The Semantics of Adjectival Modification

(1) **Our Current Assumptions Regarding Adjectives and Common Ns**

a. *Both adjectives and common nouns denote functions of type* \(<e,t>\)

   (i) \[ \text{[[ male ]] } = \lambda x : x \in D_e. x \text{ is male} \]
   (ii) \[ \text{[[ politician ]] } = \lambda x : x \in D_e. x \text{ is a politician} \]

b. *The copula and the indefinite article are ‘semantically vacuous’*

   (i) \[ \text{[[ is ]] } = \lambda f : f \in D_{<e,t>}. f \]
   (ii) \[ \text{[[ a ]] } = \lambda f : f \in D_{<e,t>}. f \]

(2) **Truth-Conditional Statements These Assumptions Derive**

a. “Barack is male” is T iff Barack is male.
b. “Barack is a politician” is T iff Barack is a politician.

(3) **PROBLEM**

Our semantic system is unable to interpret sentences like those in (4), which are quite common, canonical structures for adjectives.

(4) **Adjectival Modification Structure in English**

a. Barack is a **male politician**.

b. \[
    S \\
    \rightarrow \\
    NP_1 \\
    \rightarrow \\
    N_1 \\ V \\ D \\
    Barack is \\
    AP <e,t> \\
    A male \\
    N_4 <e,t> \\
    politician
\]

---

1 These notes are based on the material in Heim & Kratzer (1998: 63-73).
What’s the Problem with (4)?

- $NP_3$ is a node whose two daughters both denote $<e,t>$ functions…
- Thus, neither daughter of $NP_3$ can take the other as argument…
- Thus, Function Application can’t apply to interpret $NP_3$…
- Thus, we have no rule for interpreting $NP_3$.

The Plan Towards a Solution

a. Let’s try to think up a rule that will allow us to interpret $NP_3$, and which will derive the correct $T$-conditions for sentence (4a).

b. Let’s start off, then, by doing the following:

   (i) Let’s choose an accurate truth-conditional statement for (4a) which we want to derive.

   (ii) Given that truth-conditional statement, let’s figure out what the extension of $NP_3$ has to be.

   (iii) Given the extension we figure out for $NP_3$, let’s figure out a rule which will derive that extension from the extensions of $NP_4$ (politician) and the AP (male).

Targeted Truth-Conditional Statement

“Barack is a male politician” is $T$ iff Barack is male and Barack is a politician.

Side-Note:

Why this $T$-conditional statement?

- It’s accurate.
- As a matter of fact, it ends up putting us on the right track for a number of adjectives.

Now, let’s take the $T$-conditional statement in (7) as given, and then figure out what the extension of the NP “male politician” has to be in order to derive it!
1. **Deducing the Extension of “male politician”**

First, let’s deduce the semantic type of the complex, modified NP *male politician*.

(8) **Deducing the Type of NP₃**

But what kind of $<$e,t$>$ function is the extension of NP₃??

| a. Given that the type of [[S]] is $t$, and the type of NP₁ is $e$, we can conclude (via familiar reasoning), that the type of [[VP]] is $<$e,t$>$.
| b. Given that the type of [[VP]] is $<$e,t$>$, and the type of [[V]] is $<$et,et$>$, we can conclude (via familiar reasoning), that the type of [[NP₂]] is $<$e,t$>$.
| c. Given that the type of [[NP₂]] is $<$e,t$>$, and the type of [[D]] is $<$et,et$>$, we can conclude (via familiar reasoning), that the type of [[NP₃]] is $<$e,t$>$.

(9) **Some Reasoning, Part 1**

**CLAIM:** [[VP]] = [[ NP₃ ]]  

a. $[[\text{is a male politician}]] = [[\text{is}]] ( [[\text{a male politician}]] )$ (by FA, NN)  

b. $[[\text{is}]] ( [[\text{a male politician}]] ) = [\lambda f_{<e,t>} : f ] ([[\text{a male politician}]] )$ (by TN)  

c. $[\lambda f_{<e,t>} : f ] ([[\text{a male politician}]] ) = [[\text{a male politician}]]$ (by LC)  

d. $[[\text{a male politician}]] = [[\text{a}]] ( [[\text{male politician}]] )$ (by FA, NN)  

e. $[[\text{a}]] ( [[\text{male politician}]] ) = [\lambda f_{<e,t>} : f ] ([[\text{male politician}]] )$ (by TN)  

f. $[\lambda f_{<e,t>} : f ] ([[\text{male politician}]] ) = [[\text{male politician}]]$ (by LC)
(10) Some Reasoning, Part 2

a. Given our rule of FA and the types deduced above:

\[[ \text{Barack is a male politician} ]\] = \[[ \text{is a male politician} ]\](Barack)

b. Given our reasoning in (9), it follows that the three truth-conditional statements below are all equivalent.

\( (i) \quad [[\text{Barack is a male politician}]] = T \text{ iff } \text{B. is male and B. is a politician.} \)

\( (ii) \quad [[\text{is a male politician}]](\text{Barack}) = T \text{ iff } \text{B. is male and B. is a politician.} \)

\( (iii) \quad [[\text{male politician}]](\text{Barack}) = T \text{ iff } \text{B. is male and B. is a politician.} \)

c. CONCLUSION: The extension of NP\(_3\) “male politician” is a function which takes an entity x as argument, and returns T iff x is male and x is a politician.

(11) The Deduced Extension for NP\(_3\)

\[[ \text{male politician} ]\] = \[ λx : x ∈ D_e . x \text{ is male and x is a politician} \]

2. Developing a Rule that Will Derive the Extension

(12) What We’ve Deduced So Far

\[
\begin{array}{c}
\text{NP}_3 \\
\text{AP} \quad \text{NP}_4 \\
\text{A} \quad \text{N}_4 \\
\text{male} \quad \text{politician}
\end{array}
\]

= \[ λx : x ∈ D_e . x \text{ is male and x is a politician} \]

(13) What We Need

- A rule which will derive the equation in (12).
- This rule will relate the extension of NP\(_3\) to the extensions of its two daughter nodes.
- The rule will thus take [[AP]] and [[NP\(_4\)]]\], and give us back [[NP\(_3\)]]\], as schematized below:

RULE ( [[AP]] ) ( [[NP\(_4\)]] ) = [[NP\(_3\)]] = \[ λx_e : x \text{ is male and x is a politician} \]
(14) Some Reasoning

a. As mentioned above, our rule should set up the following equation:

\[
\text{RULE} \left( \left[ \text{AP} \right] \right) \left( \left[ \text{NP} \right] \right) = \left[ \lambda x : x \text{ is male and } x \text{ is a politician} \right]
\]

b. However, given NN and TN, we know that:

(i) \( \left[ \text{AP} \right] \) = \( \left[ \lambda y : y \text{ is male} \right] \)

(ii) \( \left[ \text{NP} \right] \) = \( \left[ \lambda y : y \text{ is a politician} \right] \)

c. Thus, our rule should set up the following equation:

\[
\text{RULE} \left( \left[ \lambda y : y \text{ is male} \right] \right) \left( \left[ \lambda y : y \text{ is a politician} \right] \right) = \left[ \lambda x : x \text{ is male and } x \text{ is a politician} \right]
\]

d. Given our notation, we know that the following equivalences hold in our metalanguage:

(i) \( x \text{ is male} \approx \left[ \lambda y : y \text{ is male} \right](x) = T \)

(ii) \( x \text{ is a politician} \approx \left[ \lambda y : y \text{ is a politician} \right](x) = T \)

e. Thus, given c and d, we know that our rule should set up the following equation:

\[
\text{RULE} \left( \left[ \lambda y : y \text{ is male} \right] \right) \left( \left[ \lambda y : y \text{ is a politician} \right] \right) = \left[ \lambda x : \left[ \lambda y : y \text{ is male} \right](x) = T \text{ and } \left[ \lambda y : y \text{ is a politician} \right](x) = T \right]
\]

f. CONCLUSION:
What our rule should do is take two \(<e, t>\) functions -- \(f\) and \(g\) -- and give back the \(<e, t>\) function which maps and entity \(x\) to \(T\) iff \(f(x) = T\) and \(g(x) = T\)

(15) The Rule of ‘Predicate Modification’ (PM) [Heim & Kratzer (1998: 65)]

If \(X\) is a branching node that has two daughters -- \(Y\) and \(Z\) -- and if both \([Y]\) and \([Z]\) are in \(D_{<et>}\), then:

\[
\left[ \text{X} \right] = \left[ \lambda x : x \in D_{e} \cdot \left[ Y \right](x) = T \text{ and } \left[ Z \right](x) = T \right]
\]

The \(<et>\) function which takes an entity \(x\), and yields \(T\) iff \([Y]\) applied to \(x\) is \(T\) and \([Z]\) applied to \(x\) is \(T\).
Let’s make sure this rule of ‘Predicate Modification’ does the work we want it to, by trying it out in a truth-conditional derivation!...

(16) A Quick, Sample Derivation

a. “S” is T iff (by notation)

b. [[S]] = T

c. Subproof:
(i) [[ NP₁ ]] = (by NN x2, TN)
(ii) Barack

d. Subproof:
(i) [[ V ]] = (by NN, TN)
(ii) [ λf : f ∈ Dₑₜₚ. f ]

e. Subproof:
(i) [[ D ]] = (by NN, TN)
(ii) [ λf : f ∈ Dₑₜₚ. f ]

f. Subproof:
(i) [[ AP ]] = (by NN x2, TN)
(ii) [ λy : y ∈ Dₑ. y is male ]

g. Subproof:
(i) [[ NP₄ ]] = (by NN x2, TN)
(ii) [ λy : y ∈ Dₑ. y is a politician ]

h. Subproof:
(i) [[ NP₃ ]] = (by PM, f, g)
(ii) [ λx : x ∈ Dₑ. [[ AP ]](x) = T and [[ NP₄ ]](x) = T ] = (by f )
(iii) [ λx : x ∈ Dₑ. [ λy : y ∈ Dₑₜₚ. y is male ](x) = T and [[ NP₄ ]](x) = T ] = (by LC)
(iv) [ λx : x ∈ Dₑ. x is male and [[ NP₄ ]](x) = T ] = (by g)
(v) [ λx : x ∈ Dₑ. x is male and [ λy : y ∈ Dₑₜₚ. y is a politician ](x) = T ] = (by LC)
(vi) [ λx : x ∈ Dₑ. x is male and x is a politician ]

Proof continued on the following page...
i. **Subproof:**

   (i) $[[\text{NP}_2]] = (\text{by FA, e, h})$

   (ii) $[[\text{D}]] ([[\text{NP}_3]]) = (\text{by e})$

   (iii) $[\lambda f : f \in D_{\text{ex,p}} . f] ([[\text{NP}_3]]) = (\text{by LC})$

   (iv) $[[\text{NP}_3]] = (\text{by h})$

   (v) $[\lambda x : x \in D_e . \text{x is male and x is a politician}]$

j. **Subproof:**

   (i) $[[\text{VP}]] = (\text{by FA, d, i})$

   (ii) $[[\text{V}]] ([[\text{NP}_2]]) = (\text{by d})$

   (iii) $[\lambda f : f \in D_{\text{ex,p}} . f] ([[\text{NP}_2]]) = (\text{by LC})$

   (iv) $[[\text{NP}_2]] = (\text{by i})$

   (v) $[\lambda x : x \in D_e . \text{x is male and x is a politician}]$

k. $[[\text{S}]] = T \iff (\text{by FA, c, j})$

l. $[[\text{VP}]] ([[\text{NP}_1]]) = T \iff (\text{by c})$

m. $[[\text{VP}]] (\text{Barack}) = T \iff (\text{by j})$

n. $[\lambda x : x \in D_e . \text{x is male and x is a politician}] (\text{Barack}) = T \iff (\text{by LC})$

o. Barack is male and Barack is a politician.

(17) **Conclusions**

a. Our rule of Predicate Modification (PM) is able to successfully derive the following T-conditional statement:

   “Barack is a male politician” is $T \iff$ Barack is male and Barack is a politician.

b. Our rule of PM will similarly derive the following T-conditional statements, all of which seem to be accurate:

   (i) “Muffy is a **pregnant cat**” is $T \iff$ Muffy is **pregnant** and Muffy is a **cat**.

   (ii) “Joe is a **married man**” is $T \iff$ Joe is **married** and Joe is a **man**.

   (iii) “Tor is a **dead dinosaur**” is $T \iff$ Tor is **dead** and Tor is a **dinosaur**.
3. A Problem for Our Rule of Predicate Modification: Subsective Adjectives

(18) General T-Conditional Statement Derived by Our Rule of PM

We just saw in the previous section that our rule of predicate modification derives T-conditional statements of the following general form:

“\text{name} \text{ is an adjective noun}” is T iff \text{name} is adjective and \text{name} is a noun

(19) PROBLEM!

Unfortunately, not all T-conditional statements of the form in (18) are accurate. The following T-conditional statements don’t seem to be accurate:

a. “Barack is a young president” is T iff Barack is young and Barack is a president.
   • Barack is 53, and so is a young president (\textit{i.e.}, he’s young for a president).
   • However, it isn’t true that Barack is young (in any absolute sense).

b. “Allen Iverson is a short basketball player” is T iff Allen Iverson is short and Allen Iverson is a basketball player.
   • Allen Iverson is only 6’ tall, and so he is a short basketball player.
   • However, it isn’t true that Iverson is short (in any absolute sense).

c. “Howard Lasnik is a famous linguist” is T iff Howard Lasnik is famous and Howard Lasnik is a linguist.
   • Every linguist knows who Howard Lasnik is, and so he is a famous linguist (\textit{i.e.}, he’s famous for a linguist)
   • However, it isn’t true that Howard Lasnik is famous (in any absolute sense).

(20) Conclusion

There seem to be adjectival modification structures where our rule of PM makes the wrong predictions.

\textit{So, how can we handle cases like those in (19)?}
(21) **Guiding Intuition**

a. There’s a very crucial and fundamental difference between the meanings of the adjectives in (i) below (for which our rule of PM works), and the meanings of adjectives in (ii) below (for which our rule of PM doesn’t work):

(i) *male, pregnant, married, dead, (female, unmarried, widowed…)*

(ii) *young, short, famous, (tall, old, happy, angry…)*

b. The key difference between these two classes of adjectives seems to be the following:

- Something can be *young/short/famous/etc.* in a **relative sense**. That is, it makes sense to say things like the following:

  “He is young/short/famous/etc. for an X.”

- Something **can’t** be *male/pregnant/married/dead/etc.* in a **relative sense**. That is, it makes **no** sense to say things like the following:

  “He is male/pregnant/married/dead/etc. for an X.”

c. Consequently…

If you say: then you could mean: and not:

*young president*  
young *for a president*  
young (absolutely)

*short b.ball player*  
short *for a b.ball player*  
short (absolutely)

*famous linguist*  
famous *for a linguist*  
famous (absolutely)

But…

If you say: then you **couldn’t** mean: you could only mean:

*male politician*  
*male for a politician*  
*male (absolutely)*

*pregnant cat*  
*pregnant for a cat*  
*pregnant (absolutely)*

*dead dinosaur*  
*dead for a dinosaur*  
*dead (absolutely)*

(22) **Terminology**

a. **Intersective Adjective**

Adjectives like *male, pregnant, etc.*, for which our rule of PM works perfectly.

b. **Subsective Adjective**

Adjectives like *young, short, famous, etc.*, for which our rule of PM doesn’t work.
So, our rule of PM works perfectly for the so-called ‘intersective adjectives’...
How, then, do we incorporate the so-called ‘subsective adjectives’ into our system?

(23) Proposal for Subsective Adjectives

a. \[ [\text{young}] = \]
   \[ \lambda f_{<et>} : [\lambda x_e : f(x) = T \text{ and } x \text{ is below the average age for the entities in } \{ y : f(y) = T \} ] \]
   the function which takes an <et> function \( f \), and an entity \( x \), and returns \( T \) iff:
   \( f(x) = T \) and \( x \) is below the average age for entities of which \( f \) is \( T \)

b. \[ [\text{short}] = \]
   \[ \lambda f_{<et>} : [\lambda x_e : f(x) = T \text{ and } x \text{ is below the average height for the entities in } \{ y : f(y) = T \} ] \]
   the function which takes an <et> function \( f \), and an entity \( x \), and returns \( T \) iff:
   \( f(x) = T \) and \( x \) is below the average height for entities of which \( f \) is \( T \)

Consider the semantics that these lexical entries predict for NPs like “young president” or “short basketball player”...

(24) Semantics of “Young President” (Informal Demonstration)

a. \[ \begin{array}{c}
        \text{NP}_1 \\
        \text{AP} \\
        \text{NP}_2 \\
        \lambda \\
        \text{young} \\
        \text{president}
    \end{array} = \quad \text{(by FA, NN)} \]

b. \[ [[\text{young}]] ( [[\text{president}]] ) = \quad \text{(by TN)} \]

c. \[ \lambda f_{<et>} : [\lambda x_e : f(x) = T \text{ and } x \text{ is below the average age for the entities in } \{ y : f(y) = T \} ] \]
   \[ [[[\text{president}}]] = \quad \text{(by LC)} \]

d. \[ \lambda x_e : [[\text{president}}])(x) = T \text{ and } x \text{ is below the average age for the entities in } \{ y : [[\text{president}}](y) = T \} ] = \quad \text{(by TN)} \]

e. \[ \lambda x_e : [\lambda y_e : y \text{ is a president }] (x) = T \text{ and } \]
   \[ x \text{ is below the average age for the entities in } \{ y : [\lambda y_e : y \text{ is a president }] (y) = T \} ] = \quad \text{(by LC)} \]

f. \[ \lambda x_e : x \text{ is a president and } x \text{ is below average age for the entities in } \{ y : y \text{ is a president } \} \]
(25) **Preliminary Result, Part 1**

\[
\text{[[ young president ]] } = \[
\lambda x. \text{is a president and } x \text{ is below average age for the entities in } \{ y : y \text{ is a president } \} \]
\]

the function which takes an entity \( x \), and returns \( T \) iff

\( x \text{ is a president, and } x \text{ is below average age of a president} \)

(26) **Preliminary Result, Part 2**

Via a similar proof to that in (24), we can derive the equation below:

\[
\text{[[ short basketball player ]] } = \[
\lambda x. \text{is a basketball player and } \]
\]

\[
\lambda x. \text{is a basketball player and } x \text{ is below average height for the entities in } \{ y : y \text{ is a basketball player } \} \]
\]

the function which takes an entity \( x \), and returns \( T \) iff

\( x \text{ is a basketball player, and } x \text{ is below the average height for a basketball player} \)

(27) **Key Consequence**

It’s easy to see that (25) and (26) together entail that our semantic system is able to derive the following T-conditional statements.  

a. “Barack is a young president” is \( T \) iff Barack is president, and Barack is below average age for the entities in \( \{ y : y \text{ is a president} \} \)

b. “Iverson is a short basketball player” is \( T \) iff Iverson is a basketball player, and Iverson is below average height for the entities in \( \{ y : y \text{ is a basketball player} \} \)

(28) **Some Commentary**

a. Unlike the T-conditional statements in (19a) and (19b), the ones in (27a) and (27b) seem to be accurate.

b. *Moreover, the accuracy of (27a) and (27b) show why (19a,b) are incorrect:*

   • Just because Barack is below the average age for a president, it doesn’t follow that Barack is *young* (below the average age for people in general).

   • Just because Iverson is below average height for a b.ball player, it doesn’t follow that he is *short* (below average age for people in general)

---

\(^2\) The reader is encouraged to sketch the proof out for themselves, to confirm that what I say in (27) is accurate.
So, it looks like our proposal in (23) nicely incorporates subsective adjectives into our system…

Wait, what’s that?...

(29) **PROBLEM**

- According to (23), subsective adjectives like *young/short* are of type `<et,et>`…

- But, if this is the case, how is our system supposed to interpret structures like the following, where these subsective adjectives occupy predicate position?...

```
S
  NP
  Barack
  VP
  V `<e,t>` `<e,t>`
  is
  AP `<e,t>` `<e,t>`
  young
```

TYPE MISMATCH: VP is not interpretable!

Several solutions to the problem in (29) are discussed by Heim & Kratzer (1998: 70-73). For our purposes here, we will consider the most basic of the solutions they mention…

(30) **Solution 1: Systematic Ambiguity**

- Let’s suppose that so-called ‘subsective adjectives’ are systematically ambiguous between an ‘absolute’ reading and a ‘relative’ reading.

- This idea is illustrated below for the subsective adjective ‘*young*’.

```
a. [[ young<sub>ABS</sub> ]] = [ λx e : x is young ]
b. [[ young<sub>REL</sub> ]] = [ λf, eq : [ λx e : f(x) = T and x is below the average age for the entities in { y : f(y) = T } ] ]
```

(31) **Key Consequence**

- If we make the assumption in (30), then sentences like (31a) are interpretable, just as long as we assume that the adjective is receiving its ‘absolutive reading’.

- Moreover, such sentences are (accurately) predicted to have the T-conditions in (31b)

```
a. Barack is young<sub>ABS</sub>
b. “Barack is young<sub>ABS</sub>” is T iff Barack is young
```
(32) Further Prediction

- If subsective adjectives like *young/tall/etc.* really do have ‘absolute readings’ as in (30a), then we predict that structures like (32a) should be possible, where the ‘absolute’ version of the adjective is modifying a noun.

```
a. S
   | NP          VP
   |   | N         V       NP
   |   | Barack     is     D
   |   |   a        AP <et> NP <et>
   |   |     A      N

young<sub>ABS</sub> president
```

- As illustrated above, it follows from (30) that a subsective adjective under its ‘absolute reading’ will be of type <et>.

- Therefore, in order to interpret an NP modified by a subsective adjective under its ‘absolute reading’, we need to employ the rule of Predicate Modification.

- When we do, we compute that structures like (32a) should have the following truth-conditions:\(^3\)

```
b. \([[[B\text{arack is a young}<sub>ABS</sub> president]] = T iff B. is young and B. is a president.\]
```

- **PREDICTION:**
  Sentences like (19a) (repeated below) have a reading where the following T-conditional statement *does* hold:

```
c. “B\text{arack is a young president}” is T iff B. is young and B. is a president.
```

**But does this prediction indeed hold?...**

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**Key Observation:** The reading in (32c) can be *false* even if Barack is young *for a president.* (see (19))

\(^3\) The reader is encouraged to perform the computation themselves, to confirm that what I say here is accurate.
(33) **Test of the Prediction in (32)**

Does the following sentence make any sense at all? Does it have an interpretation where it is not a flat-out contradiction:

*Barack is not a young president, but he is young for a president.*

a. **If it does:**
   - Then it follows that the sentence “Barack is a young president” can be false, *even when Barack is young for a president.*
   - This would entail that the sentence “Barack is a young president” has a reading where it means something *other* than *Barack is young for a president.*
   - Thus, this would support our prediction in (32)

b. **If it doesn’t:**
   - Then it follows that the sentence “Barack is a young president” *has to be true* when Barack is young for a president.
   - This would support the notion that the sentence “Barack is a young president” has only the ‘relative’ T-conditions in (27)

... so what are the facts?...

(34) **Summary: The General Picture that Emerges about Adjectival Modification**

There are (at least) two different types of adjectives in natural language:

a. **Intersective Adjectives** *[male, married, dead]*
   - These adjectives are uniformly predicative (of type <e,t>)
   - In modification structures, they are interpreted via a special rule (PM)

b. **Subsective Adjectives** *[young, short, famous]*
   - These adjectives are uniformly modificational (of type <<e,t><e,t>>)
   - When they are the main predicate of a sentence, ‘something special’ happens (maybe they have a null argument, maybe they receive a special <et> reading)