langle \text{Dave}, \text{Sue} \rangle + \langle \text{Bill}, \text{Frank} \rangle = \langle \text{Dave}+\text{Bill}, \text{Sue}+\text{Frank} \rangle \)

b. \( \langle \text{Joe}, \text{Bill}, \text{Sue} \rangle + \langle \text{Bill}, \text{Dave}, \text{Frank} \rangle = \langle \text{Joe}+\text{Bill}, \text{Bill}+\text{Dave}, \text{Sue}+\text{Frank} \rangle \)

c. \( \langle e_1, \text{Dave}, \text{piano}_1 \rangle + \langle e_2, \text{Bill}, \text{piano}_2 \rangle = \langle e_1 + e_2, \text{Dave}+\text{Bill}, \text{piano}_1+\text{piano}_2 \rangle \)

(12) **Major Consequence**

Given our redefinition of ‘+’ in (10), our ‘**’-operator can now apply to predicates of any arbitrary arity! (Note too that we’ve not actually redefined ‘*’ in any way here…)

**Illustration:**

- Suppose that \( [[\text{lifted}]] = \{ \langle \text{Dave}, \text{piano}_1 \rangle, \langle \text{Bill}, \text{piano}_2 \rangle \} \)
- Then clearly, \( *[[\text{lifted}]] = \{ \langle \text{Dave}, \text{piano}_1 \rangle, \langle \text{Bill}, \text{piano}_2 \rangle, \langle \text{Dave}, \text{piano}_1 \rangle + \langle \text{Bill}, \text{piano}_2 \rangle \} = \{ \langle \text{Dave}, \text{piano}_1 \rangle, \langle \text{Bill}, \text{piano}_2 \rangle, \langle \text{Dave}+\text{Bill}, \text{piano}_1+\text{piano}_2 \rangle \} \)

(13) **A Side-Note on Notation**

- Some authors use a slightly different notation for ‘*’ when it applies to a polyadic predicate.
- That is, rather than re-define ‘+’ as we do in (10), some authors prefer to introduce a new operator ‘**’, often called ‘double-star’, and define it as follows:

**Definition of ‘****’**: 
Let \( P \) be a set of pairs.
\( **P \) is the smallest set such that
a. \( P \subseteq **P \)

b. If \( \langle x, y \rangle, \langle s, t \rangle \in **P \), then \( \langle x+s, y+t \rangle \in **P \)

- Since it saves on notational symbols, we’ll just be using ‘*’ instead of ‘**’…
2.2 Step Two: Deriving Cumulative Readings with our ‘*-Operator

The redefinition in (10) is all we need to derive the main fact in (1). Consider the LF in (14a), where our ‘*-operator applies to the two-place predicate graded. It will be assigned the T-conditions in (14b).

(14) Deriving the Cumulative Reading

a. 

\[
\begin{array}{c}
\text{DP} \quad \text{TP}_5 \\
\text{three TAs} \\
\text{TP}_4 \\
\text{1} \\
\text{DP} \\
\text{thirty exams} \\
\text{TP}_3 \\
\text{TP}_2 \\
\text{TP}_1 \\
\text{t}_1 \\
\text{VP} \\
\text{V} \\
\text{t}_2 \\
\text{*} \\
\text{V} \\
\text{graded}
\end{array}
\]

b. \[\exists x : x \in [[[TA]]] \land |\text{ATOMS}(x)| = 3 \land \exists y : y \in [[[\text{exam}]]] \land |\text{ATOMS}(y)| = 30 \land <x,y> \in [[[\text{graded}]])\]

‘There is a group of three TAs x, and a group of thirty exams y, and <x,y> is a tuple that can be formed from ‘+-ing together the tuples in [[[graded]]]

(15) Key Fact: These T-Conditions are the ‘Cumulative Reading’

The T-conditions in (14b) would hold in a situation like (a) below. After all, in such a situation, [[[graded]]] is as in (b), and so *[[graded]] contains the pairs in (c).


b. \[[\text{graded}]] = \{ <\text{Dave, exam1}>, \ldots , <\text{Dave, exam10}>, <\text{Mary, exam11}>, \ldots , <\text{Mary, exam15}>, <\text{Sue, exam16}>, \ldots , <\text{Sue, exam30}> \}

c. *[[\text{graded}]] = \{ \ldots <\text{Dave, exam1}+\ldots+\text{exam10}>, <\text{Mary, exam11}+\ldots+\text{exam15}>, <\text{Sue, exam16}+\ldots+\text{exam30}>, \ldots <\text{Dave+Mary+Sue, exam1}+\ldots+\text{exam30}> \ldots \}
Thus, the redefinition of ‘+’ in (10) allows us to derive the ‘cumulative reading’ by applying our ‘*’-operator to polyadic predicates!...
Moreover, as we see below, we can now also derive (certain) ‘co-distributive readings’...

(16) Deriving the ‘Co-Distributive Reading’

a. The LF Structure of (2b)
   For simplicity’s sake, we assume that parallel to is a single V (it is certainly a single predicate in the lexicon).

```
TP
   /   \
DP   VP
   /     \
V     DP
     /   \
   V     the sides of square-2
     /   \
   parallel to
```

b. The T-Conditions of LF (16a)

If the sides of square-1 are a,b,c and d, and the sides of square-2 are e,f,g and h, then the LF in (16) has the following T-conditions.

\[ <a+b+c+d, e+f+g+h> \in *[\parallel]\]

(17) Key Fact: These T-Conditions are the ‘Co-Distributive Reading’

The T-conditions in (16b) would hold in situations like (a) below. After all, in such situations, *[\parallel] is as in (b), and so *[*[\parallel]] contains the pairs in (c)...

a.

```
+---+---+---+---+
|   |   |   |   |
| a | e | c | g |
|   | b |   | h |
+---+---+---+---+
```

b. *[\parallel] = \{ ... <a,e> ... <b,f> ... <c,g> ... <d,h> ... \}

c. *[*[\parallel]] = \{ ... <a+b+c+d, e+f+g+h> ... \}
Our system can now also capture the fact in (18), \textit{which receives no discussion or analysis in Scha (1981/1984)}.

(18) **Cumulative Readings in Conjunctions**
Sentence (a) is true in situation (b).

\begin{enumerate}
\item [a.] \textbf{Sentence:} Dave and Bill moved piano1, piano2 and piano3.
\item [b.] \textbf{Situation:} Dave moved piano1. Bill moved piano2 and piano3 (collectively).
\end{enumerate}

(19) **Deriving the Cumulative Reading with Conjunctions**

\begin{enumerate}
\item [a.] The LF Structure of (18a)

\begin{center}
\begin{tikzpicture}
  \node (TP) {TP};
  \node (ConjP) at (0,1) {ConjP};
  \node (VP) at (1,1) {VP};
  \node (Dave and Bill) at (-1,0) {Dave and Bill};
  \node (V) at (0,0) {V};
  \node (ConjP2) at (1,0) {ConjP};
  \node (Piano1 and Piano2 and Piano3) at (2,0) {Piano1 and Piano2 and Piano3};
  \node (moved) at (0,-1) {* V moved};
  \draw (TP) -- (ConjP);
  \draw (ConjP) -- (VP);
  \draw (Dave and Bill) -- (ConjP);
  \draw (VP) -- (ConjP2);
  \draw (ConjP2) -- (Piano1 and Piano2 and Piano3);
  \end{tikzpicture}
\end{center}

\item [b.] The T-Conditions of LF (19a)
\[
< \text{Dave+Bill}, \text{piano1+piano2+piano3} > \in *[[\text{moved}]]
\]
\end{enumerate}

(20) **Key Fact: These T-Conditions are the ‘Cumulative Reading’**

The T-conditions in (19b) would hold in situation (18b). After all, in such a situation, \[[\text{moved}]\] is as in (a), and so \[*[[\text{moved}]]*\] is as in (b).

\begin{enumerate}
\item [a.] \[[\text{moved}]\] = \{ \langle \text{Dave, piano1} \rangle, \langle \text{Bill, piano2+piano3} \rangle \}
\item [b.] \*[[\text{moved}]] = \{ \ldots \langle \text{Dave+Bill, piano1+piano2+piano3} \rangle \ldots \}
\end{enumerate}

(21) **Summary**

\begin{itemize}
\item By redefining ‘+’ so that it applies to arbitrary tuples, our ‘*’-operator is able to apply to predicates of any arity.
\item We’ve seen that the readings that result from applying ‘*’ to non-monadic predicates are essentially those of the ‘cumulative’ and ‘co-distributive’ readings.
\item Thus, this system meets the key desideratum in (3): \textit{one single mechanisms – our ‘*’-operator is able to account for all the phenomena in (1) and (2)!}
\end{itemize}
2.3 Step Three: The Universality of Cumulativity

In our system above, cumulative and co-distributive readings are generated by applying our phonologically null ‘*’-operator to polyadic predicates...

...However, Krifka (1989, 1992, 1999) goes one step further, and proposes that these readings are actually due to an inherent and universal property of predicates in natural language...

(22) The Universality of Cumulativity (Krifka 1989, 1992, 1999)

Let P be a natural language predicate of any arity. If x, y ∈ [[P]], then x+y ∈ [[P]].

(23) Commentary

- Thus, in a sense, Krifka proposes that all natural language lexical items come pre-packaged with a ‘*’-operator.

- Thus, the relative naturalness of the cumulative readings follows: *we are able to generate these readings without adding any additional null structure to the LF whatsoever!*

- Note that inclusion of nouns under this generalization may be problematic, *unless we assume the semantics for number developed by Sauerland (2003, 2005).*

- For notational convenience, we will still write ‘*’ in our meta-language representations of the extensions of verbs, just to make clear that the predicates in question are closed under ‘+’

(24) Question: What About the Other Readings?

Will the principle in (22) rule out the possibility that (a) can receive a ‘collective reading’ under which it is true in situation (b)? Or, a ‘distributive reading’ under which it is true in situation (c)?

a. Three TAs graded thirty exams.

b. Collective Reading Situation:
   Dave, Mary and Sue (as a group) graded 30 exams (simultaneously).

c. Distributive Reading Situation:
   Dave graded 30 exams, Mary graded another 30 exams, and Sue graded still another 30 exams.
(25) **The Collective Reading is Still Possible**

Under the principle in (22), the sentence in (24a) receives the LF in (a), which is assigned the T-conditions in (b).

a.  

```
         TP5
        /   \
DP      TP4
      /   \
three TAs 1
        /   \
TP3
        /   \
DP      TP2
      /   \
thirty exams 2
        /   \
TP1
```

b. \[\exists x : x \in *[[TA]] \land |\text{ATOMS}(x)| = 3 \land \\
\quad \exists y : y \in *[[\text{exam}]] \land |\text{ATOMS}(y)| = 30 \land <x,y> \in *[\lambda s [\lambda t : t \text{ graded } s ]]\]

These T-conditions in (b) clearly hold in a situation like (24b). After all, in such a situation, the function \text{‘*}[\lambda s [\lambda t : t \text{ graded } s ]]\text{’}\ will be equivalent to the set in (c), and so \text{‘*}[\lambda s [\lambda t : t \text{ graded } s ]]\text{’}\ will be equivalent to the set in (c).

d. \{ <\text{Dave}+\text{Mary}+\text{Sue}, \text{exam}1+\ldots+\text{exam}30> \}

(26) **The Distributive Reading is Still Possible**

Our theory still admits of a phonologically null ‘*’-operator. Thus, sentence (24a) may be assigned the LF below.

a. **Possible LF for (24a):** \[ \text{Three TAs [ * [ 1 [ thirty exams [ 2 [ t_1 \text{ graded } t_2 ] \ldots ] ] ] ]} \]

This LF will be assigned the T-Conditions below. *Clearly these T-conditions will hold in situation (24c)!*

b. \[\exists x : x \in *[[TA]] \land |\text{ATOMS}(x)| = 3 \land \\
\quad x \in *[\lambda z : \exists y : y \in *[[\text{exam}]] \land |\text{ATOMS}(y)| = 30 \land \\
\quad <z,y> \in *[\lambda s [\lambda t : t \text{ graded } s ]]\]

‘There is a group of three TAs x, and x can be formed from the things z such that there is a group of thirty exams y such that 
<z,y> is a tuple that can be formed from the tuples <s,t> such that s graded t.’
3. Some Challenges for the Picture Above

The system is able to provide a unified account for the existence of cumulative, co-distributive and cover readings. It also does so in a way that predicts that these readings should be especially salient and common across languages…

...Nevertheless, it does fact some interesting (though not insuperable) challenges...

3.1 Other Co-Distributive Readings

We’ve seen that our ‘*-operator is now able to capture the ‘co-distributive’ reading of (2b). Recall, though, that Scha (1981/1984) groups (2b) with the following phenomena.

(27) Other ‘Co-Distributive’ Readings?

a. Sentence (i) is true in situation (ii).
   (i) Sentence: The squares contain the circles.
   (ii) Situation:

   ![Diagram of squares containing circles]

b. Sentence (i) is true in situation (ii)
   (i) Sentence: The sides of square1 cross the sides of square2.
   (ii) Situation:

   ![Diagram of squares crossing sides]
(28) **Problem 1**

Even assuming the principle in (22), our account does not predict the facts in (27a). After all, (27ai) would be assigned the T-conditions in (a), and those T-conditions don’t hold in situation (27a(ii)).

- In situation (27a(ii)), \{ <x,y> : x contains y \} will be as in (b).
  - Thus, \*\{ <x,y> : x contains y \} will be as in (c).

- Thus, in situation (27a(ii)), \*\{ <x,y> : x contains y \} does not contain the pair \(< sq1+sq2+sq3 , crl1+crl2+crl3 >, and so the T-conditions in (a) do not hold.

a. \(< sq1+sq2+sq3 , crl1+crl2+crl3 > \in \*\{ <x,y> : x contains y \}

b. \{ <sq1, crl1+crl2> , <sq2, crl3> \}

c. \{ <sq1, crl1+crl2> , <sq2, crl3> , <sq1+sq2, crl1+crl2+crl3> \}

(29) **Problem 2**

Even assuming the principle in (22), our account does not predict the facts in (27b). After all, (27bi) would be assigned the T-conditions in (a), and those T-conditions don’t hold in situation (27b(ii)).

- In situation (27b(ii)), \{ <x,y> : x cross y \} will be as in (b).
  - Thus, \*\{ <x,y> : x cross y \} will be as in (c).

- Thus, in situation (27b(ii)), \*\{ <x,y> : x cross y \} does not contain the pair \(< a+b+c+d, e+f+g+h >, and so the T-conditions in (a) do not hold.

a. \(< a+b+c+d, e+f+g+h > \in \*\{ <x,y> : x crosses y \}

b. \{ <c,f>, <f,c> , <b,e> , <e,f> \}

c. \{ ... <c+b, f+e > ... <c+f+b+e, c+f+b+e > ... \}

(30) **The General Problem**

- Under our approach to cumulative/co-distributive readings, the truth of such a reading requires that it be possible to ‘form’ the pair \(<\text{SUBJECT}, \text{OBJECT}>\) from the set of pairs \(<x,y>\) in the extension of the V…

- But in the examples in (27), the pair \(<\text{SUBJECT}, \text{OBJECT}>\) contains parts that are *not* in *any* of the pairs \(<x,y>\) in the extension of the V…

- That is, in these examples \(<\text{SUBJECT}, \text{OBJECT}>\) is strictly bigger than MAX([[[V]]]), and so cannot be a member of [[V]]!
So, what could be going on in these sentences?...

(31) **One Line of Thought (Scha 1981/1984)**

- The truth of (27a/bi) in situation (27a/bii) does not reflect the *cumulative* reading of those sentence, but rather simply reflects their *collective* reading.

- That is, in addition to our principle in (22), the lexical items *contain* and *cross* are subject to the following conditions, which (in some way) follow from their lexical semantics.

  a. \( <x, y> \in [[contain]] \iff \forall z \leq y . \exists s \leq x . s \text{ contains } z \)

  b. \( <x, y> \in [[cross]] \iff \exists z \leq y . \exists s \leq x . s \text{ crosses } z \)

- Clearly, with these ‘meaning postulates’ in place, we predict the truth of (27ai) in situation (27aii)

  o In that situation, \( \forall z \leq crl1+crl2+crl3 . \exists s \leq sq1+sq2+sq3 . s \text{ contains } z \)

  o Thus, given (31a), \( <sq1+sq2+sq3, crl1+crl2+crl3> \in [[contain]] \), as desired.

- Clearly, with these ‘meaning postulates’ in place, we predict the truth of (27bi) in situation (27bii)

  o In that situation, \( \exists z \leq e+f+g+h . \exists s \leq a+b+c+d . s \text{ crosses } z \) (c crosses f)

  o Thus, given (31b), \( <a+b+c+d, e+f+g+h> \in [[cross]] \), as desired

(32) **Another, Vaguer Line of Thought**

- Suppose we agree with Scha (1981/1984) that the truth of (27a/bi) in situation (27a/bii) simply reflects the core ‘collective’ meaning of the predicates in question.

- Recall the following, funny property of collective readings.

  a. **The Vagueness of Collective Predication (Brisson 2003)**

     Under a collective reading, a sentence can be true without *every* part of the plurality in question ‘taking part’ in the event/state described.

     (i) **Sentence:** The boys are smiling.

     (ii) **Vague T-Conditions:** Some significant portion of the boys are smiling.

- Something similar could be at work in (27). That is, (27ai) might be true in situation (27aii) because *some significant portion* of the squares contain the circles.

- Similarly, perhaps (27bi) is true in situation (27bii) because *some significant portion* of the sides cross…
3.2 ‘Maximality’ in the Cumulative Reading?

(33) Initial Observation, Part 1

The T-conditions our system assigns to (a) are those in (b). These T-conditions actually hold in a situation like (c), where the sum total of TAs grading and exams graded is greater than 3 and 30, respectively.

a. Three TAs graded thirty exams.

b. \[ \exists x : x \in [[[TA]]] & |\text{ATOMS}(x)| = 3 & \exists y : y \in [[[exam]]] & |\text{ATOMS}(y)| = 30 & <x,y> \in [[[graded]]] \]


Exercise to the Reader: Verify the claim made in (33).

(34) Initial Observation, Part 2

The T-conditions that Scha (1981/1984) assigns to (33a) are those below. These T-conditions don’t hold in a situation like (33c).

\[ |\text{ATOMS}( +\text{proj}_1( \{ <x,y> \in [[[TA]]] \times [[[exam]]] : <x,y> \in [[[graded]]] \} ) ) | = 3 & |\text{ATOMS}( +\text{proj}_2( \{ <x,y> \in [[[TA]]] \times [[[exam]]] : <x,y> \in [[[graded]]] \} ) ) | = 30 \]

‘The sum total of TAs involved in grading exams is 3, and the sum total of exams graded by some TA or TAs is 30.’

Exercise to the Reader: Verify the claim in (34)

(35) Obvious Question

Which theory is right? Is (33a) true in situation (33c) or not?

(36) Krifka’s Answer, Vaguely Stated

(33a) is true in a situation like (33c). Any feeling that we have that it might not be true (or might be misleading) is due a pragmatic inference…

- However, while it’s clear that this is what, generally speaking, Krifka (1999) proposes, his actual view is a bit hard to pin down in terms of its specifics…

- Here’s what I think his view might be (and in at any rate, here’s what I think the truth probably is…)
First Intuition

Sentence (33a) is (for me) unquestionably true in situation (33c). That is, a statement like the following does not seem to imply that only three TAs graded exams.

a. **Sentence:** Hey! Guess what? Three TAs graded (over) thirty exams.

b. **Intuition:** Is not at all misleading if (say) there is a fourth TA who graded exams as well.

Second Intuition

The straightforward truth of sentence (33a) in a situation like (33c) is subtly different from the truth of (a) in a situation like (b). That is, (a) really does feel to be misleading in a situation like (b).

a. **Sentence:** Hey! Guess what? Three TAs graded exams.

b. **Intuition:** Is misleading if (say) there is a fourth TA who also graded exams.

Third Intuition

However, sentence (33a) does feel misleading in situation (33c) if it’s offered as an answer to the question *How many TAs graded how many exams?*

What could be going on here?...

Accounting for the Second Intuition: Classic Gricean Reasoning

Sentence (a) is clearly logically weaker than (b), as (b) entails (a), but not vice versa!

a. Three TAs graded exams.

b. Four TAs graded exams.

Thus, by standard Gricean reasoning, uttering (a) in a situation where (b) is true would violate the Maxim of Quantity.

Accounting for the First Intuition: Classic Gricean Reasoning

Sentence (a) is actually *not* logically weaker than (b). Note that (b) does not entail (a), as situation (c) illustrates.

a. Three TAs graded over thirty exams.

b. Four TAs graded over thirty exams.

c. Dave graded eight exams, Mary graded eight exams, Sue graded eight exams, John graded eight exams.
(42) **Key Observation**

a. Three TAs graded (over) thirty exams.
b. Four TAs graded (over) thirty exams.

• Even in a situation where both (a) and (b) are true, (b) is not *strictly speaking* logically *stronger* than (a)

• Therefore, the classic Gricean maxim of Quantity does not actually require one to assert (a) in situations where (b) is true…

• Thus, we (perhaps) predict the observed contrast between (37) and (38)…

OK… but what about our intuition in (39), that (33a) is misleading in situation (33c), if it is offered as answer to the question *How many TAs graded how many exams?*

(42) **Krifka’s View (Possibly)**

“…if the main interest is in the number of boys that ate apples and the number of apples eaten by boys, which is a very typical background for such sentences, then the maxim of Quantity will lead us to assume the greatest such numbers that still yield a true proposition” (Krifka 1999: 266)

• When a person asks *How many TAs graded how many exams?*, they most likely are most likely wanting to know the *sum total* of TAs that graded exams, and the *sum total* of exams graded. (Why this is so is left unexplained.)

• Thus, in order to provide them the information they seek, one should utter that sentence of the form *n TAs graded m exams* such that there are no greater numerals *n’* and *m’* such that *n’ TAs graded m exams* is true

• Thus, if asked *How many TAs graded how many exams*, in a situation where four TAs total graded 38 exams total, it would be misleading to say “Three TAs graded thirty exams.”

(43) **Side-Note**

The general contours of Krifka’s story can be cashed out in the framework for the semantics of questions and (exhaustive) answers developed by Kratzer (2009)…

… however, in the interests of time I won’t do that here…
3.3 Cumulative Readings Based on Non-Lexical Predicates

Note that sentence (44a) can be interpreted as true in situation (44b). This suggests that (44a) should allow for a reading with the ‘cumulative’ T-conditions in (44c).

(44) **Another Cumulative Reading**

a. Three columnists write movie reviews for 50 newspapers.


c. \[ \exists x : x \in *[[\text{columnist}]] \& |\text{ATOMS}(x)| = 3 \& \exists y : y \in *[[\text{newspaper}]] \& |\text{ATOMS}(y)| = 50 \& <x,y> \in *[\lambda s : [\lambda t : t \text{ writes movie reviews for } s ]] \]

(45) **The Puzzle**

• The predicate \( [\lambda s : [\lambda t : t \text{ writes movie reviews for } s ]] \) is not the extension of a lexical item.
• Rather, it must be derived somehow in the syntactic derivation of the LF for (44a).
• So… how do we do that?!?

(46) **The Solution: Counter-Cyclic Movement (Sauerland 1998)**

Suppose we create the LF in (a) by QR-ing the subject *three columnists*.

a. \[ [\text{three columnists} \[ 1 \[ t_1 \text{ write movie reviews for 50 newspapers } ] ] ] \]

We can then create the LF in (b) by (counter-cyclically) QR-ing the DP *50 newspapers* to a position directly below *three columnists*.

b. \[ [3 \text{ columnists} [50 \text{ newspapers} \[ 2 [1 [ t_1 \text{ write movie reviews for } t_2 ] ] ] ] ] \]

We can then obtain the desired cumulative reading by inserting the ‘*’-operator as sister to the derived two-place predicate.

c. \[ [3 \text{ columnists} [50 \text{ newspapers} \[ * [2 [1 [ t_1 \text{ write movie reviews for } t_2 ] ] ] ] ] ] \]

Finally, each of the two quantificational DPs undergo an additional step of QR (in order to ensure that the types will compose correctly).

A Quick Critique of (46)

- The syntactic derivation in (46) seems quite complicated, despite the fact that the cumulative reading in question is extremely natural (like all cumulative readings…)
- Perhaps most problematic is the fact that the derivation in (46) requires an instance of counter-cyclic movement (which syntacticians would not typically approve of):

### 3.4 A Quick Note on the New Importance of Event Semantics

If we hold to the ‘Universality of Cumulativity’ in (22), there’s a somewhat unfortunate consequence for the way we informally relate VP extensions to ‘states of affairs’.

An Unfortunate Ambiguity in our Notation

Previously, if a V like lift had the extension in (a), this corresponded to a particular state of affairs, (b). Now, given (22), it can also correspond to the state-of-affairs in (c).

a. \[
[[ \text{lift} ]] = \{ <\text{Dave, piano1}>, <\text{Bill, piano2}>, <\text{Dave+Bill, piano1+piano2}> \}
\]
b. Dave lifts piano1. Bill lifts piano2. Then, Dave and Bill (as a group) lift piano1 and piano2 (simultaneously).


It’s not clear that the consequence in (48) has any negative empirical impact. However, if we find it distasteful to have this kind of ‘ambiguity’ in our meta-theory, there is a solution.

The Solution: Reintroduce Events as Explicit Arguments

Once we move back to an event-based semantics, the consequence in (48) goes away. Now, in a state-of-affairs like (48c), the extension of lift is as in (49a). Most importantly, in a state-of-affairs like (48b), the extension of lift is as in (49b).

a. \[
[[ \text{lift} ]] = \{ <e_1, \text{Dave, piano1}>, <e_2, \text{Bill, piano2}>, <e_1+e_2, \text{Dave+Bill, piano1+piano2}> \}
\]
b. \[
[[ \text{lift} ]] = \{ <e_1, \text{Dave, piano1}>, <e_2, \text{Bill, piano2}>, <e_1+e_2, \text{Dave+Bill, piano1+piano2}>, <e_3, \text{Dave+Bill, piano1+piano2}>, \ldots \}
\]

Since (49a,b) are different sets of tuples, the ‘ambiguity’ in (48) is eliminated!
4. **Cumulative Readings with the D-Operator?**

A recurring issue in our discussions has been the way that both our cumulative ‘*-operator and the distributive ‘D’-operator can be made to do similar empirical work.

(50) **An Obvious Question**

- We’ve just seen how a generalization of our ‘*-operator is able to provide a unified account of the phenomena in (1)-(2a).

- We’ve also seen how a D-operator can – *if made sensitive to a contextually supplied ‘cover’* – capture the possibility of ‘cover readings’ like (2a).

- Thus, *can the D-operator be tweaked so that it generates cumulative and co-distributive readings as well?*

(51) **Answer: YES!**

- As with our ‘*-operator, the trick will be to adjust our definitions so that ‘D’ can apply to *polyadic predicates*.

- To simplify our presentation of the main ideas, I’ll mainly focus on binary predicates here (see Schwarzschild 1996:Ch. 6 for a generalization to predicates of any arity…)

- I will also aid the presentation of these ideas by showing step-by-step how we can capture a single, concrete example (52)

(52) **Our Targeted Example**

Sentence (a) is true in situation (b).

a. **Sentence:** Dave and Bill moved piano1 and piano2.

b. **Situation:** Dave moved piano1. Bill moved piano2.

(53) **Step 1: Extending the Definition of a ‘Cover’**

Let \(< S_1, S_2 >\) be an ordered pair of sets. Let A be a set of *pairs* of sets.

(e.g. \(A = \{ <A_1, A_2 >, <B_1, B_2 >, <C_1, C_2 >, \ldots \} \))

A is a *cover* of \(< S_1, S_2 >\) if the following conditions hold:

a. For every pair \(< X_1, X_2 > \in A, \ X_i \subseteq S_i \)

   *(i.e., A_1 \subseteq S_1, A_2 \subseteq S_2, B_1 \subseteq S_1, B_2 \subseteq S_2, \ldots)*

b. For every \(x \in S_1\), there is some pair \(< X_1, X_2 > \in A\), such that \(x \in X_i\)

   *(Each set in \(< S_1, S_2 >\) is exhausted by the sum total of pairs in A).*
(54) **In Other Words…**

A set of pairs of sets \((A = \{ <A_1, A_2> , <B_1, B_2> , <C_1, C_2> .... \} )\) is a cover of some pair of sets \(<S_1 , S_2>\) if:

a. \(\text{proj}_1(A)\) is a cover of \(S_1\)

b. \(\text{proj}_2(A)\) is a cover of \(S_2\)

(55) **Step 2: The Concept of a ‘Paired Cover’**

Let \(S\) be a set (of entities). \(\text{COV}\) is a paired cover of \(S\) if \(\text{COV}\) is a cover of \(<S,S>\).

(56) **Illustration**

Let \(D\) be the set \(\{\text{Dave, Bill, piano1, piano2}\}\). The following are paired covers of \(D\) (i.e. covers of the pair \(<\{\text{Dave, Bill, piano1, piano2}\} , \{\text{Dave, Bill, piano1, piano2}\}>\)).

a. \(\{ <\{\text{Dave, Bill}\} , \{\text{piano1, piano2}\}> , <\{\text{piano1, piano2}\} , \{\text{Dave, Bill}\}> \} \)

b. \(\{<\{\text{Dave}\},\{\text{Dave}\}> , <\{\text{Bill}\},\{\text{Bill}\}> , <\{\text{piano1}\},\{\text{piano1}\}> , <\{\text{piano2}\},\{\text{piano2}\}>\}\)

c. \(\{ <\{\text{Dave},\{\text{piano1}\}> , <\{\text{Bill}, \{\text{piano2}\}> \} <\{\text{piano1, piano2}\}, \{\text{Dave,Bill}\}> \} \)

(57) **Step 3: Now Paired Covers are Contextually Supplied as Well**

Let us assume that the context provides a salient ‘paired cover’, which we will indicate with the superscript ‘PCov’ on our assignment function:

\([ [ . ] ]^{\text{PCov}}\)

(58) **A New Distributivity Operator**

- In addition to our earlier ‘D’-operator, let us posit the existence of a phonologically null operator ‘D2’.

- The ‘D2’-operator is a distributivity operator that takes *binary* predicates as argument. It has the following semantics:

\([ [ D2 ] ]^{\text{PCov}} = [ \lambda P : [ \lambda x : [ \lambda y : \text{For all } <s,t> \text{ such that } s \leq y \text{ and } t \leq x, \text{ and } <\text{ATOMS}(s),\text{ATOMS}(t)> \in \text{PCov} , <s,t> \in P ] ] ]\)
This is now sufficient to capture the targeted facts in (52)...
The reader is encouraged to explore the ways that (58) can also capture the facts in (1)-(2)...

(59) **Generating the Cumulative/Co-Distributive Reading of (52a)**

- Sentence (52a) can be mapped to the LF in (a) below. If we interpret (a) relative to the PCov in (56c), we obtain the T-conditions in (b).

```
a. TP
   / \
  ConjP V  VP
     Dave and Bill V Piano1 and Piano2
                        / \
                       V    ConjP
                        D2  moved

b. [[ TP ]]^{(33c)} =
```

For all <s,t> such that s \leq Dave+Bill and t \leq piano1+piano2, and <ATOMS(s), ATOMS(t)> \in

\{ <\{Dave\},\{piano1\}>, <\{Bill\},\{piano2\}> <\{piano1, piano2\}, \{Dave,Bill\}> \}

\< s, t> \in [[ moved ]]\)

- Now, there are only two pairs <s,t> such that s is part of Dave+Bill, t is part of piano1+piano2 and <ATOMS(s), ATOMS(t)> is some pair in the set (33c):
  - < Dave, piano1 > and <Bill, piano2>

- Thus, the T-conditions in (b) are equivalent to the T-conditions in (c), which certainly hold in situation (52b)!

```
c. < Dave, piano1 > \in [[ moved ]] and <Bill, piano2> \in [[ moved ]]
```

(60) **Summary**

- It is possible (though relatively difficult) to devise a D-operator that applies to polyadic predicates and is contextually sensitive to a ‘cover’.

- Just as allowing ‘*’ to apply to polyadic predicates yields cumulative, co-distributive and cover readings... allowing the D-operator to apply to polyadic predicates (in the way above) yields cumulative, co-distributive and cover readings...

- Thus, it’s again very difficult to empirically distinguish analyses based upon ‘*’ from those based upon ‘D’.
5. Some Final Comments on the ‘Primacy of the Collective Reading’

We began our study of plural predication with the question in (61). Until now, we had been assuming the basic perspective in (62).

(61) **Fundamental (Metaphysical) Question**

What, exactly, are the conditions under which a plural entity lies within the extension of a basic verbal predicate (lexical verb)?

(62) **The Primacy of the Collective Reading**

If a plural entity satisfies an argument position of a lexical verb, then it does so 'collectively' (the plural entity satisfies that argument position ‘as a group’).

We noted early on that the principle in (62) seems to vastly under-generate the range of readings a sentence containing plural DPs can have.

- In order to mitigate those problems, we introduced our null ‘*’-operator an the null ‘D’-operator.

Note, however, that once we adopt the principle in (22) – repeated below – the principle in (62) now no longer holds.

(63) **The Universality of Cumulativity (Krifka 1989, 1992, 1999)**

Let P be a natural language predicate of any arity. If x, y ∈ [[P]], then x+y ∈ [[P]].

We noted also that once we have (63) on the table, we no longer need ‘*’ or ‘D’ to generate certain observed readings of sentences with plurals.

- But also note that we do still need ‘*’ and ‘D’ to generate certain readings (see (26))

Since we no longer hold to (62), what answer do we now offer to (61)? Basically, if we assume (63), then our answer to (61) is the following.

(64) **The Nature of Plural Predication, Part 1**

If a plural entity x satisfies an argument position of a lexical verb, then either

a. It does so collective (‘as a group’), or

b. There is some way of dividing up x into sub-parts, each of which satisfies the argument position either singly or ‘as a group’
Moreover, since every entity $x$ is a sub-part of itself, we could just as easily answer (61) with the following.

(65) **The Nature of Plural Predication, Part 2**

If a plural entity $x$ satisfies an argument position of a lexical verb, then there is some way of dividing up $x$ into sub-parts, each of which satisfies the argument position either singly or ‘as a group’.

Thus, the ‘Primacy of the Collective Reading’ in (62) was something of a misstep, but one that we find carried through much of the literature on plurals, and one that was perhaps necessary to bring us to our current state of understanding...