The Basics of Plurals: Part 1
The Meaning of Plural NPs and the Nature of Predication Over Plurals

1. Introductory Questions and Guesses

(1) Blindingly Obvious Fact about Natural Language

There is number marking (singular, plural) on NPs, and this number seems to have an effect on the meaning of the sentence.

a. Some boy just ran into the room ≠

b. Some boys just ran into the room.

(2) Overarching Question

• What is the meaning of plural number marking?
• That is, what is the contribution of plural number marking to the truth-conditions of a sentence?

(3) Some Very Basic Data to Get Us Started

The inference from (3a) and (3b) to (3c) seems valid.

a. Bill is a boy.

b. Frank is a boy.

c. Bill and Frank are boys.

(4) A Very Basic Intuition to Get Us Started

If we think of “boy” as denoting a set of (individual) boys... perhaps the plural “boys” denotes the set of all possible ‘groups’ of boys.

Then the inference in (3) would go through!

If Bill is a boy and Frank is a boy, then Bill and Frank together are a possible ‘group’ of boys, and so should be in the extension of “boys”.

---

1 As complementary reading for this set of notes, I refer the reader to Lasersohn (to appear) and Link (1998: Chapters 1 and 4).
2. Plurals and Plural Entities

So, to start out, let’s try to build enough into our semantics to capture the validity of (3) following the ‘intuition’ in (4).

(5) Opening Caveat

- In these notes, we will develop a system based upon the influential work of Goedehard Link in the early 1980s.
- However, Link’s system is not the only system for plurality that has been proposed.

(6) Fundamental Question

*What do we mean when we say that the plural NP “boys” denotes groups of boys?*

a. Answer 1: Sets (Schwarzschild 1996, et alia.)
   - “Boys” denotes all the possible sets of boys.
   - For a given plural NP “NP-pl”, $[[NP\text{-}pl]] = P([[NP]])$ (power set of $[[NP]]$)

b. Answer 2: Plural Entities (Link 1983, et alia)
   A ‘group of boys’ is a kind of complex entity, a thing of type ‘e’ made out of other things of type ‘e’ (namely, individual boys)…

(7) Deciding Between The Two Answers

- As far as I am aware, there are no empirical arguments favoring (6a) over (6b), or vice versa.
- At one time, there were thought to be purely conceptual / metaphysical arguments for (6b), but the consensus now is that those arguments are not very compelling.
- Moreover, my own sense is that (6a) – particularly as it is developed by Schwarzschild (1996) – is conceptually simpler in a number of ways
- However, since more works seem to adopt answer (6b), for didactic reasons it’s best to get everyone used to that way of doing things.
Plural Entities: The Background Metaphysics

a. **Obvious, Common-Sense Observation**  
   Clearly, objects can be made out of other objects.

b. **Huge Metaphysical Leap**  
   For any two objects A and B, there is the abstract ‘plural object’ constructed from A and B.

c. **Notation**  
   ‘A+B’ = the plural object made from A and B.

d. **Examples**  
   Suzi+Noah, Seth+HulkHogan, (Suzi+Noah)+Seth  
   ((Seda+Jason)+Minta)+(Angelika+Kathy)

Let’s start to clarify this idea some, by adding some additional concepts and formalisms to our ‘meta-language’

The Group Forming Operator ‘+’

- The (meta-language) operator ‘+’ is a function of type <e,<e,e>>. It takes two entries of type ‘e’, and yields the (plural) entity of type ‘e’.

- The operator ‘+’ is idempotent, commutative and associative. Thus:
  
  \[x+x = x\]
  
  \[x+y = y+x\]
  
  \[x+(y+z) = (x+y)+z\]

- Consequently, parentheses are not required in representing pluralities constructed from other pluralities.
  
  \[x+(y+z) = (x+y)+z = x+y+z\]

The Domain of Individual and Plural Entities

Let \(D\) be our domain of ‘individual entities’.

**New Notation:**  
\[*D = \text{‘The domain of plural entities.’}\]

*The closure of \(D\) under the ‘group forming’ operator ‘+’*

\[= \text{The smallest set such that}\]

a. \(D \subseteq *D\)

b. If \(x, y \in *D\), then \(x+y \in *D\)
(11) **A Picture of What the Domains Look Like Now**

a. **The Domain of Individuals, D:** Sue, Frank, Bill

b. **The Domain of Plural Entities, *D:**

\[
\begin{align*}
S+F+B & \\
S+F & S+B & F+B \\
Sue & Frank & Bill
\end{align*}
\]

Finally, before we start using this ontology of ‘plural entities’ to do natural language semantics, we’ll want to have some vocabulary for describing certain key properties and relations that the ‘denizens of *D’ can bear…

(12) **Two Key Concepts: ‘Part-of’ and Atomicity**

a. **The Relation ‘Part of’**
   (i) **Notation**
   \( x \leq y \) \ ‘x is part of y’
   
   (ii) **Definition**
   \( x \leq y \iff \exists z. x+z = y \)
   
   *There is some z such that y is a group consisting of x and z.*

   **Side-Note**
   • A key consequence of this definition is that for all \( x, x \leq x \)

b. **The Property of ‘Atomicity’**
   (i) **Notation**
   \( \text{AT}(x) \) \ ‘x is an atom’
   
   (ii) **Definition**
   \( \text{AT}(x) \iff \forall y. y \leq x \rightarrow y=x \)
   
   *The only object that is a ‘part of x’ is x itself.*

   **Side-Note**
   • Consequently, in the system as we have it here, \( \text{AT}(x) \iff x \in D \)

*With the metaphysical machinery laid out here, we can now start to provide a semantics for plural NPs that captures the very basic data in (3)…*
3. A Basic Semantics for Plural NPs

Remember our idea that “boys” denotes all the possible groups of (individual boys)? Here’s a way to precisely state that idea in our formal metalanguage.

(13) **Interpretation of Plural NPs**

\[
[[ \text{NP pl} ]] = *[[\text{NP}]] = \text{‘the closure of [[NP]] under +’}
\]

(14) **Pluralization in a Picture**

Suppose that the boys are Frank, Bill and Dave. Consequently:

a. \[[\text{boy}]\] = \{Frank, Bill, Dave\}

Moreover, given the rule in (13), it follows that:

b. \[[\text{boy pl}]\] = *[[\text{boy}]] = \{ Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, Frank+Dave+Bill \}

To complete the picture, we need a new rule for interpreting conjunctions of phrases of type ‘e’

(15) **Plural Conjunction**

If ‘DP1’ is type e and ‘DP2’ is type e, then [DP1 and DP2] is of type e, then:

\[
[[ \text{DP1 and DP2} ]] = [[\text{DP1}]] + [[\text{DP2}]].
\]

*The group formed from DP1 and DP2*

We now have enough for our semantics to predict the inference in (3), repeated below:

(16) **Targeted Valid Inference**

a. Bill is a boy.
b. Frank is a boy.
c. Bill and Frank are boys.

(17) **Proof of Validity of Inference**

a. Suppose “Bill is a boy” and “Frank is a boy” are true.
b. Clearly, our semantics predicts that Bill, Frank ∈ \{ x : x is a boy \}
c. Consequently, Bill+Frank ∈ *\{ x : x is a boy \}
d. Thus, \[[ \text{Bill and Frank} ]] ∈ *[[\text{NP pl}]]
e. Therefore, \[[ \text{Bill and Frank are boys} ]] = T
4. **The Semantics of Definite Plurals**

So far our system can handle simple sentences where the plural NP is a predicate…
But what about sentences where the plural NP is in an *argument of a verb*?

(18) **Definite Plurals as Arguments to Vs**

a. The boys are running.
b. Sue saw the boys.

(19) **Overarching Questions**

a. What is the meaning of such plural DPs like “the boys”?
b. How can the meaning of the definite article “the” combine with the meaning of the plural NP “boys” to produce the observed meaning of “the boys”?

(20) **Some Very Basic Data to Get Us Started**

a. The inference from (i) to (ii) seems valid.

(i) Frank, Bill and Dave are the boys.
(ii) Every boy is either Frank or Bill or Dave.

b. The singular definite in (i) is undefined if there is *more than one boy*, while the plural definite in (ii) obviously isn’t.

(i) The boy is nice. (Undefined if more than one boy)
(ii) The boys are nice. (Completely fine if there’s more than one boy)

(21) **A Very Basic Intuition to Get Us Started**

a. Like “Bill and Frank”, the plural DP “the boys” refers to a particular *plural* entity
b. The plural entity that “the boys” refers to is *the group consisting of all the boys*

(22) **The Maximization Operator, MAX**

Let S be a set of entities from *D.

\[
\text{MAX}(S) = \text{the unique } x \text{ in } S \text{ such that } \forall y. y \in S \rightarrow y \leq x
\]

*That element in S which all other things in S are a part of*

(otherwise undefined)
Note: In some work, the maximization operator is called the ‘sum’ operator, and is notated as $\sigma$

(23) **A New Interpretation of the Definite Article**

You’re probably familiar with a semantics for definite articles that looks something like the following:

a. Basic Semantics for Definite Article (Heim & Kratzer 1998):

$$[[ \text{the} ]] = \left[ \lambda P . \ P \text{ is an } \langle \text{et} \rangle \text{ function and is true of exactly one } x : \text{ the unique } y \text{ such that } P(y) = T \right]$$

Throughout this seminar, however, we’ll be employing a very different semantics for the definite article:

b. Maximization Semantics for the Definite Article

$$[[ \text{the} ]] = \lambda P_{<\text{et}>} \cdot \text{MAX}(P)$$

As we will see presently, this semantics provides the right meaning for definite plurals…

(24) **The Semantics of Definite Plurals**

Suppose that Frank and Bill and Dave are all the boys in the world. All the following hold in our system:

a. $$[[ \text{boy} ]] = \{ \text{Frank}, \text{Bill}, \text{Dave} \}$$

b. $$[[ \text{boys} ]] = \{ \text{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, Frank+Dave+Bill } \}$$

c. $$[[ \text{the boys} ]] = \text{MAX}([[\text{boys}]])$$

$$= \text{MAX}(\{ \text{Frank, Bill, Dave, Frank+Bill, Frank+Dave, Dave+Bill, Frank+Dave+Bill } \}$$

$$= \text{Frank+Bill+Dave}$$

Thus, our system captures the ‘intuition’ that “the boys” refers to the group (plurality) consisting of all the boys.

Moreover, with a fairly simple assumption regarding DPs in predicate position (25), we accurately predict the validity of the inference in (20a).
(25) **Special Rule for Predicative DPs**

\[ [[\text{is/are DP}]] = \lambda x . x = [[\text{DP}]] \]

(26) **Validity of Inference (20a)**

a. Suppose that “Frank, Bill and Dave are the boys” is true.

b. By our rule in (25), it follows that \([[\text{Frank and Bill and Dave}]] = [[\text{the boys}]]

c. Thus, Frank+Bill+Dave = MAX(*\{x: x \text{ is a boy}\})*

d. Thus, Frank+Bill+Dave contain as parts all the possible groups of boys.

e. Thus, if anything is a boy, then its either Frank or Bill or Dave

But, does our new semantics in (23b) extend to singular definites?...
In fact, it does! It predicts that a singular definite like “the boy” is **undefined** is there is more than one boy (just as in (23a))…

(27) **The Semantics of Singular Definites**

a. **When There is Only One Entity Satisfying the NP**

(i) Suppose that there is exactly one boy, Dave.

(ii) Consequently, \( [[\text{boy}]\] = \{Dave\}

(iii) Recall that **for all** \( x, x \leq x \) (see p. 4)

(iv) Consequently, \( \text{MAX}(\{\text{Dave}\}) = \text{Dave} \)

(v) Consequently, \( [[\text{the boy}]\] = \text{Dave} \)

b. **When There is More than One Entity Satisfying the NP**

(i) Suppose that there are **two** boys, Dave and Bill.

(ii) Consequently, \( [[\text{boy}]\] = \{Dave, Bill\}

(iii) Note that neither Dave \( \leq \) Bill nor Bill \( \leq \) Dave.

(iv) Consequently, there is no element in \{Dave, Bill\} that contains all other elements in the set as component parts.

(v) Consequently, \( \text{MAX}(\{\text{Dave, Bill}\}) \) is **undefined**

(vi) Consequently, “the boy” is **undefined**
5. Pluralities as Arguments of Predicates

Our semantics for definite plurals seems pretty good so far…

… but how exactly do such plural DPs function as the arguments of verbs?

(28) Definite Plurals as Arguments to Vs

a. The boys are running.
b. Sue saw the boys.
c. The boys ate a pizza.

Well, in a certain sense, there’s no great mystery here. Recall that in our semantic system, even super-mega-complex plural entities like “the things in my house or in my yard” are all of type e

Consequently, there should be no problem with their combining with predicates of type <et>, <e<et>>, or what have you!

Thus, it’s possible for the extensions of verbal predicates to contain plural entities as well…

(29) Possible Extensions in Our System

<table>
<thead>
<tr>
<th>Expression</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>[[ run ]]</td>
<td>{ Tom, Frank, Bill+Dave }</td>
</tr>
<tr>
<td>[[ see ]]</td>
<td>{ &lt;Frank, Bill&gt;, &lt;Sue, Frank+Bill+Dave&gt; }</td>
</tr>
<tr>
<td>[[ eat a pizza ]]</td>
<td>{ Tom, Frank, Bill+Dave }</td>
</tr>
</tbody>
</table>

However, the possibility of extensions like (29) raises the following key (metaphysical) question:

(30) Fundamental (Metaphysical) Question in the Theory of Plurals

- What, exactly, are the conditions under which a plural entity lies in the extension of a verbal predicate?
- That is, what ‘state of affairs’ (loosely speaking) do extensions like (29) represent in our theory?

It would be fair to say that many of the important controversies in the theory of plurals in one way or another boil down to this fundamental question in (30)…

… and one reason why this question remains such a font of controversies is that different predicates seem to behave differently…
(31) Predicates that (Always) ‘Distributively’ Predicate Over Pluralities

Some predicates, like “running”, seem to hold of a plural entity iff they hold of every atomic member of that entity. (They distributively predicate over the plural entity.)

a. Valid Inference:
   (i) Dave, Bill and Frank are the boys.
   (ii) The boys are running.
   (iii) Therefore, Dave is running, Bill is running, and Frank is running.

So, could the general answer to (30) be that ‘a predicate holds of a plural entity iff it holds of every atomic member of that entity’?...

   NO! Because there are predicates like the following:

(32) Predicates that Can Hold ‘Collectively’ of a Plurality

Some predicates, like “lift a piano” or “meet”, can hold collectively of a plural entity; That is, they hold of the plural entity without necessarily holding of all (or any) atomic member of that entity.

a. Invalid Inference
   (i) Dave, Bill and Frank are the boys.
   (ii) The boys lifted a piano. / The boys met.
   (iii) Thus, Dave lifted a piano, Bill lifted a piano, and Frank lifted a piano /
   Thus, * Dave met, * Bill met, and * Frank met.

b. Consistent Statement
   (i) Dave, Bill and Frank are the boys.
   (ii) The boys lifted a piano / The boys met.
   (iii) But, Dave (alone) didn’t lift a piano, nor Bill, nor Frank. /
   But, * Dave (alone) didn’t meet, nor Bill, nor Frank.

So, how do we make sense of these facts? What is the answer to (30)?

• Well, it’s fair to say that (since at least Scha 1981), people have taken one of these two patterns above as basic, and have treated the other as involving some ‘special’ property of the meaning of the predicate.

• Most of the literature assumes that predicates like “run” in (31) are the ‘special case’, the one that involves some additional ‘special’ factor to their meaning.

• Relatedly, most of the literature assumes that predicates like “lift a piano” or “meet” in (32) are the ‘basic case’, the one that ultimately reveals how plural predication at base functions in natural language.  
(But not everyone accepts this view (Landman 2000: Chapters 5 – 6))
• In our discussion here, we will assume the more common (though not necessarily correct) view that plural predication is – at base – collective predication….

• For the moment, then, we will put aside predicates like “run” (but we’ll come back to them again soon with an explanation for their particular behavior)…

OK… so what does this mean regarding a clear answer to (30)? The view we will adopt here (which you will find in many other works) can be summarized as in (33).

(33) **The Classic Answer to (30): Collective Predication**

- For a basic VP predicate like “eat a pizza”, a singular (atomic) entity lies in the extension of the predicate *iff* that entity is the **agent** of the act/event in question.

- Thus, for such basic predicates, a plural (non-atomic) entity lies in the extension of the predicate *iff* that entity is **collectively** (together) the agent of the act/event in question.

(34) **Illustration with Pizza Eating**

a. Suppose that Frank ate pizza-1 completely on his own, while Dave and Bill shared pizza-2, completely eating the whole thing.

b. Frank alone is clearly the eater of pizza-1. *But who is the ‘eater’ of pizza-2?*

c. Dave alone didn’t eat pizza-2. Bill alone didn’t eat pizza-2. Rather, **Dave and Bill together ate pizza-2.**

d. This is nicely summarized in the following picture (adapted from Kratzer 2005).

<table>
<thead>
<tr>
<th>Event-of-Pizza-Eating</th>
<th>Pizza Eaten</th>
<th>Pizza Eaters</th>
</tr>
</thead>
<tbody>
<tr>
<td>e₁</td>
<td>pizza-1</td>
<td>Frank</td>
</tr>
<tr>
<td>e₂</td>
<td>pizza-2</td>
<td>Dave+Bill</td>
</tr>
</tbody>
</table>

e. Thus, in this kind of state of affairs, we should consider the extension of the VP “eat a pizza” to be as follows:

$$[[ \text{eat a pizza } ]] = \{ \text{Frank, Dave+Bill } \}$$

f. And, moreover, we will assume that verbal extensions of the kind in (34e) obtain in states of affairs like that indicated in the box under (34d).
(35) **Summary Regarding Collective Predication**

For basic predicates,\(^2\) a plural entity is in the extension of the predicate *iff* that entity *collectively* satisfies the predicate (i.e., that plural entity satisfies the predicate *as a group*).

(36) **Important Note**

- This way of modeling ‘collective predication’ is much more transparent if we employ an event-based semantics for VPs.
- After all, in an event-based semantics, the extension of the VP like “eat a pizza” will be the following function:

\[
\lambda x . \left( \lambda e . \exists y \left( \text{eat}(e) \land y \text{ is a pizza and Theme}(e) = y \land \text{Agent}(e) = x \right) \right)
\]

\(e\) is an event of eating, there is a pizza that is the theme of \(e\), and \(x\) is the agent of \(e\)
- Thus, in situation (34d), the extension of this VP will be the following set of pairs.

\{ < Frank, e_1 >, < Dave+Bill, e_2 > \}
- And this kind of extension corresponds rather clearly to the ‘intuitive picture’ in (34d), *as each event is paired with its particular (unique) Agent.*

That is, the intuitive notion of ‘collective prediction’ that everyone seems to go by is that there is a single event of the type in question whose agent is the entire plural entity.

Thus, it’s in retrospect a bit odd that (as we’ll see) so much of the literature on plurality – particularly that relating to the nature of collective readings – doesn’t employ an explicitly event-based semantics.

(This same point is made briefly and elegantly by Kratzer (2005)…)

... So, (35) lays out a view of what it means for a plural entity to lie in the extension of a VP... ... but what about plural entities that are in other argument positions?...

\(^2\) The importance of making this restriction to ‘basic predicates’, will appear once we discuss so-called ‘distributive’ and ‘cumulative’ readings.
Definite DPs in Other Argument Positions

a. Dave kissed the girls.
b. Dave judged the boys.

Here, the situation is rather similar to what we saw above for plural DPs in subject position.

Some Verbs ‘Distributively’ Predicate Over Plural Direct Objects

Intuitively “Dave kissed the girls” is T iff for every individual girl x, Dave kissed x.

Some Verbs Can Predicate ‘Collectively’ Over Plural Direct Objects

- “Dave judged the boys” can be T without Dave judging each individual boy.
- Consider a dance competition involving group performances.

Recalling the perspective articulated in (33)-(35), we might again take the ‘basic’ case here to be verbs like those described in (39)...

(verbs like those described in (38) have some kind of ‘special property’ we’ll come back to...)

General Summary Regarding Pluralities as Arguments

a. Plural DPs are of type e, and therefore can easily function as arguments to basic verbal predicates (of type <et>, <eet> or what have you).
b. For basic predicates, if a plural entity satisfies an argument position, it does so ‘collectively’. That is, the entire plurality satisfies the argument position (and not necessarily any individual atom in the plurality).

Again... not everybody shares this view in (40) (cf. Landman 2000)... but it seems to be a common assumption...

To test our comprehension of these basic notions regarding plural arguments, let’s look at some sample extensions and then describe ‘informally’ what kind of situations they correspond to...
(41) **Some Illustrative Extensions**

Informally speaking, what kinds of ‘states of affairs’ do the following extensions correspond to?

(As noted above, under the event-based semantics, it’s a bit clearer how one is supposed to understand these extensions…)

a. $[[\text{lift}]]$

   (i) **Classic Semantics:**

   \[
   \{ <\text{Dave}, \text{piano-1}>, <\text{Dave}+\text{Bill}, \text{piano-2}>, <\text{Tom}, \text{piano-1}+\text{piano2}> \}
   \]

   (ii) **Event-Based Semantics:**

   \[
   \{ <\text{Dave}, \text{piano-1}, e_1>, <\text{Dave}+\text{Bill}, \text{piano-2}, e_2>, <\text{Tom}, \text{piano-1}+\text{piano2}, e_3> \}
   \]

b. $[[\text{eat}]]$

   (i) **Classic Semantics:**

   \[
   \{ <\text{Dave}, \text{cookie-1}>, <\text{Dave}+\text{Bill}, \text{cookie-2}>, <\text{Frank}, \text{cookie-3}+\text{cookie4}>, <\text{Frank}+\text{Bill}, \text{cookie5}+\text{cookie6}> \}
   \]

   (ii) **Event-Based Semantics:**

   \[
   \{ <\text{Dave}, \text{cookie-1}, e_1>, <\text{Dave}+\text{Bill}, \text{cookie-2}, e_2>, <\text{Frank}, \text{cookie-3}+\text{cookie4}, e_3>, <\text{Frank}+\text{Bill}, \text{cookie5}+\text{cookie6}, e_4> \}
   \]

c. $[[\text{give}]]$

   (i) **Classic Semantics:**

   \[
   \{ <\text{Dave}, \text{toy-1}+\text{toy2}, \text{Paul}>, <\text{Bill}+\text{Frank}, \text{toy3}, \text{Joy}>, <\text{Tom}, \text{toy-4}, \text{Judy}+\text{Jen}>, <\text{Bill}+\text{Frank}, \text{toy-5}+\text{toy6}, \text{Lucy}+\text{Sue} > \}
   \]

   (ii) **Event-Based Semantics**

   \[
   \{ <\text{Dave}, \text{toy-1}+\text{toy2}, \text{Paul}, e_1>, <\text{Bill}+\text{Frank}, \text{toy3}, \text{Joy}, e_2>, <\text{Tom}, \text{toy-4}, \text{Judy}+\text{Jen}, e_3>, <\text{Bill}+\text{Frank}, \text{toy-5}+\text{toy6}, \text{Lucy}+\text{Sue}, e_4 > \}
   \]