1. Our Current Picture of Plurals

At the conclusion of ‘Part 1’, we had built a semantics for plural NPs and DPs that had the following key ingredients.

(1) Ontology of Plural Entities

In addition to the domain D of ‘singular entities’, we have the extended domain *D, which contains all the ‘singular entities’ and all the possible ‘plural entities’ (pluralities) that one can construct from the set of singular entities.

Picture of *D

(i) The Domain of Individuals, D: Sue, Frank, Bill

(ii) The Domain of Plural Entities, *D:

\[
\begin{array}{c}
\text{S+F+B} \\
\text{S+F} & \text{S+B} & \text{F+B} \\
\text{Sue} & \text{Frank} & \text{Bill}
\end{array}
\]

(2) The Interpretation of Plural NPs

A plural NP is interpreted as the set of all the possible pluralities that could be formed from the extension of the singular NP.

Picture of Plural NP Semantics

(i) \([ [ \text{ boy } ] ] = \{ \text{Frank, Bill, Dave} \} \)

(ii) \([ [ \text{boys } ] ] = *[ [ \text{boy} ] ] = \{ \text{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, Frank+Dave+Bill} \} \)

We also added a new interpretation rule for the conjunction of type e expressions:

New Rule for Conjunction:

If ‘DP1’ is type e and ‘DP2’ is type e, then \([ [ \text{DP1 and DP2 } ] ] = [[\text{DP1}]]+[[\text{DP2}]]. \)
The Interpretation of Plural Definates

A plural definite denotes the ‘maximal plurality’ from the extension of the plural NP. This is the plurality that contains all the entities in the extension of the singular NP.

a. Semantics of the Definite Article

\[ [[ \text{the} ]] = \lambda P_{<et>} \cdot \text{MAX}(P) \]

b. The Semantics in a Picture

(i) \[ [[ \text{boy} ]] = \{ \text{Frank, Bill, Dave} \} \]

(ii) \[ [[ \text{boys} ]] = \{ \text{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, Frank+Dave+Bill} \} \]

(iii) \[ [[ \text{the boys} ]] = \text{MAX}([[\text{boys}]]) = \text{MAX}(\{ \text{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, Frank+Dave+Bill} \}) = \text{Frank+Bill+Dave} \]

Predication of Plural Entities is Collective Predication

- In our semantic system, all plural entities are of type ‘e’.
- Thus, in this system, verbs can directly take plural entities as arguments.
- We assume that when a plural entity satisfies an argument position, it does so ‘collectively’; the whole plural entity fills the ‘argument role’, and not necessarily any individual members of that entity.

a. Illustration:

(i) A Possible Extension in Our System

\[ [[ \text{lift} ]] = \{ <\text{Dave, piano1}>, <\text{Dave+Bill, piano2}>, <\text{Dave, piano1+piano2}> \} \]

(ii) The State of Affairs this Intuitively Corresponds To:

Dave lifted piano1 on his own. Dave and Bill acting together lifted piano2. Dave alone lifted piano1 and piano2 together (say, with one stacked on top of the other).
However, there are some obvious inadequacies with this simple, initial picture.

(5) **Outstanding Questions to Answer Here**

a. **What About Other Plural DPs?**

- There are many other determiners that plural NPs can combine with besides just the definite determiner…
- What should the semantics of these DPs be?

b. **What About Non-Collective Readings of Sentences with Plural DPs?**

- As stated in (4), we assume that predication of plural entities is always collective.
- But what about predicates like *run* or *sleep*, which seem to involve distributive predication over plural arguments?
- AND – most importantly – what about the fact that a sentence like (i) below can be understood as true in a situation like (ii)? *This doesn’t follow from the current semantics.*

(i) **Sentence:** Dave and Frank lifted a piano.

(ii) **Situation:** Dave (alone) lifts piano-1; Frank (alone) lifts piano-2.

In these notes, we will put forth a common, basic (though controversial) set of answers to these questions…

2. **Distributive Predication Over Plurals**

Recall the fact discussed below.

(6) **There are Predicates that (Always) ‘Distributively’ Predicate Over Pluralities**

Some predicates, like “running”, seem to hold of a plural entity *iff* they hold of every atomic member of that entity. (They *distributively* predicate over the plural entity.)

a. **Valid Inference:**

(i) Dave, Bill and Frank are the boys.
(ii) The boys are running.
(iii) Therefore, Dave is running, Bill is running, and Frank is running.
As discussed earlier, many theories of plurality assume that predicates like those in (6) have some ‘special property’ that separates them from other predicates…

But, what is that property?...

It’s fair to say that there are two basic views, both of which are based upon the following intuitive observation:

(7) **Key Observation: No Sensible Collective Meaning**

- Predicates like “run” have the quirk that – *simply due to the kind of action they describe* – there is just no sense in which a plural entity could be their agent ‘collectively’.

- After all, given what running *is* (moving briskly on your two legs), what would it mean for some group of entities to run without each individual entity running?

How this key idea gets implemented, though, differs from author-to-author…

(8) **View 1: Distributivity Lies in the Meaning of the Predicate (Scha 1981)**

- Plural entities can be direct arguments of predicates like “run”. Thus, the extension of run can contain a plural entity like ‘Dave+Bill+Frank’

- However, for these predicates, their lexical entry also contains a ‘special statement’ – called a **meaning postulate** – that states that (i) if the predicate holds of a plural entity, then (ii) the predicate holds of every atomic member of that entity.

  **Meaning Postulate for ‘Run’**

  For all x, if \([\text{run}](x) = T\), then for all y such that \(y \leq x\) and \(\text{AT}(y)\), \([\text{run}](y) = T\).

*Thus, under one view, the special property seen in (6) is due to a special statement – called a ‘meaning postulate’ – within the lexical entry of the predicate.*

*Under the second major view, however, the special properties of predicates like “run” follow from a primal inability to take plural entities as arguments.*
(9) **View 2: Distributivity is the Result of an Operator (Link 1998: Chapter 1)**

a. Let us assume that predicates like “run” cannot (at base) take plural entities as arguments. Thus, their extensions will only ever contain singular, atomic entities. (Because running is, by its nature, something only singular entities can do.)

[[ run ]] = [ \lambda x . x \in \text{ATOMS} : x \text{ runs} ]

b. **Obvious Question:**
How, then, can a plural DP like “Dave and Frank and Joe” ever serve as the subject to such a predicate?

c. **Highly Influential Answer**
- Remember our little ‘*’-operator? It takes a set of entities and forms all the pluralities that you can make from that set.

- *Let’s suppose that it can take VPs as argument too (and is phonologically null when it appears with VPs)*.

- Thus, the surface predicate “run” in English actually corresponds to two different underlying structures, each with two different meanings:

(i) ‘Singular’ run : [[ run ]] = { Dave, Frank, Joe }

(ii) ‘Plural’ run : [[ * run ]] = { Dave, Frank, Joe, Dave+Frank, Dave+Joe, Frank+Joe, Dave+Frank+Joe }

- Clearly, then the plural subject “Dave and Frank and Joe” can be argument to the pluralized version of “run”.

d. **How This Gets the Targeted Facts in (6)**

- Recall that the basic lexical item “run” can only hold of atomic entities (9a).

- Thus, if ever the ‘surface verb’ “run” appears with a plural subject, it must be that that surface form “run” is actually the *pluralized version* of the basic predicate (as in (9ci)).

- Moreover, since the basic predicate “run” can only hold of atomic entities, it follows that if a plural entity is in the extension of [[*run]], every atomic element of that entity must therefore be in the extension of [[run]].

- Thus, if the surface predicate “run” ever truly predicates a plural entity, then it must be that [[*run]] truly predicates that plural entity, and so [[run]] truly predicates every atomic member of that plural entity. (QED)
A Quick Note on This ‘*-Operator Idea

• Thus, under ‘View 2’ – which seems to be more widely accepted – the distributive predication in sentences like “The boys are running” follows from the presence of a null *-operator taking the VP as argument…

• This notion – that all predicates in natural language can be ‘pluralized’ via ‘*’ – has been a highly influential and widely adopted one….

• As we will see in a moment, it can also do work for us in understanding the range of readings available for sentences like “Dave and Frank lifted a piano…

A Quick Note on Subcategorization for Atomic Entities vs. Pluralities

• A key component of ‘View 2’ is the notion that some basic predicates / lexical items can only take atomic entities as argument.

• This idea is lent some independent plausibility by the fact that there are clearly basic predicates / lexical items that can only take plural entities as arguments.

Examples of Strictly ‘Plurality-Seeking’ Predicates

meet, gather, elect, ostracize, outnumber, disperse, mix, etc.

So, thus far, we have an account of the behavior of predicates like “run” which is consistent with our more general view in (12) below.

Predication of Plural Entities is Collective Predication

For any basic predicate / lexical item P, if a plural entity satisfies an argument position of P, it does so ‘collectively’.

Recall, however, that the view in (12) is also challenged by the following fact.

The General Availability of Distributive Readings

A sentence like (a) below can be true in a situation like (b).

a. Sentence: Dave and Frank lifted a piano.

b. Situation: Dave lifted piano-1. Frank lifted piano-2.
(14) **The Problem**

- Under our current system, a sentence like (13a) is only T in a situation corresponding to the extension in (a) below.

  \[
  \begin{align*}
  \text{[[ lift a piano ]] } &= \{ \ldots \text{ Dave+Frank } \ldots \} \\
  &= \{ \ldots < \text{Dave+Frank, e} > \ldots \}
  \end{align*}
  \]

- Under our view in (12), though, such an extension only holds in a situation where Dave and Frank together, collectively lift a piano… (where there is a single event of lifting a piano, with Dave+Frank as its collective agent).

- However, such a situation clearly doesn’t hold in (13b)…and yet (13a) can be interpreted as true in such a situation...

---

So, how do we make sense of these facts? Do we just give up our general view in (12)?

(15) **A Possible Nihilistic View**

- Look, what exactly it means for a plural entity to be in the extension of a predicate just varies from predicate to predicate.

- For some predicates, a plural can only satisfy a given argument position if every atomic entity satisfies that position. (e.g. *run*)

- For other predicates, a plural can only satisfy a given argument position if the whole plural entity does so collectively. (e.g. *meet*)

- For others still, a plural can satisfy a given argument position under either set of conditions (e.g. *lift a piano*).

- Thus, whether a sentence containing a plural is understood as involving ‘distributive’ or ‘collective’ predication just simply isn’t a matter for the grammar to decide… (but for the pragmatic/cognitive system…)

For now, we’re going to put aside this ‘nihilist view’, but it’s good to have it clearly articulated for our consideration…

… Later, we’ll see that Winter (2000) puts forth some empirical arguments against it…
Happily, though, there are multiplicity of solutions to the problem in (13)/(14). It’s fair to say that there are three main types of approach…

(16) **View 1: The ‘Distributive Reading’ is the Result of an Ambiguity in the DP**

- Suppose that a plural DP like “Dave and Frank” is actually ambiguous!
- Under one reading, it refers to a plural entity. But, under a second reading, it denotes the following generalized quantifier.
  
  \[
  [[ \text{Dave and Frank} ]] = \lambda P . P(\text{Dave}) = T \text{ and } P(\text{Frank}) = T
  \]

- Thus, under this reading of the subject, the sentence in (a) below is computed to have the T-conditions in (b).

  a. **Sentence:** Dave and Frank lifted a piano.

  b. **Computation of T-Conditions**

  \[
  [[ \text{Dave and Frank lifted a piano} ]] = \\
  [[ \text{Dave and Frank} ]] ( [[ \text{lifted a piano} ]] ) = \\
  [\lambda P . P(\text{Dave}) = T \text{ and } P(\text{Frank}) = T] ([[ \text{lifted a piano} ]] ) = \\
  [[\text{lifted a piano}]](\text{Dave}) \& [[\text{lifted a piano}]](\text{Frank}) = \\
  (\text{vaguely}) \text{ Dave lifted a piano and Frank lifted a piano}.
  \]

- Clearly, these T-conditions hold in the situation described in (13b).

The approach in (16) was historically the first; Lasersohn (1995: Chapter 5) lays out the history of this kind of analysis,…

… *He also lays out a variety of arguments against it in Chapter 7.*

A well-known and widely-accept argument is against (16) the following:
Argument Against (16) from VP Conjunction

Consider the following sentence:

a. Dave and Frank met in the bar and had a beer.

• Note that the subject “Frank and Dave” is understood to ‘collectively’ satisfy the first VP “met in the bar” (since a single person can’t ‘meet’)

• Note also that – under the most salient interpretation – the subject “Frank and Dave” ‘distributively’ satisfies the second VP (since they are each having separate beers).

• QUESTION:
Could the ‘distributive’ predication of “have a beer” over “Frank and Dave” be due to “Frank and Dave” having the GQ denotation in (16)?

• ANSWER:
NO! If we gave the subject the denotation in (16), we would incorrectly predict the truth-conditions in (b), which are nonsense.

b. $[[\text{Dave and Frank}]]( [[\text{met and had a beer}]] ) = [
\lambda P . P(\text{Dave}) = T \land P(\text{Frank}) = T ] ( [[\text{met and had a beer}]] ) =
[[\text{met and had a beer}]](\text{Dave}) \land [[\text{met and had a beer}]](\text{Frank}) =$

(??) Dave met and had a beer, and Frank met and had a beer.

• Consequently, the ‘distributive’ predication of “have a beer” over “Frank and Dave” must be due to something other than the ability of “Frank and Dave” to have the denotation in (16).

• Consequently, whatever is responsible for the distributive predication in (17a) is also likely the source of the distributive reading of (13a) [and so the analysis in (16) isn’t necessary…]

For this reason (and several others), people tend to now put aside the ‘DP-ambiguity’ approach in (16)…

… the more popular approach is to assume that there is an ambiguity in the syntax/semantics of the $VP$ predicate…

… here there are two main sub-types of approach…
(18) View 2: The Distributive Reading is the Result of a ‘*-Operator

- Recall again our idea in (9c) that ‘*’ can apply to VPs.
- Now, consider the situation in (13b), where Dave lifts piano-1 and Frank lifts piano-2.
- Clearly, in this situation, the extension of the basic VP “[lifted a piano]” will be as follows:

  a. \[ \text{[[ lifted a piano ]] = \{ Dave, Frank \} } \]

- Thus, the extension of the ‘pluralized’ VP “*[lift a piano]” will be as follows:

  b. \[ \text{[[ * [ lifted a piano ] ]]] = \{ Dave, Frank, Dave+Frank \} } \]

- Clearly, then the following syntactic structure will come out as true in situation (13b):

  c. \[ [ [ Dave and Frank ] [ * [ lifted a piano ] ] ] ]

- However, under the assumption that the pluralizing ‘*’ operator is phonologically null, the syntactic structure in (18c) will just be pronounced as the sentence in (13a), repeated below.

  d. \[ Dave and Frank lifted a piano. \]

- Thus, the ability for (13a) to be true in (13b) may be due to a structural ambiguity: the optional presence of a phonologically null pluralizing ‘*-operator taking the VP as argument.

The view in (18) is well represented in the literature…

… but so is its subtly different competitor in (19).
(19) **View 3: The Distributive Reading is the Result of a ‘D’-Operator**

- Let’s suppose that there exists a phonologically null operator ‘D’ (for ‘distributive’), with the following semantics:

  a. \[ [[ D ]] = [[ \lambda P . [ \lambda x . [ \text{For all } y \leq x \text{ and } AT(y), P(y) = T ] ] ]] \]
  \[ ‘\text{all ATOMS } y \text{ in } x \text{ are such that } P(y) \text{ is true of them’} \]

- Then, a surface VP like “lifted a piano” could be structurally ambiguous, and could have the form in (b), which would be computed to have the meaning in (c).

  b. \[ [ D \ [ \text{lifted a piano} \ ] ] \]

  c. \[ [[ D \ [ \text{lifted a piano} \ ] ]] = \]

  \[ [[ D ]] \ ( [[ \text{lifted a piano} \ ] ]) = \]

  \[ [ \lambda P . [ \lambda x . [ \text{For all } y \leq x \text{ and } AT(y), P(y) = T ] ] ] \ ( [[ \text{lifted a piano} \ ] ]) = \]

  \[ [ \lambda x . [ \text{For all } y \leq x \text{ and } AT(y), [[\text{lifted a piano}]](y) = T ] ] = \text{(vaguely)} \]

  \[ [ \lambda x . [ \text{For all } y \leq x \text{ and } AT(y), \text{y lifted a piano} ] ] \]

  \[ ‘\text{all ATOMS } y \text{ in } x \text{ are such that y lifted a piano’} \]

- If this predicate were to take the plural subject “Dave and Frank” as argument, we would get the T-conditions in (d).

  d. \[ \text{For all } y \leq \text{Dave+Frank} \text{ and } AT(y), \text{y lifted a piano} \]

- Finally, since the atoms in ‘Dave+Frank’ are just Dave and Frank, it follows that the T-conditions we predict in (d) are equivalent to those in (e).

  e. \[ \text{Dave lifted a piano and Frank lifted a piano.} \]

- Clearly, these T-conditions hold in situation (13b), where Dave lifts piano-1 and Frank lifts piano-2.

  - **Thus, the ability for (13a) to be true in (13b) may be due to a structural ambiguity: the optional presence of a phonologically null *distributive* operator ‘D’ taking the VP as argument.**
(20) **Burning Question**

How, exactly, can we empirically distinguish between (18) and (19)? Are these really different views?

- Complicating the matter even further is that sometimes authors refer to our “pluralizing” operator ‘*’ as a *distributive* operator…

  …whereas that term is more commonly used to refer to an operator with the semantics in (19a)…

(21) **One Imaginable Line of Argumentation**

Well, we need the ‘*’-operator anyway, for plural NPs… so why not adopt (18) as the more parsimonious view?...

**PROBLEM:**

We might also independently need the ‘D’ operator, as it might be a plausible analysis of distributive particles like *each* in English.

(22) **Another Imaginable Line of Argumentation**

- There seems to be a ‘reading’ of (a) where it actually *entails* (b) [this is highly controversial: see Schwarzschild (1995: 60)]

  a. Dave and Frank lifted a piano.
  b. Dave lifted a piano and Frank lifted a piano.

- Note that this entailment is valid if we assume that, under the reading in question, (22a) has the structure in (22c), and so the T-conditions in (22d).

  c. \[ [ \text{Dave and Frank} \] [ D [ lifted a piano ] ] ]
  d. For all \( y \leq \text{Dave+Frank} \) and AT\( (y) \), \( y \) lifted a piano

- Moreover, note that this entailment would *not* be valid if (22a) had the structure in (22e), and so the T-conditions in (22f). (After all, suppose that \([\text{[lifted a piano]}] = \{\text{Dave+Frank}\}\)

  e. \[ [ \text{Dave and Frank} \] [ * [ lifted a piano ] ] ]
  f. \( \text{Dave+Frank} \in \{ * [ \text{lifted a piano} ] \} \)

- Therefore, if the ‘distributive’ reading does indeed license the validity of (22a)-(22b), it must be that the distributive reading is obtained via the structure in (22c), with the D-operator, and not the structure in (22e), with the *-operator.
Whether or not there’s a real difference between (18) and (19) is [I believe] a difficult outstanding issue…

… in this class, we’ll not worry that much about it, and will often swing back-and-forth between both views (but we’ll probably tend to adopt the D-operator analysis…)

(23) **General Summary**

- It is indeed possible to maintain our general view in (12):

  For any basic predicate / lexical item P, if a plural entity satisfies an argument position of P, it does so ‘collectively’.

- Apparent counter-examples to this view can be analyzed as due to a structural ambiguity, namely, the optional presence of phonologically null operators taking the VP as argument.

- These operators – either our ‘pluralizing *-operator’ or our distributive ‘D-operator’ – have the effect of creating complex, derived predicates that ‘distribute’ the predication of the basic [[VP]] over the atomic members of the plurality…

So much for the problem in (5b)… *Now what about the problem in (5a)?*…

3. **Plural DPs Beyond Definites**

It’s patently obvious that plural NPs can combine with all kinds of Ds besides “the”.

(24) **Some Other Plural DPs**

a. Some dogs  
b. No dogs  
c. Five dogs  
d. Many dogs  
e. Most dogs  
f. Exactly five dogs  
g. Few dogs  
h. All dogs

(25) **Outstanding Question**

How do we analyze each of these Ds so that they can combine with the meaning of their plural NP complements, *and yield the correct truth-conditions for the sentences containing them?*
First, let’s note that the classic ‘GQ’ semantics for the determiners ‘some’ and ‘no’ will still obtain the right results.

(26)  **The Indefinite *Some***

Under a classic ‘GQ’ analysis, an indefinite determiner like ‘some’ or ‘a’ has the following meaning:

a.  \[ [[ \text{some / a}] = \lambda P_{<et>} \cdot \lambda Q_{<et>} \cdot | P \cap Q | \neq \emptyset \]
\[ \lambda P_{<et>} \cdot \lambda Q_{<et>} \cdot \exists x. P(x) = 1 & Q(x) = 1 \]

This lexical entry clearly captures the meaning of singular indefinites like (26b). **It also performs perfectly for plural indefinites like (26c).**

b.  Some dog is barking.

c.  Some dogs are barking.

The T-conditions that (26a) predicts for sentences like (26c) is as follows:

d.  \[ | *\{x: x \text{ is a dog}\} \cap *\{y: y \text{ is barking}\} | \neq \emptyset \]
\[ \exists x. x \in \{y : y \text{ is a dog}\} & [[*\text{barking}])(x) \]
Some group of dogs is barking.

(27)  **The Quantifier *No***

Under a classic GQ treatment, the D *no* receives an analysis under the following lines:

a.  \[ [[ \text{no}] = \lambda P_{<et>} \cdot \lambda Q_{<et>} \cdot | P \cap Q | = \emptyset \]
\[ \lambda P_{<et>} \cdot \lambda Q_{<et>} \cdot \neg \exists x. P(x) = 1 & Q(x) = 1 \]

This lexical entry clearly captures the meaning of sentences like (27b). **It also performs perfectly for sentences like (27c).**

b.  No dog is barking.

c.  No dogs are barking.

The T-conditions that (27a) predicts for sentences like (27c) is as follows:

d.  \[ | *\{x: x \text{ is a dog}\} \cap \{y: y \text{ is barking}\} | = \emptyset \]
\[ \neg \exists x. x \in *\{y : y \text{ is a dog}\} & [[*\text{barking}])(x) \]
No group or individual dog is barking.
(28) **Important Side Note**

- First, note that since \(*\{y : y \text{ is a dog}\}*\) includes individual dogs as well as pluralities of dogs, (27c) is indeed predicted to be false if a single dog is barking…

- However, note that since \(*\{y : y \text{ is a dog}\}*\) includes individual dogs, (26c) is predicted to be true as long as a single dog is barking…

- However, this seems to be wrong; (26c) seems to commit the speaker to the presence of *multiple* dogs barking…

- If, however, we were to try to fix the problem by removing individual, atomic dogs from the extension of \([[\text{dogs}]]\), *then we would fail to predict that (27c) is false just as long as a single dog barks*…

- There is a certain amount of literature regarding this seeming paradox: Sauerland 2003, Sauerland *et al.* 2005, Spector 2007, Farkas & De Swart 2010

- The general view that people take is that the feeling that (26c) is false when only a single dog barks is due to a *scalar implicature* of some sort:
  (e.g., if you knew that only a single dog was barking, you would have said ‘dog’)

- Formulating such an analysis in detail, however, is quite tricky, since under our semantics, (26b) and (26c) are *actually materially equivalent*…
  … in our system, (26b) is true iff (26c) is true…

- For more info on this interesting puzzle, I refer you to the works listed above…

Putting aside the puzzle in (28), so far a classic GQ semantics for determiners interacts well with our theory of plural NPs…

… however, things become more complicated once we come to numerals…

### 3.1 The Semantics of Numerals with Plural NPs

Recall the following, classic ‘GQ’ semantics for numerals like *five*.

#### (29) Semantics of Numerals in Classic Generalized Quantifier Theory

\[
[[ \text{five} ]] = \lambda P_{\leq 5} \cdot \lambda Q_{\leq 5} \cdot | P \cap Q | \geq 5
\]

*The function that takes a set P and a set Q, and returns T iff the cardinality of their intersection is at least five.*

In an (over-)simplified semantics, where we ‘ignore’ the plurality of the NP, and treat all NPs as sets of atomic individuals, this semantics delivers the correct truth-conditions.
(30) **Truth-Conditions Predicted by (8), Ignoring Plurality**

\[
[[ \text{Five boys smoke} ]] = T \iff | \{x: x \text{ is a boy}\} \cap \{x: x \text{ smokes}\} | \geq 5
\]

*The set of boys that smoke is at least five.*

**Crucial Issue:**
In a (more realistic) system where the plural number of the NP complement of “five” is interpreted the semantics in (29) yields the wrong truth-conditions!

(31) **Truth-Conditions Predicted by (29), Interpreting Plurality**

\[
[[ \text{Five boys [* smoke ]} ]] = T \iff | *\{x: x \text{ is a boy}\} \cap *\{x: x \text{ smokes}\} | \geq 5
\]

*The set of pluralities of boys each of whose members smoke is at least five.*

*(i.e. Five groups of boys smoke.)*

(32) **The Inadequacy of the Truth-Conditions in (31)**

a. Suppose that Frank, Bill and Dave smoke.

b. Intuitively, then, all the following are groups of boys each of whose members smoke.

(i) Frank+Bill \quad (cf. Frank and Bill smoke)

(ii) Frank+Dave \quad (cf. Frank and Dave smoke)

(iii) Bill+Dave \quad (cf. Bill and Dave smoke)

(iv) Frank+Bill+Dave \quad (cf. Frank and Bill and Dave smoke)

c. Moreover recall that the extension of “boys” in our system will be the following:
\{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Bill+Dave, Frank+Bill+Dave\}

d. Consequently, the following set S will be *\{x: x \text{ is a boy}\} \cap *\{x: x \text{ smokes}\}:

\[
S = \{\text{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Bill+Dave, Frank+Bill+Dave}\}
\]

e. Clearly | S | \geq 5

f. Thus, the truth-conditions in (31) hold, and so the lexical entry in (29) wrongly predicts that “five boys smoke” should be true in a situation where (only) Frank, Bill and Dave smoke.

So... how do we fix the lexical entry in (29)?...

*Let’s start off by trying to characterize what we feel might be a more accurate set of T-conditions for “Five boys smoke.”*
More Accurate T-Conditions, First Pass

\[
[[ \text{Five boys smoke }]] = T \quad \text{iff} \quad \text{Some group of five boys is such that each individual member smokes.}
\]

If we accept this informal statement of the T-conditions, we write it more formally as follows:

More Accurate T-Conditions, Formalized Statement

\[
[[ \text{Five boys smoke }]] = T \quad \text{iff} \quad \exists x. x \in \{y : y \text{ is a boy}\} \land | \{z : z \leq x \land AT(z)\} | \geq 5 \land [[ \text{*smokes }]](x) = T
\]

There is some group of boys x, whose number of atoms is at least five, and every atom in x smokes...

Side-Note

The truth-conditions in (34) would correctly predict that “five boys smoke” will be false in the scenario sketched in (32)… as there isn’t any group of five boys in the extension of “*smokes”

Now let’s craft a lexical entry for “five” that will deliver the T-conditions in (34)...

Semantics of Five in Our System for Plurals

\[
[[ \text{five }]] = \lambda P_{<et>}. \lambda Q_{<et>}. \exists x. P(x) = T \land | \{z : z \leq x \land AT(z)\} | \geq 5 \land Q(x) = T
\]

We can, of course, generalize this approach to all numerals:

Semantics of Numerals in Our System for Plurals

a. \[
[[ \text{one }]] = \lambda P_{<et>}. \lambda Q_{<et>}. \exists x. P(x) = T \land | \{z : z \leq x \land AT(z)\} | \geq 1 \land Q(x) = T
\]

b. \[
[[ \text{two }]] = \lambda P_{<et>}. \lambda Q_{<et>}. \exists x. P(x) = T \land | \{z : z \leq x \land AT(z)\} | \geq 2 \land Q(x) = T
\]

c. \[
[[ \text{three}]] = \lambda P_{<et>}. \lambda Q_{<et>}. \exists x. P(x) = T \land | \{z : z \leq x \land AT(z)\} | \geq 3 \land Q(x) = T
\]

d. \[
[[ \text{four}]] = \ldots
\]
Side Comment Regarding the Existential Force

- In our lexical entries in (35)-(36), we retain the notion that numerals are generalized quantifiers (type <et,<et,t>>).
- It is actually more common (and more correct) to view numerals as simply being modifiers of the plural NPs (type <et>), as follows:
  a. \[ [[ \text{one} ]] = \lambda x . \{ z : z \leq x \land AT(z) \} \geq 1 \]
  b. \[ [[ \text{two} ]] = \lambda x . \{ z : z \leq x \land AT(z) \} \geq 2 \quad \ldots \text{etc.} \]
- In order to get the existential semantics of sentences like (34), we might for present purposes assume that there exists a null indefinite determiner, as in (c) below.
  c. \[ [[ \varnothing \text{D} ]] = \lambda P_{<et>} \cdot \lambda Q_{<et>} . \exists x. P(x) = T \land Q(x) = T \]
- The ‘modifier’ analysis of numerals is able to explain the possibility of sentences where numerals are predicates (d), as well as structures where numerically modified NPs are arguments to true Ds, such as demonstratives.
  d. Those boys are \textbf{five} (in number).
  e. Those \textbf{five} boys ate a pizza.

So far, we’ve found that our semantics for plural NPs can be combined with a plausible semantic analyses for ‘no’, ‘some’, and numerals…

**Exercise for the Reader:** What about ‘many’ (24d) and ‘most’ (24e)?

However, while simple numerals could all be interpreted as existential quantifiers over pluralities, not all numeral phrases admit of such an analysis…

What follows are a list of ‘determiners’ for which our analysis of numerals cannot be easily extended…

*Note that all the following quantifiers share the property of NOT being upward monotone (on their right argument)*
(38) **Exactly N**

Suppose we were to analyze the quantifier ‘exactly 3’ in the following fashion:

\[
[[ \text{exactly three} ]] = \lambda P. \lambda Q. \exists x. P(x) \land | \{z : z \leq x \land AT(z)\} | = 3 \land Q(x)
\]

Some group of exactly three \([^\text{NP}][^\text{QP}]\) are \([^\text{QP}][^\text{QP}]\).

The T-conditions of the sentence ‘Exactly 3 boys smoked’ would be the following:

b. \(\exists x. x \in \{y : y \text{ is a boy}\} \land | \{z : z \leq x \land AT(z)\} | = 3 \land [[\text{*smoked}}](x) = T\)

Some group of exactly three boys smoked.

However, this set of T-conditions would hold in a situation where five boys smoked – Bill, Dave, Frank, Tom and Lou – since there is a group of exactly three boys (Bill, Dave and Frank) each of whose members smoke.

(39) **Less than N**

Suppose we were to analyze the quantifier ‘less than 3’ in the following fashion:

\[
[[ \text{less than three} ]] = \lambda P. \lambda Q. \exists x. P(x) \land | \{z : z \leq x \land AT(z)\} | \leq 3 \land Q(x)
\]

Some group of less than three \([^\text{NP}][^\text{QP}]\) are \([^\text{QP}][^\text{QP}]\).

Then the T-conditions of the sentence ‘Less than 3 boys smoked’ would be the following:

b. \(\exists x. x \in \{y : y \text{ is a boy}\} \land | \{z : z \leq x \land AT(z)\} | \leq 3 \land [[\text{*smoked}}](x) = T\)

Some group of less than three boys smoked.

However, this set of T-conditions would hold in a situation where five boys smoked – Bill, Dave, Frank, Tom and Lou – since there is a group of less than three boys (Dave and Frank) each of whose members smoked.

(40) **Few**

Suppose we were to analyze the quantifier ‘few’ in the following fashion:

\[
[[ \text{few} ]] = \lambda P. \lambda Q. \exists x. P(x) \land | \{z : z \leq x \land AT(z)\} | \leq n \land Q(x)
\]

Some group of fewer than \(n\) \(\text{P}\) (where \(n\) is a contextually given ‘small’ amount) are \(Q\).

Following the reasoning laid out above, these T-conditions would wrongly predict that a sentence like ‘Few boys smoked’ could be true in a situation where many boys smoked… …since there will always be some small (sub-)group of boys each of whose members smoked…
**Issue:** For quantifiers that are not upward monotone (on their right argument), a simple analysis where they are existential quantifiers over pluralities seems not to work...

... so, it seems that some other approach to their meaning must be adopted...

*For more info on the meaning of plural DPs headed by ‘exactly N’, ‘less than N’ and ‘few’, see the following works:


Landman, Fred (2000) *Events and Plurality*. Dordrecht: Kluwer (Chapters 6, 7)

Hackl, Martin (2000) *Comparative Quantifiers*. PhD Dissertation. MIT (Chapters 1, 2)

... and what about DPs like “all dogs” (24h)?...  
... we’ll talk about those in the next set of notes...