The Basics of Plurals: Part 3
Cumulative Readings and ‘Grid’ in Distributives

1. Our Current Picture of Plurals

At the conclusion of ‘Part 2’, we had built a semantics for plural NPs and DPs that had the following key ingredients.

(1) Ontology of Plural Entities

In addition to the domain D of ‘singular entities’, we have the extended domain *D, which contains all the ‘singular entities’ and all the possible ‘plural entities’ (pluralities) that one can construct from the set of singular entities.

a. The Domain of Individuals, D: Sue, Frank, Bill

b. The Domain of Plural Entities, *D: Sue, Frank, Bill, Sue+Frank, Sue+Bill…

(2) The Interpretation of Plural NPs

A plural NP is interpreted as the set of all the possible pluralities that could be formed from the extension of the singular NP.

a. [[ boy ]] = {Frank, Bill, Dave}

b. [[boys ]] = *[[boy]] = { Frank, Bill, Dave, Frank+Bill, … }

(3) New <e,<ee>> Interpretation for Conjunction

If ‘DP1’ is type e and ‘DP2’ is type e, then [[ DP1 and DP2 ]] = [[DP1]]+[[DP2]].

(4) The Interpretation of Plural Definites

A plural definite denotes the ‘maximal plurality’ from the extension of the plural NP. This is the plurality that contains all the entities in the extension of the singular NP

a. [[ the ]] = λP<et> . MAX(P)

b. Illustration:
   (i) [[boy]] = {Frank, Bill, Dave, Tom}
   (ii) [[the boys]] = Frank+Bill+Dave+Tom

(5) Plural Entities Satisfy Argument Positions ‘Collectively’

When a plural entity satisfies an argument position of a lexical item, it does so ‘collectively’: the whose plural entity satisfies the argument role ‘as a group’ (and its individual members do not necessarily satisfy that role individually).
The Existence of ‘*’ and ‘D’ Operators

- A natural language contains (one of) the following two operators, which are phonologically null and can be freely inserted as sister to any <et> predicate.

a. $$[[ * ]] = \lambda P_{<et}> \cdot *P$$ maps an <et> predicate to the closure of that predicate under ‘+’

b. $$[[ D ]] = [ \lambda P \cdot [ \lambda x \cdot [ \text{For all } y \leq x \text{ and AT}(y), P(y) = T ] ] ] \ \text{'}all the atoms } y \text{ in } x \text{ are such that } P(y) = T '\text{'}

- The existence of these operators explains the possibility of ‘distributive’ readings of sentences like (c), whereby they are true in situations like (d).

- Moreover, lexical items that seem to only allow for such distributive readings (e.g. run, sleep, smoke, kiss) can be thought of as subcategorizing for atomic entities.

  - Thus, the only way such lexical items can combine semantically with plural entities is if they are sister to a */D operator, which thereby obtains the ‘distributive’ meaning of the sentence.

Plural DPs Besides Definites

- A standard semantics for the indefinite determiner some and the negative indefinite determiner no interacts fine with our semantics for plural NPs.

- A ‘GQ’ semantics for numerals, however, will not. Numerals may instead be thought of as being mere modifiers of the NP.

a. $$[[ \text{two} ]] = \lambda x . | \{ z : z \leq x \text{ & AT}(z) \} | \geq 2$$

- The existential quantificational force associated with numeral phrases like ‘five dogs’ can be thought of as coming from some null structure (as is now commonly assumed for indefinites). For our purposes, we might appeal to a null existential determiner.

b. $$[[ \emptyset_D ]] = \lambda P_{<et>}. \lambda Q_{<et>}. \exists x. \text{P}(x) = T \text{ and } \text{Q}(x) = T$$

- This approach will not work for numeral phrases that are not upward monotone, like exactly three or less than three. For these phrases, more sophisticated treatment is necessary (Krifka 1999, Landman 2000, Hackl 2000).
In this set of notes, we will discuss two key challenges to the system developed so far.

- One of these challenges (the problem of ‘cumulative readings’) will be treated in more depth in the second sub-unit of this course.
- For the other challenge (the problem of ‘grid’ in distributive readings) we will sketch the two main, competing solutions in the literature.

2. The Problem of ‘Cumulative’ Readings

Consider the sentence in (8) below.

(8) Two boys lifted two pianos.

It’s long been noted that this sentence is true in a wide variety of situations. Given the additions of (i) the operators ‘D/*’ in (6), and (ii) our semantics for numerals in (7), we can actually generate a ton of distinct ‘readings’ for (8), each of which correctly allows it to be true in a different type of situation.

Side-Note:
A foundational problem in the semantics of plurals, which few have given much extended thought to, is whether the multiplicity of situations that (8) is true in really reflects rampant ambiguity (as assumed here), or simply an inherent vagueness/underspecificity of its meaning…

2.1 Reading 1: The Doubly-Collective Reading

If we add no ‘D/*’ operators to the representation of the sentence, (8) is predicted to have the LF structures in (9), and thus the T-conditions in (10).

(9) The Double-Collective Reading of (8)

a. [ [ two boys ]_2 [ [ two pianos ]_1 [ t_2 lifted t_1 ] ] ]

b. [ [ two pianos ]_1 [ [ two boys ]_2 [ t_2 lifted t_1 ] ] ]
(10) **The Double-Collective Reading of (8)**

a. \[ \exists x. [[\text{boys}]](x) = T \land | \{z : z \leq x \land \text{AT}(z)\} | \geq 2 \land \exists y. [[\text{pianos}]](y) = T \land | \{z : z \leq y \land \text{AT}(z)\} | \geq 2 \land [[\text{lift}]](y)(x) = T \]

‘There is a group of two boys \(x\), and a group of two pianos \(y\), and \(x\) (collectively) lifted \(y\) (collectively).’

b. \[ \exists y. [[\text{pianos}]](y) = T \land | \{z : z \leq y \land \text{AT}(z)\} | \geq 2 \land \exists x. [[\text{boys}]](y) = T \land | \{z : z \leq x \land \text{AT}(z)\} | \geq 2 \land [[\text{lift}]](y)(x) = T \]

‘There is a group of two pianos \(y\), and a group of two boys \(x\), and \(x\) (collectively) lifted \(y\) (collectively).’

Of course, given that both of the arguments in (9) are indefinites, the two readings in (10) are equivalent.

- Under both readings, both the plurality of two boys and the plurality of two pianos are directly the arguments of the lexical item ‘lift’.
- Given our core ‘collectivity’ assumption in (5), it follows that this LF is true in precisely the following kinds of situations.

(11) **Situation Where the Double-Collective Reading is True**

There’s a single event where a group of two boys (together) lift a group of two pianos (together, perhaps with one on top of the other…)

\[
\text{boy1+boy2} \quad \text{LIFTING} \quad \text{piano1+piano2}
\]

- Happily, sentence (8) does feel to be true in a situation of this kind…

2.2 **Readings 2 and 3: Subject-Distributive, Object-Collective Readings**

Given that we can freely insert phonologically null ‘D/*’ operators into our LFs, we can also associate sentence (8) with the LFs in (12), which differ minimally from that in (9b). These LFs are computed to have the T-conditions in (13).

(12) **The Subject-Distributive, Object-Collective, Object Wide-Scope Reading**

a. \[
[[ \text{two pianos} ]_1 \ [ [ \text{two boys} ]_2 \ [ \ast \ [ t_2 \text{lifted} t_1 ] \ ] ] ]
\]

b. \[
[[ \text{two pianos} ]_1 \ [ [ \text{two boys} ]_2 \ [ \text{D} \ [ t_2 \text{lifted} t_1 ] \ ] ] ]
\]
The Subject-Distributive, Object-Collective, Object Wide-Scope Reading

a. \( \exists x. [[\text{pianos}]](x) = T \& \{ z : z \leq x \& \text{AT}(z) \} \geq 2 \&\)
\[ \exists y. [[\text{boys}]](y) = T \& \{ z : z \leq y \& \text{AT}(z) \} \geq 2 \& \]
\[ y \in * \lambda z : [[\text{lift}]](x)(z) \]

‘There is a group of two pianos \( x \), and a group of two boys \( y \), and \( y \) is a plurality that can be formed from the things that lifted \( x \).’

b. \( \exists x. [[\text{pianos}]](x) = T \& \{ z : z \leq x \& \text{AT}(z) \} \geq 2 \& \)
\[ \exists y. [[\text{boys}]](y) = T \& \{ z : z \leq y \& \text{AT}(z) \} \geq 2 \& \]
\[ \text{For all } z \leq y \text{ and } \text{AT}(z), [[\text{lift}]](x)(z) \]

‘There is a group of two pianos \( x \), and a group of two boys \( y \), and for every atomic member \( z \) of \( y \), \( z \) lifted \( x \).’

- Note that under this reading, only the plurality of two pianos \( x \) is a direct argument of the lexical item “lift”.
- The plurality of two boys \( y \) is argument to the derived predicates ‘\( *[\lambda z : [[\text{lift}]](x)(z)] \)’ (the groups that can be formed from the things that lifted \( x \)), or ‘\( D [ \lambda z : [[\text{lift}]](x)(z)] \)’ (the groups each of whose members lifted \( x \)).
- Thus, it follows that the LFs in (13) are true in situations like the following.

<table>
<thead>
<tr>
<th>Situation Where the Subject-Distributive, Object-Collective, Object Wide-Scope Reading is True</th>
</tr>
</thead>
<tbody>
<tr>
<td>There’s a single event where one boy lifted the group piano1+piano2, and there’s a single event where another boy (alone) lifted the group piano1+piano2.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boy1</th>
<th>LIFTING</th>
<th>piano1+piano2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boy2</td>
<td>LIFTING</td>
<td>piano1+piano2</td>
</tr>
</tbody>
</table>

Note that the LFs in (12) give the direct object “two pianos” widest scope. We can also generate a minimally different LF where the direct object has scope just below the ‘\( */D \)’ operator.

The Subject-Distributive, Object-Collective, Object Narrow-Scope Reading

a. \( [[\text{two boys}]]_2 [ * [ [ \text{two pianos} ]_1 [ t_2 \text{lifted } t_1 ] ] ] ] \)

b. \( [[\text{two boys}]]_2 [ D [ [ \text{two pianos} ]_1 [ t_2 \text{lifted } t_1 ] ] ] \)
These LFs are predicated to have the T-conditions in (16) below.

(16) The Subject-Distributive, Object-Collective, Object Narrow-Scope Reading

a. \[ \exists y. [[\text{boys}]](y) = T \land \{ z : z \leq y \land \text{AT}(z) \} \geq 2 \land y \in \ast [ \lambda z : \exists x. [[\text{pianos}]](x) = T \land \{ z : z \leq x \land \text{AT}(z) \} \geq 2 \land [[\text{lift}]](x)(z) ] \]

‘There is a group of two boys \( y \), and \( y \) is a group that can be formed from the things that lifted a group of two pianos (in a single event).’

b. \[ \exists y. [[\text{boys}]](y) = T \land \{ z : z \leq x \land \text{AT}(z) \} \geq 2 \land \forall z \leq y \land \text{AT}(z), \exists x. [[\text{pianos}]](x) = T \land \{ u : u \leq x \land \text{AT}(u) \} \geq 2 \land [[\text{lift}]](x)(z) ] \]

‘There is a group of two boys \( y \), and for all the atomic members \( z \) of \( y \), there is a group of two pianos \( x \) such that \( z \) lifted \( x \).’

It is clear from the informal paraphrases above that these readings are true in situations of the following sort.

(17) Situation Where the Subject-Distributive, Object-Collective, Object Narrow-Scope Reading is True

There’s a single event where one boy (alone) lifted the group piano1+piano2, and there’s a single event where another boy (alone) lifted the group piano3+piano4.

<table>
<thead>
<tr>
<th>Boy1</th>
<th>LIFTING</th>
<th>piano1+piano2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boy2</td>
<td>LIFTING</td>
<td>piano3+piano4</td>
</tr>
</tbody>
</table>

Happily, sentence (8) is indeed felt to be true in situations (14) and (17).

2.3 Readings 4 and 5: Subject-Collective, Object-Distributive Readings

In the LFs in (12) and (15), there is a single ‘*/D’ operator that is adjacent to the subject. We could also minimally change these LFs so that the single ‘*/D’ operator is adjacent to the object. Consider first the LFs below.

(18) The Subject-Collective, Object-Distributive, Subject Narrow-Scope Reading

a. \[ [ [ \text{two pianos} ]_1 [ * [ [ \text{two boys} ]_2 [ t_2 \text{lifted } t_1 ] ] ] ] \]

b. \[ [ [ \text{two pianos} ]_1 [ \text{D} [ [ \text{two boys} ]_2 [ t_2 \text{lifted } t_1 ] ] ] ] \]
These LFs are predicated to have the T-conditions in (19) below.

(19) **The Subject-Collective, Object-Distributive, Subject Narrow-Scope Reading**

a. \( \exists x. [[\text{pianos}]](x) = T \ & \ | \{ z : z \leq x \ & \ AT(z) \} | \geq 2 \ & \ x \in * [ \lambda z : \exists y. [[\text{boys}]](y) = T \ & \ | \{ z : z \leq y \ & \ AT(z) \} | \geq 2 \ & \ [[\text{lift}]](z)(y) ] \)

‘There is a group of two pianos x, and \( x \) is a member of all those pluralities you can form from things that were lifted (collectively) by a group of two boys’

b. \( \exists x. [[\text{pianos}]](x) = T \ & \ | \{ z : z \leq x \ & \ AT(z) \} | \geq 2 \ & \ \\
\forall z \leq x \ \& \ AT(z), \exists y. [[\text{boys}]](y) = T \ & \ | \{ z : z \leq y \ & \ AT(z) \} | \geq 2 \ & \ [[\text{lift}]](z)(y) ] \)

‘There is a group of two pianos x, and for every atomic member z of x, there exists a group of two boys y such that y (collectively) lifted z.’

It is clear from these informal paraphrases that these LFs are true in situations like the following.

(20) **Situation Where the Subject-Collective, Object-Distributive, Subject Narrow-Scope Reading is True**

There’s a single event where one group of two boys lifted piano1, and there’s a single event where a different group of two boys lifted piano2.

```
Boy1+Boy2 LIFTING piano1
Boy3+Boy4 LIFTING piano2
```

In the LFs in (18), the object has widest scope. We can also generate the minimally different LFs below, where the subject ‘two boys’ has widest scope.

(21) **The Subject-Collective, Object-Distributive, Subject Wide-Scope Reading**

a. \([ [ \text{two boys} ]_2 \ [ [ \text{two pianos} ]_1 \ [ * \ [ t_2 \ \text{lifted} \ t_1 ] ] ] ] \)

b. \([ [ \text{two boys} ]_2 \ [ [ \text{two pianos} ]_1 \ [ \text{D} \ [ t_2 \ \text{lifted} \ t_1 ] ] ] ] \)

These LFs are predicated to have the T-conditions in (22).
The Subject-Collective, Object-Distributive, Subject Wide-Scope Reading

a. \( \exists y. [[\text{boys}}](y) = T \land |\{z : z \leq y \land AT(z)\}| \geq 2 \land \exists x. [[\text{pianos}}](x) = T \land |\{z : z \leq x \land AT(z)\}| \geq 2 \land x \in * \lambda z : [[\text{lift}}](z)(y) \]

‘There is a group of two boys y, and a group of two pianos x, and x is a plurality that you can form from the things that y (collectively) lifted.’

b. \( \exists y. [[\text{boys}}](y) = T \land |\{z : z \leq y \land AT(z)\}| \geq 2 \land \exists x. [[\text{pianos}}](x) = T \land |\{z : z \leq x \land AT(z)\}| \geq 2 \land \forall z \leq x \land AT(z), [[\text{lift}}](z)(y) \]

‘There is a group of two boys y, and a group of two pianos x, and for every atomic member z of x, y (collectively) lifted x.’

From the paraphrases above, it is clear that the LFs in (21) are T in situations like the following:

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
(23) & Situation Where the Subject-Collective, Object-Distributive, Subject Wide-Scope Reading is True \\
\hline
There’s a single event where one group of two boys lifted piano1, and there’s a single event where that same group of two boys lifted piano2. \\
\hline
Boy1+Boy2 & LIFTING & piano1 \\
\hline
Boy1+Boy2 & LIFTING & piano2 \\
\hline
\end{tabular}
\end{center}

Happily, it’s again true that sentence (8) feels to be true in situations like (20) and (23)… (though people often have difficulty perceiving it as true in situation (20), where there’s 4 boys)

2.4 Reading 6 and 7: The Doubly-Distributive Readings

The final set of LFs our system generates are ones where a ‘*/D’ operator appears adjacent to both the subject and the object. Consider first the LFs below.

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
(24) & The Doubly-Distributive, Object Wide-Scope Reading \\
\hline
a. & \[ [ \text{two pianos} ]_1 \ [ \ast [ [ \text{two boys} ]_2 \ [ \ast [ t_2 \text{ lifted } t_1 ] ] ] ] \] \\
\hline
b. & \[ [ \text{two pianos} ]_1 \ [ \text{D} [ [ \text{two boys} ]_2 \ [ \text{D} [ t_2 \text{ lifted } t_1 ] ] ] \] \\
\hline
\end{tabular}
\end{center}

These LFs are predicted to have the T-conditions below.
The Doubly-Distributive, Object Wide-Scope Reading

a. \( \exists x. [[\text{pianos}]](x) = T \land | \{ z : z \leq x \land \text{AT}(z) \} | \geq 2 \land \)
\[ x \in * [ \lambda z : \exists y. [[\text{boys}]](y) = T \land | \{ z : z \leq y \land \text{AT}(z) \} | \geq 2 \land \]
\[ y \in * [ \lambda s : [[\text{lift}]](z)(s) ] \]

‘There exists a group of two pianos \( x \), and \( x \) is a group that can be formed from all those things \( z \) such that
there exists a group of two boys \( y \) and \( y \) is group that can formed from all those things \( s \) such that \( s \) lifted \( z \).’

b. \( \exists x. [[\text{pianos}]](x) = T \land | \{ z : z \leq x \land \text{AT}(z) \} | \geq 2 \land \)
For all \( z \leq x \) and \( \text{AT}(z) \),
\[ \exists y. [[\text{boys}]](y) = T \land | \{ z : z \leq y \land \text{AT}(z) \} | \geq 2 \land \]
For all \( s \leq y \) and \( \text{AT}(s) \), [[\text{lift}]](z)(s) \]

‘There exists a group of two pianos \( x \) such that for every atomic member \( z \) of \( x \),
there exists a group of two boys \( y \) such that for every atomic member \( s \) of \( y \)
\( s \) lifted \( z \).’

Admittedly, the logical formula in (25b) is a bit easier to understand. The key aspect of both these formulae is that only atomic entities are argument to the lexical item “lift”.
Consequently, both formulae end up being true in situations like the following.

Situation Where the Doubly-Distributive, Object Wide-Scope Reading is True

There are four individual events of lifting. In the first event, boy1 lifts piano1. In the second event, boy2 lifts piano1. In the third, boy3 lifts piano2. In the fourth, boy 4 lifts piano 2.

As with (20), speakers often disagree over whether (8) is actually true in situations like (26), where there are a total of four boys… let us assume for the moment, however, that this ‘reading’ does exist…
In the LFs in (24), the object has widest scope. Interestingly, we get a different set of T-conditions if we allow the subject to have widest scope. The LFs in (27) will get the T-conditions in (28).

(27)  The Doubly-Distributive, Object Narrow-Scope Reading


(28)  The Doubly-Distributive, Object Narrow-Scope Reading

a.  \[ \exists x. [[\text{boys}])(x) = T \ & \ \{ z : z \leq x \ & \ \text{AT}(z) \} \geq 2 \ & \
\text{x } \in \ast \{ \lambda z : \exists y. [[\text{pianos}]](y) = T \ & \ \{ z : z \leq y \ & \ \text{AT}(z) \} \geq 2 \ & \\text{y } \in \ast \{ \lambda s : [[\text{lift}]](s)(z) \} \]

‘There exists a group of two boys x, and x is a group that can be formed from all those things z such that there exists a group of two pianos y and y is group that can formed from all those things s such that z lifted s.’

b.  \[ \exists x. [[\text{boys}])(x) = T \ & \ \{ z : z \leq x \ & \ \text{AT}(z) \} \geq 2 \ & \
\text{For all } z \leq x \text{ and } \text{AT}(z), \exists y. [[\text{pianos}]](y) = T \ & \ \{ z : z \leq y \ & \ \text{AT}(z) \} \geq 2 \
\text{For all } s \leq y \text{ and } \text{AT}(s), [[\text{lift}]](s)(z) \]

‘There exists a group of two boys x such that for every atomic member z of x, there exists a group of two pianos y such that for every atomic member s of y z lifted s.’

Again, the logical formula in (28b) is a bit easier to understand. However, both are true in situations like the following.

(29)  Situation Where the Doubly-Distributive, Object Narrow-Scope Reading is True

There are four individual events of lifting. In the first event, boy1 lifts piano1. In the second event, boy1 lifts piano2. In the third, boy2 lifts piano1. In the fourth, boy 2 lifts piano 4.

```
Boy 1  Boy 2
  |   |   
  |   |   
  |   |   
Piano 1  Piano 2  Piano 3  Piano 4
```

Happily, sentence (8) is definitely perceived as true in situations like (29).

2.5 The Problem of Cumulative Readings

Thus far, we’ve seen that our system for plurals, augmented with the ‘*/D’ operator, generates a variety of readings for sentence (8), most of which are easily perceived to exist.

Thus far, if anything, our system seems to overgenerate readings for (8)…

Unfortunately, it can easily be shown that our system actually undergenerates readings…

There are situations where (8) is perceived to be true, but where our theory predicts it should be false…

Consider the situation in (30) below.

<table>
<thead>
<tr>
<th>(30)</th>
<th>Situation Where the ‘Cumulative’ Reading is True</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>There are two individual events of lifting. In the first, Boy1 lifts Piano1. In the second, Boy2 lifts piano2.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>LIFTING</th>
<th>piano1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boy1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boy2</td>
<td></td>
<td>piano2</td>
</tr>
</tbody>
</table>

Clearly, sentence (8) “Two boys lifted two pianos” is felt to be true in situations like (30). However, as the reader may check, none of the LFs generated above is predicted to be true in such a situation…

• This kind of reading of sentences like (8), where it is true in situations like (30) are called ‘cumulative readings’.

• They are called ‘cumulative readings’ because, intuitively, they seem to be about the ‘total number’ of entities involved….

… and they don’t ‘care’ how those entities necessarily relate to one-another

Paraphrase of the Cumulative Reading of (8):

If you add up all the piano-liftings done by boys, there were a total of two boys, and a total of two pianos…(and nothing is said about how those boys relate to those pianos)

• These kinds of readings are very easy to perceive, but notoriously difficult to generate. Here are some other examples:
Illustrations of Cumulative Readings

a. 600 Dutch firms use 5000 American computers (Scha 1981)
   ‘The total number of Dutch firms using American computers is 600, and the total number of American computers used by those firms is 5000.’

b. Four TA’s graded 300 exams.
   ‘The total number of TAs grading exams was four, and the total number of exams graded was 300.’

c. Seven chickens laid 49 eggs today.
   ‘The total number of chickens laying eggs was seven, and the total number of eggs laid was 49’.

So, despite all our complex machinery of QR and ‘*/D’-operators, we are still not generating one rather clear and obvious reading for sentences like (8)…

• How are we going to solve this problem?

• Should we finally give up our commitment to the assumption in (5), that all plural predication by lexical items is ‘collective’ predication?

• Should we move to a totally different theory of what plural DPs denote?

• Should we give up on plurals entirely and go back to studying something sane like wh-questions?

… all these questions will be addressed in the next sub-unit of this course, which will deal exclusively with (some of) the literature surrounding these cumulative readings…

3. The Problem of ‘Grid’ (or ‘Granularity’) in the Semantics of Distributives

Let us for the moment consider our ‘D’-operator, which is assumed to have the semantics below. (Let us also temporarily put aside our ‘*’-operator…)

The Distributivity Operator

\[
[[D]] = [[\lambda P . [\lambda x . [\text{For all } y \leq x \text{ and } AT(y), P(y) = T ] ] ]]
\]
It has long been noted that this semantics for ‘D’ is challenged by sentences like (33a) below. Under one salient interpretation, (33a) has the T-conditions in (33b,c).

(33) **The Problem of ‘Grid’ in the Semantics of Distributives**

a. The boys and the girls (each) met in the park.

b. The boys met in the park, and the girls (separately) met in the park.

c. A PICTURE OF THE READING IN QUESTION

\[
\begin{array}{ccc}
\text{BOYS} & \text{PARK} & \text{GIRLS} \\
\text{Boy1} & \text{Boy2} & \text{Boy3} & \text{Girl1} & \text{Girl2} & \text{Girl3} \\
\end{array}
\]

Unfortunately, our semantics for D in (32), which distributes over *atomic* members of a plurality, does not allow (32a) to be true in (32b,c).

(34) **The Problem for Our Semantics**

- In the situation in (33c), the extension of “the boys” is Boy1+Boy2+Boy3. Moreover, the extension of “the girls” is Girl1+Girl2+Girl3.

- Therefore, given our semantics for “and”, the extension of “the boys and the girls” is Boy1+Boy2+Boy3+Girl1+Girl2+Girl3.

- Therefore, the LF in (34a) below is assigned the T-conditions in (34b,c), which are nonsense.

a. \[
[ [ \text{the boys and the girls} ] [ D / ‘each’ [ met in the park ] ] ]
\]

b. For all \( y \leq \text{Boy1+Boy2+Boy3+Girl1+Girl2+Girl3} \) and \( AT(y) \), \( y \) met in the park.

c. Boy1 met in the park, Boy2 met in the park, Boy3 met in the park, Girl1 met in the park, Girl2 met in the park, Girl3 met in the park.
The Overarching Problem

- We want a system whereby sentence (33a) can have the T-conditions below.
  a. Boy1+Boy2+Boy3 ∈ [[met in the park]] & Girl1+Girl2+Girl3 ∈ met in the park.

- Thus, we want our distributive operator to distribute over certain sub-pluralities of the plural subject

- In the terms used by Fred Landman, there’s the possibility of grid or granularity in the semantics of the distributive.

Some Obviously Incorrect Answers

- What if we minimally changed our semantics of ‘D’ to the following… this way, it won’t necessarily distribute over atoms anymore...
  a. [[ D ]] = [ λP . [ λx . [ For all y ≤ x , P(y) = T ] ] ]

- PROBLEM: But now it quantifies over all sub-groups of the plural subject, and so we would still predict it to be false in situation (33c) [since Boy1+Girl1 didn't meet].

- What if we weaken the quantificational force to existential?
  b. [[ D ]] = [ λP . [ λx . [ For some y ≤ x , P(y) = T ] ] ]

- PROBLEM: Now we predict that (33a) can be true in a situation where only the boys meet in the park…

So, how do we fix this problem?

There are two main approaches in the literature. Interestingly, both solutions have been found to do interesting work besides solving the problem in (33).

Also, both ideas appear in some of the works we’ll read this term, and so we’ll consider each in turn…

3.1 The Theory of ‘Groups’

So far, we’ve been occasionally using the term ‘group’ informally to mean ‘plurality’. Let’s now stop doing that. We’ll refer to the pluralities in our system as pluralities, and we’ll reserve the term group for something else...

A key idea in some of the literature on plurals (Link 1984; Landman 1989, 2000) is that, in addition to the ‘pluralities’ we’ve been employing thus far, there is another, special kind of non-individual: the group.
(37) The Main Intuition Behind the Notion of a ‘Group’

- The linguistics department is clearly, in one sense, a plurality of individuals: Seth, Angelika, Barbara, Jason, Andrew, Suzi, etc.

- However, in another sense, the linguistics department is something more. It’s bigger than the sum or its parts.

- After all, all of us in the linguistics department could form a kick-ball team (The South College Slayers), and in a certain sense that’s a different group from the group that we form when we are linguists.

- Indeed, in a certain sense, you could say that the UMass Linguistics Department and the South College Slayers are two different, primitive entities.

  [Consider: the following two bands – or groups – consist of the same members, but are intuitively two different groups: Spinal Tap and The Folksmen]

- These entities correspond to pluralities of people, but they are not identical to them.

- So, maybe, there’s two different kinds of atomic entities:
  a. Individuals: Seth, Angelika, Barbara, Andrew, Suzi, Minta, …
  b. Groups: The South College Slayers, The UMass Linguistics Department, …

- And, similarly, maybe there are two kinds of non-individual entities
  c. Pluralities: Seth+Angelika, Barbara+Andrew+Minta, …
  d. Groups: The South College Slayers, The UMass Linguistics Department, …

- That is, groups are neither individuals nor pluralities, but something distinct. They are atomic ‘thingies’ (and so are elements of $D$) that ‘correspond’ (in some sense) to pluralities.

The Upshot:

Let’s move to an ontology where, in addition to individuals and pluralities, we also have ‘groups’, which are atomic entities that ‘correspond’ to pluralities.
(38) Adding Groups to Our Ontology

a. The Domain of Entities

Our atomic domain $D$ is now made up of two different sets of things: Individuals and Groups.

(i) $D = \text{INDIVIDUALS} \cup \text{GROUPS}$

Our full domain of individuals, $^*D$, is as before, the closure of $D$ under ‘+’.

Side-Note: Notice that we can now form pluralities out of groups, as well as groups taken with individuals…

b. The Relationship Between Groups and Pluralities

There is a function ‘⇑’ which maps pluralities ($^*D − D$) into the set of GROUPS.
There is also a function ‘⇓’ which maps groups into the set of pluralities ($^*D − D$)

- That is, for every plurality in $^*D$, there is a corresponding group in $^*D$, which is the group corresponding to the plurality.
- Moreover, for every group in $^*D$, there is a corresponding plurality in $^*D$, which is the plurality constituting the members of the group.
- Note that because ‘⇑’ is a function, we’ve departed somewhat from the intuitive notion of ‘group’ in (37).
- Note also that, since both ‘⇑’ and ‘⇓’ are functions (and assumed to be complete and the inverse of one another), it follows that both the set of GROUPS and the set of pluralities must be of the same size.
- Finally, since we allow pluralities to consist of groups and individuals, it follows that GROUPS must be an infinite set.

c. Illustration:

INDIVIDUALS: { Seth, Summer, Hazel, Tommy }
GROUPS: { THE-CABLES, THE-KIDS, etc. … }
$^*D$: { Seth+Summer, Seth+THE-KIDS, Hazel+Tommy, … }

⇑(Seth+Summer+Hazel+Tommy) = THE-CABLES
⇓(THE-KIDS) = Hazel+Tommy
So, that’s the ontology (roughly speaking)…

...How does it offer a solution to the problem in (33)?

(39) The Key Idea

- Let’s suppose that natural language has two special operators, UP and DOWN, with the following semantics:

  a. \([[[ \text{UP} ]] = \lambda x . \uparrow(x)\).
  
  b. \([[[ \text{DOWN} ]] = \lambda x . \downarrow(x)\).

- Let’s also suppose that these operators are (as usual) phonologically empty.

- Thus, the LF of a sentence like (c) might be that in (d).

  c. The boys and the girls (each) met in the park.
  
  d. \( [[[ \text{UP} [\text{the boys}]] \text{ and } [[ \text{UP} [\text{the girls}]]]} \text{ met in the park } ]\]

- Given our semantics above, this LF will have the following T-conditions (note that we don’t have to do anything at all to our semantics for ‘D’)

  e. For all \(y \leq \uparrow(\text{boy1+boy2+boy3})+\uparrow(\text{girl1+girl2+girl3})\) and \(\text{AT}(y),\)

  \(\downarrow y \text{ met in the park}\)

- Note that the \textit{atomic} elements of \(\uparrow(\text{boy1+boy2+boy3})+\uparrow(\text{girl1+girl2+girl3})\) are the \textit{groups} \(\uparrow(\text{boy1+boy2+boy3})\) and \(\uparrow(\text{girl1+girl2+girl3})\).

- Thus, the T-conditions in (e) are equivalent to those in (f)

  f. \(\downarrow \uparrow(\text{boy1+boy2+boy3}) \text{ met in the park and } \downarrow \uparrow(\text{girl1+girl2+girl3}) \text{ met in the park}\)

- Finally, since ‘\(\uparrow\)’ and ‘\(\downarrow\)’ are assumed to be the inverse of one another, it follows that these T-conditions are equivalent to:

  g. boy1+boy2+boy3 met in the park and girl1+girl2+girl3 met in the park

- And, these T-conditions in (g) clearly hold in the situation in (33c). (QED)

Of course, ‘groups’ have been found to be useful for other things as well, and this notion is especially well developed in the work of Fred Landman (1989, 2000).

But, there are also other ways of solving the problem in (33)…
3.2 The Theory of Covers

Roger Schwarzschild (1996: Chapter 4) has provided a number of widely-regarded empirical arguments against the use of ‘groups’ in the semantics of plurals.¹

Building on earlier work by Gillon (1987), Schwarzschild argues that the phenomena in (33) are best analyzed via appeal to the notion of a ‘cover’.

(40) The Main Intuition Behind the Notion of a ‘Cover’

• In every context, there’s a salient way (or multiple salient ways) of ‘dividing up’ the domain of discourse.

• For example, if we’re talking about the grad students, in one context, a natural division might be according to year. In another context, though, a natural division might be according to subject area.

• One way of characterizing this ‘dividing up’ is by saying that every context makes salient a particular (set of) ‘covers’ for the domain of discourse.

(41) The Definition of a Cover (Schwarzschild 1996: 64)

Let $S$ be a set of entities. A set $C$ is a cover of $S$ if and only if:

a. $C$ is a set of subsets of $S$.

b. Every member of $S$ is in some set in $C$ ($C$ exhausts the members of $S$)

c. The null set is not in $C$.

(42) Illustration

a. $S = \{ \text{Dave, Frank, Joy, Sue} \}$
b. A Cover of $S = \{ \{ \text{Dave, Frank} \}, \{ \text{Frank, Joy} \}, \{ \text{Sue} \} \}$
c. Another Cover of $S = \{ \{ \text{Dave} \}, \{ \text{Frank, Joy, Sue} \} \}$
d. Another Cover of $S = \{ \{ \text{Dave} \}, \{ \text{Frank} \}, \{ \text{Joy} \}, \{ \text{Sue} \} \}$
e. Another Cover of $S = \{ \{ \text{Dave, Frank} \}, \{ \text{Joy, Sue} \} \}$

How does this concept of a ‘cover’ help us with (33)?

Let’s first introduce a few more pieces of technology.

¹ It would take us too far afield to review those arguments, but you are encouraged to read the chapter cited above.
The Operator ATOMS

Let us add to our metalanguage the operator ATOMS, which maps a given plurality to the set of its atoms.

\[ \text{ATOMS}(x) = \{ y : y \leq x \text{ and } \text{ATOM}(y) \} \]

The Contextual Parameter ‘COV’

Relative to every context, there is a single salient cover of the domain \(*D\). This contextually salient cover will be referred to as \(\text{COV}\).^2

\[[[\ldots]]_{\text{g, w, i, speaker, addressee, COV}} \]

With these ideas in place, we can offer a novel semantics for our distributive operator.

New Semantics for our Distributive Operator

\[[[D]]_{\text{COV}} = \left[ \lambda P . \left[ \lambda x . \left[ \text{For all } y \text{ such that } y \leq x \text{ and } \text{ATOMS}(y) \in \text{COV}, P(y) = T \right] \right] \right] \]

‘every subpart \(y\) of \(x\) whose atoms are part of the contextually-given cover \(\text{COV}\) are such that \(P(y)\)’

How does this new semantics for ‘\(D\)’ solve the problem in (33)?

The Solution Involving Covers, Part 1

• Let us suppose that, as in (33), \(D = \{ \text{Boy1, Boy2, Boy3, Girl1, Girl2, Girl3} \}\)

• Thus, the set \(BOYSvGIRLS\) below is a possible cover of \(D\)

\[BOYSvGIRLS = \{ \{ \text{Boy1, Boy2, Boy3} \}, \{ \text{Girl1, Girl2, Girl3} \} \}\]

• Thus, we could interpret the sentence “The boys and the girls (each) met in the park” relative to a context where the contextually salient cover is \(BOYSvGIRLS\).

---

^2 This is an oversimplification for our purposes here in class. In order to allow there to be multiple salient covers in a given context, we would assume that ‘COV’ is a function from indices to those covers.
The Solution Involving Covers, Part 2

- Clearly, the following holds:
  a. \[ \text{[[} D \text{]}^{\text{BOYSvGIRLS}} = \]
  \[ \lambda P . [ \lambda x . [ \text{For all } y \text{ such that } y \leq x \text{ and } \text{ATOMS}(y) \in \text{BOYSvGIRLS}, P(y) = T ] ] ] \]

- It follows, then, that the LF in (b) below will be computed in context (33c) to have the T-conditions in (c) below.

  b. \[ \text{[[ } \text{[[ the boys and the girls ] [ D [ met in the park ]]] } ]^{\text{BOYSvGIRLS}} \]
  c. For all y such that y ≤ Boy1+Boy2+Boy3+Girl1+Girl2+Girl3
     and ATOMS(y) ∈ BOYSvGIRLS
     y met in the park.

- Given the definition of the set BOYSvGIRLS, the T-conditions in (c) are equivalent to those in (d) below.

  d. For all y such that y ≤ Boy1+Boy2+Boy3+Girl1+Girl2+Girl3
     and ATOMS(y) ∈ \{ { Boy1, Boy2, Boy3 }, { Girl1, Girl2, Girl3 } \}
     y met in the park.

- Note, however, that there are only two subparts of ‘Boy1+…+Girl3’ whose set of atomic members are elements of \{ { Boy1, Boy2, Boy3 }, { Girl1, Girl2, Girl3 } \}

  (i) Boy1+Boy2+Boy3
  (ii) Girl1+Girl2+Girl3

- Therefore, the T-conditions in (d) are equivalent to those in (e). (QED)

  e. Boy1+Boy2+Boy3 met in the park, and Girl1+Girl2+Girl3 met in the park.

Quick Note About Distributivity Down to Atoms

- Note that the set of singletons \{ { Dave }, { Frank }, { Joy }, { Sue } \} is also a cover of the set \{ Dave, Frank, Joy, Sue \}

- Thus, if we were to interpret the ‘D’-operator relative to a cover of this sort, we would predict a reading akin to what we obtain with our original semantics for ‘D’, where the distributivity is down to the atoms.
Schwarzschild (1996) further develops this notion of ‘cover’ and the role that it plays in a variety of phenomena related to plurals…

… and, as we will see, this notion of a ‘cover’ has been found to be especially useful in a variety of other phenomena besides those discussed by Schwarzschild (1996).

<table>
<thead>
<tr>
<th>(48)</th>
<th><strong>Burning Question: Groups or Covers?</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>So, which is the right approach to the phenomenon in (33), the ability for ‘distributive readings’ to ‘distribute’ over sub-pluralities?</td>
</tr>
</tbody>
</table>

This is an outstanding question, and it will take us too far afield to cover all the arguments for and against each view….

- I’ve already mentioned that Schwarzchild (1996: Chapter 4) provides some rather convincing arguments against the ‘group’ analysis.

- The argument in (49) is sometimes taken to be an equally convincing argument against the ‘covers’ analysis [but see Schwarzschild (1996: 65-66)]

<table>
<thead>
<tr>
<th>(49)</th>
<th><strong>The TAs and Their Salary</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Suppose that each of three TAs was paid $7,000 last year ³</td>
</tr>
<tr>
<td></td>
<td>Intuitively, the sentence in (a) can be read as true, as well as the sentence in (b), but not the sentence in (c)</td>
</tr>
<tr>
<td>a.</td>
<td>The TAs earned $7,000 last year. * (Atomic) Distributive Reading</td>
</tr>
<tr>
<td>b.</td>
<td>The TAs earned $21,000 last year. * Cumulative Reading</td>
</tr>
<tr>
<td>c.</td>
<td>The TAs earned $14,000 last year. * (Cover) Distributive Reading</td>
</tr>
<tr>
<td></td>
<td>However, if we allowed COV to equal { { TA1 , TA2 } and { TA2, TA3 } }, then the T-conditions of (c) would be those in (d) which actually hold in the situation described.⁴</td>
</tr>
<tr>
<td>d.</td>
<td>For all y such that ( y \leq TA1+TA2+TA3 ) \ and ( \text{ATOMS}(y) \in { { TA1 , TA2 } and { TA2, TA3 } } ) \y (collectively) earned $14,000 last year.</td>
</tr>
<tr>
<td></td>
<td>It seems, then, that an analysis involving ‘covers’ might incorrectly predict that (c) can be read as true in the situation described…</td>
</tr>
</tbody>
</table>

³ This is an extremely dated example.

⁴ This argument assumes that we have a means for capturing the ‘cumulative’ reading of (49b), which we actually at present do not. But if we did, the argument would go through.