Trajectory Reconstruction for Travel Time Estimation

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Abstract

In this paper, we proposed a Trajectory Reconstruction Model as an improvement to existing speed-based travel time estimation models. The proposed model utilizes point-based speed data collected by existing Intelligent Transportation Systems (ITS). Using the smoothing scheme proposed in this paper, it is possible to construct a speed surface as a function of space and time. Then, one can reconstruct the trajectory of an imaginary vehicle by allowing it to adopt the local speed determined by the speed surface wherever the vehicle travels. Therefore, the travel time of this vehicle can be readily determined from its trajectory. This paper developed an analytical formulation of the model. Meanwhile, a discrete version of the formulation is also provided as a computational algorithm to facilitate real world implementation.

In comparison with existing models, the proposed model accounts for continuous speed variation in both time and space. This ensures that the model preserves vehicle trajectories and provides sound estimates of travel time. Empirical studies were conducted based on comparison of the reconstructed travel time (estimated by the proposed model) against the Ground Truth travel time and the Instantaneous and Linear Model travel time. The empirical results showed that (1) the reconstructed travel time agrees well with the Ground Truth travel time; (2) the reconstructed travel time is more smooth than the Ground Truth, Instantaneous and Linear Model travel times; (3) the Instantaneous and Linear Model travel time does not exhibit much difference from the other two when traffic condition is good (e.g. low travel time for the same stretch of road); the difference is noticeable when traffic condition deteriorates; the difference reaches its peak under severe congestion. Quantitatively, the reconstructed travel time is not statistically different from the Ground Truth travel time and the corresponding mean absolute percentage error (MAPE) is 6.3 percent. In contrast, the Linear and Instantaneous travel time is statistically different from the Ground Truth travel time and the corresponding MAPE is 11.7 and 14.0 percent respectively.

Keywords: Intelligent Transportation Systems (ITS), travel time estimation, trajectory reconstruction
For the general traveling public, travel time is perhaps the most pertinent piece of information, especially in relation to personal experiences of traffic congestion. As a matter of fact, (TTI 2005) uses travel time index (the ratio of peak travel time to free travel time) as an indicator of individual traveler congestion. Unlike other traffic flow characteristics such as flow, speed, and occupancy, travel time is typically difficult to measure without special equipment such as Global Positioning System (GPS) enabled probe vehicles or license plate / toll tag recognition systems. Alternatively, travel time has to be estimated from other, perhaps easier to measure, traffic data such as spot speeds.

With the widespread of Intelligent Transportation Systems (ITS), especially automated traffic surveillance systems, a wealth of traffic data (such as flow, speed, and occupancy) has been collected and archived. This makes it possible to extract travel time and other useful traveler information from the ITS data on a regular basis. To serve such a purpose, many computational models have been developed. A number of metrics can be used to characterize these models. For example, some models predict travel time (Lin et al. 2004; Schrader et al. 2004; Wouters et al. 2005; Lam et al. 2005), some estimate travel time (Chen et al. 2003; Hobeika and Dhulipala 2004), some work on-line (Yang et al. 2004; Chen et al. 2005; Jeong and Rilett 2005; Miska et al. 2005), some work off-line (Smith and Demetsky 1994; Park and Rilett 1998), some compute link travel time (Xie et al. 2004; Yang et al. 2004), some determine route travel time (Chakroborty and Kikuchi 2004; Chen et al. 2005), some predict/estimate travel time from variables (such as speed) that directly effect travel times (Miska et al. 2005), and some predict/estimate travel times from variables (such as historical travel time) that correlate to travel time (Schrader et al. 2004; Wouters et al. 2005; Xie YC et al. 2006).

These computational models can be roughly summarized in the framework shown in Figure 1. The framework involves two types of data: (a) point-based data including ITS data collected by point sensors such as loop detectors and video cameras and (b) route-based data including mainly route travel times collected by probe vehicles or license plate / toll tag recognition systems. In relation to the timeline in the figure, our focus is travel time at the current moment and in the future. Traffic speed collected at a fixed location can be used to estimate the
travel time on the containing link and the travel time of a route is the sum of link travel times. This corresponds to the two “estimation” arrows. On the other hand, future traffic speed can be predicted based on current and historic speed data and the same prediction technique applies to travel time. This corresponds to the two “prediction” arrows. The diagonal arrow means that, starting from current speed data, one can obtain future travel time in two ways as indicated by the arrowed sides of the top-left and bottom-right triangles.

![Figure 1 A framework of travel time computation](image)

In this paper, our major focus is to estimate travel time based on speed data up to the current moment, as shaded in Figure 1. Though this topic has received a considerable amount of attention in the past and many models have been successfully proposed, we aim to explore further improvement potential by considering speed variation over both time and space. More specifically, in this paper, we propose a Trajectory Reconstruction Model to estimate travel time. This model merits particular attention because (a) it utilizes the data collected by existing ITS without requiring additional data collection efforts, (b) it adds much realism to the estimated travel time by considering the variation of speed over both time and space, (c) empirical validation shows good agreement between the outcome of the proposed model and the true measurement, (d) it exhibits superior performance to models of similar nature.

To simplify notation, we refer the travel time measured from true vehicle trajectories to the “Ground Truth” travel time, the travel time estimated from the Trajectory Reconstruction Model as the “Reconstructed” travel time, the travel time estimated from Linear Model as the “Linear” travel time, and the travel time estimated from the
Instantaneous Model as the “Instantaneous” travel time. In the next section, we present existing work that is most relevant to the proposed model. This is followed by the formulation of the proposed model. Next, empirical validation is conducted by comparing the estimates of the proposed model against the Ground Truth. It is also interesting to highlight the difference between the proposed model and existing work. Finally, the paper is concluded with a summary of our findings.

RELEVANT LITERATURE

The proposed method estimates travel time using traffic speed data collected at a set of fixed locations along a highway, e.g. ITS data. To facilitate subsequent discussion and cross-comparison, we first present the context within which relevant literature is reviewed.

Perhaps the best way to present the context is by means of a figure. In Figure 2, we show a segment of a highway, i.e., the route abcdef, which is monitored by traffic sensors, as is the case in most ITS. These sensors are capable of collecting traffic data such as speed at the sensor locations over time. Traffic speed $v$ can be represented as a function of space $x$ and time $t$, i.e. $v = v(t, x)$. Thus a speed surface can be constructed as shaded in Figure 2. Suppose a vehicle traverses the route which consists of a few links as delineated by the sensors, the trajectory of the vehicle can be traced and this is indicated as the curve ABCDEF in Figure 2. The projection of the vehicle trajectory onto the speed surface is curve A$_1$B$_1$C$_1$D$_1$E$_1$F$_1$ and the height of the curve dictates the speed that the vehicle adopts as it proceeds on its trajectory. In the discussion thereafter, we call the curve/line that indicates the speed of a vehicle the “speed trajectory”, e.g., curve A$_1$B$_1$C$_1$D$_1$E$_1$F$_1$. 
A few relevant models are identified which estimate travel time based on traffic speed observed at a fixed set of sensor locations (Li et al. 2006). Probably one of the most widely known models is the instantaneous travel time estimation model (abbreviated as the “Instantaneous Model” thereafter). This model estimates the total travel time on a route as the summation of link travel times along the route as measured at the same instant, i.e.

\[
T_i(j) = \sum_{x_i}^{x_f} \frac{2(x_{i+1} - x_i)}{v(x_i) + v(x_{i+1}, t_j)}
\]

Total travel time with trip starting time \(t_j\) : \(TT(j) = \sum_i T(i, j)\)
Where \(x_i\) \((i = 0, 1, 2 \ldots)\) are nodes/sensor locations on the route; \(t_j\) \((j = 0, 1, 2 \ldots)\) is the time when the trip starts; \(v(x_i, t_j)\) and \(v(x_{i+1}, t_j)\) are the traffic speeds measured at the upstream and downstream ends of the link at time \(t_j\). In relation to Figure 2, the vehicle trajectory reduces to line \(AB'C'D'E'F'\) under the Instantaneous Model and the corresponding speed trajectory consists of lines \(A_2B_2' + B_3'C_2' + C_3'D_2' + D_3'E_2' + E_3'F_2'\). Noticeably, this model achieves simplicity and computational efficiency by considering only speed variation in discrete space and ignoring speed variation in time.

(Li et al. 2006) also mentioned a Time Slice Model without providing its reference. This model improves the Instantaneous Model by incorporating speed variation in discrete time. In this model, the travel time on the first link, \(T(x_1, t_1)\), is determined in the same way as the Instantaneous Model. When the vehicle arrives at the second link, the model adopts the speed corresponding to link \(x_2\) and the time when the vehicle finishes the previous link, i.e.

\[
T(2, j) = \frac{2(x_3 - x_2)}{[v(x_2, t_j + T(1, j)) + v(x_3, t_j + T(1, j))]} \tag{1}
\]

Similar calculations repeat for the links there after and the total travel time is, again, the summation of all link travel times. In relation to Figure 2, the vehicle trajectory reduces to lines \(AB' + BC'' + CD'' + DE'' + EF''\) under this model and the corresponding speed trajectory consists of lines \(A_2B_2' + B_3'C_2'' + C_3'D_2'' + D_3'E_2'' + E_3'F_2''\). Notice that this model achieves a limited improvement by considering speed variation in discrete time and discrete space.

As an enhancement to the Time Slice Model, (Cortes et al. 2002) proposed a Dynamic Time Slice Model which updates speed by approximating the true vehicle trajectory. Based on a recursive formulation, this model determines some key points along the vehicle trajectory and calculates link travel times based on speeds corresponding to these key points, i.e.

\[
T(i, j) = \frac{2(x_{i+1} - x_i)}{[v(x_i, t_j)) + v(x_{i+1}, t_j + T(i, j))]} \tag{2}
\]
A similar model was reported by (Chen et al. 2003) where the recursion was saved by replacing the average speed in the above equation with the speed observed at the nearest space-time/grid point. In relation to Figure 2, the vehicle trajectory is the true trajectory, i.e. curve ABCDEF, under this model but the corresponding speed trajectory reduces to lines A3B3 + B4C3 + C4D4 + D3E4 + E3F2.

In the above models, the speed may vary between links and speed variation is discrete in time, in space, or in both. However, within each link, the speed is constant and typically takes the average of the speeds at both ends of the link. (van Lint and van der Zijpp 2003) proposed a Linear Model which allowed smooth speed variation in space. The model determines the speed at location \( x_i \) (\( x_i \leq x \leq x_{i+1} \)) as a linear interpolation of the speeds at both ends of the containing link, i.e.

\[
v(x_i, t_j) = v(x_i, t_j) + \frac{x - x_i}{x_{i+1} - x_i} \left( v(x_{i+1}, t_j) - v(x_i, t_j) \right)
\]

Travel time can be determined using a finite difference technique, i.e. \( T(j) = \sum \Delta x / v(x, t) \). Note that this model considers continuous speed variation in space but ignores speed variation in time. In relation to Figure 2, the vehicle trajectory reduces to line AB’C’D’E’F’ under this model and its speed trajectory is curve A1B1’C1’D1’E1’F1’.

All the above models deal with a speed surface/field, so they are characterized as speed-based models. It is worth noting that (Kwon and Petty 2005) proposed a non-speed-based model. In this model, rather than constructing a speed surface/field, the authors worked on a travel time field to estimate/predict link travel time. The advantages of this model are its scalability to account for flexible routes and its capability to predict future travel times, though from a practical standpoint constructing a travel time field might not be as easy and straightforward as constructing a speed surface.

As a summary, Table 1 categorizes the above models based on their characteristics in speed variation. For example, the Instantaneous Model accounts for discrete speed variation in space and the speed is invariant in time; the Dynamic Time Slice Model involves discrete speed variation in both space and time. It is interesting to note that the bottom row of the table suggests a potential model which is characterized by continuous speed variation in both
In relation to Figure 2, this potential model corresponds to the true vehicle trajectory, i.e. curve ABCDEF, and the true speed trajectory, i.e. curve A1B1C1D1E1F1. Unlike the models reviewed above, both trajectories in this potential model would be in their original forms without distortion. Thus this potential model appears particularly promising because it is more theoretically sound than the models reviewed above.

Table 1 Categorization of speed-based travel time estimation models

<table>
<thead>
<tr>
<th>Speed Variation</th>
<th>In space</th>
<th>In time</th>
<th>Discrete</th>
<th>Continuous</th>
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<tbody>
<tr>
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<tr>
<td>Time Slice Model</td>
<td>x</td>
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<tr>
<td>Dyn Time Slice Model</td>
<td>x</td>
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<td>x</td>
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<tr>
<td>Linear Model</td>
<td>x</td>
<td></td>
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<td>x</td>
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<tr>
<td>Trajectory Reconstruction</td>
<td>x</td>
<td>x</td>
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<td>x</td>
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</tbody>
</table>

TRAVEL TIME ESTIMATION BASED ON TRAJECTORY RECONSTRUCTION

In this section, we explore the above-mentioned potential to estimate travel time by working with continuous speed variation in both space and time. We assume that the speed surface, i.e. $v = v(t, x)$, is provided up to the current moment as illustrated in Figure 2. Our objective is to estimate the travel time of an imaginary vehicle which started its trip (e.g. crossed sensor a) at some early time and finished its trip (e.g. crossed sensor f) before the current moment. Note that, though we use Figure 2 as our illustrative example here, the formulation itself should stand out as generic. With the aid of the speed surface, it is possible to reconstruct the trajectory of the imaginary vehicle and its travel time can be read directly from its trajectory. The proposed Trajectory Reconstruction Model is formulated as follows.

Consider a point $(t, x)$ on the trajectory of the imaginary vehicle. If one runs a vertical line through this point and perpendicular to the space-time $(x-t)$ domain, the intersection of this line and the speed surface (i.e. the height of the line) is the instantaneous speed of the vehicle at this point, i.e. $v(t, x)$. Since space $x$ and time $t$ are
correlated for any point on the trajectory, we can conveniently express space as a function of time, i.e. \( x = x(t) \) or vice versa. In addition, the following relationship also holds for such a point:

\[
\frac{dx(t)}{dt} = v(t, x(t)) \tag{4}
\]

Boundary conditions are \( x(t_a) = x_a \) and \( x(t_f) = x_f \), see Figure 2. To find its solution, the above equation is converted to:

\[
dx(t) = v(t, x(t))dt \tag{5}
\]

Integrate both sides with respect to \( x(t) \) and \( t \):

\[
\int_{x=x_d}^{x=x_f} dx(t) = \int_{t=t_d}^{t=t_f} v(t, x(t))dt \tag{6}
\]

This equation is not easy to solve because the left hand side involves \( t \) and the right hand side contains \( t_F \) which is the unknown variable we are trying to determine. Considering that a valid trajectory should be a non-decreasing function of both time \( t \) and space \( x \), the inverse function \( t(x) \) exists and it is also non-decreasing. Thus we can find out \( t(x) \) by solving the following differential equation:

\[
\frac{dt(x)}{dx} = \frac{1}{v(t(x), x)} \tag{7}
\]

With boundary conditions \( t(x_a) = t_a \) and \( t(x_f) = t_f \). The unknown variable is \( t_f \) which is related to the travel time \( (t_b - t_a) \) of our interest. The travel time can be determined by integration:

\[
t_f - t_a = \int_{t=t_a}^{t=t_f} dt = \int_{x=x_d}^{x=x_f} \frac{1}{v(t(x), x)}dx \tag{8}
\]

The above equation serves as the analytical formulation of the proposed Trajectory Reconstruction Model. However, the equation is still difficult to solve due to the interdependent relationship between space \( x \) and time \( t(x) \) in the speed \( v(x, t(x)) \) term. However, it is possible to approximate the solution to this integration equation using a finite difference method.
As a two-dimensional version of Figure 2, Figure 3 shows only part of the space-time domain. Points \((t_i, x_j)\) \((i=0,1,2,...; j=0,1,2,...)\) represent grid points where mean traffic speed \(v(t_i, x_j)\) is observed under a given sensor configuration and aggregation interval. Equation 8 can be converted to the following summation form:

\[
t'_F - t'_L = \int_{x=x_a}^{x=x_f} \frac{1}{v(t(x), x)} d\Delta x = \sum_{k=1}^{n} \frac{1}{\bar{v}(t(x_k), x_k)} \Delta x
\]

where \(k = 1, 2, ..., n\). \(x_1 = x_A\), \(x_n = x_F\), and \(\Delta x = \frac{x_F - x_A}{n - 1}\). Now the problem is to determine the speed \(\bar{v}(t(x_k), x_k)\) as an average of the speeds at the following four points \((t(x_k), x_k), (t(x_{k+1}), x_k), (t(x_{k+1}), x_{k+1}), (t(x_k), x_{k+1})\), and \((t(x_k), x_{k+1})\), see Figure 3. However, these points may not coincide with the grid points where their speeds are known. Alternatively, \(\bar{v}(t(x_k), x_k)\) has to be calculated from the nearest grid points with known speeds such as \((t_i, x_j), (t_{i+1}, x_j), (t_i, x_{j+1}), (t_{i+1}, x_{j+1})\).

Suppose the imaginary vehicle is currently at point \((t, x)\) moving toward the next point \((t', x')\). The task now is to determine the instantaneous vehicle speed using the mean traffic speeds observed as the grid points. As some prior attempts, (Chen et al. 2003) proposed the use of the known traffic speed at one of the four grid points \((t_i, x_j), (t_{i+1}, x_j), (t_i, x_{j+1}), (t_{i+1}, x_{j+1})\) as a surrogate to the instantaneous vehicle speed; (van Lint and van der Zijpp 2003) and (Rice and Van Zwet 2004) proposed the use of the average traffic speed at points \((t_i, x_j)\) and \((t_i, x_{j+1})\). The concern here is that, by taking one of the sample speeds or by averaging two of the sample speeds along space domain, one inevitably encounters abrupt speed change when the vehicle moves from one link to the next. Sometimes such a speed change may be considerable and such a method becomes unrealistic. On the other hand, real world sensors are not perfect and sensor errors are the reality with which ITS have to overcome. It is likely that the above models accidentally pick an erroneous observation as the surrogate which introduces estimation error. In order to account for speed variation in both space and time as well as to reduce the impact of sensor errors, it is worth trying to smooth the instantaneous vehicle speed by using traffic speeds observed at the four grid points:

\[
v(t, x) = \frac{D_{i,j} v(t_i, x_j) + D_{i,j+1} v(t_i, x_{j+1}) + D_{i+1,j} v(t_{i+1}, x_j) + D_{i+1,j+1} v(t_{i+1}, x_{j+1})}{D_{i,j} + D_{i,j+1} + D_{i+1,j} + D_{i+1,j+1}}
\]
Where $D$ denotes area, i.e. $D_{i,j} = \|t - t_{i,j}\| \times \|x - x_{j} \|$, $D_{i,j+1} = \|t - t_{i,j+1}\| \times \|x - x_{j+1} \|$, $D_{i+1,j} = \|t - t_{i+1,j}\| \times \|x - x_{j} \|$, and $D_{i+1,j+1} = \|t - t_{i+1,j+1}\| \times \|x - x_{j+1} \|$. This smoothing scheme is essentially a two-dimensional interpolation in which the instantaneous vehicle speed is the weighted average of the traffic speeds observed at the four grid points.

With the instantaneous vehicle speed and the finite distance $\Delta x = x' - x$, one can determine the finite time, $\Delta t = t' - t$, that the imaginary vehicle takes to move from point $(t, x)$ to point $(t', x')$. Now that $t'$ and $x'$ become known, the instantaneous vehicle speed at the new location, $v(t', x')$, can be updated using the smoothing scheme. Repeat the above procedure, the time that the vehicle needs to travel to the next point $(t'', x'')$ can be determined. Continue the above process till the vehicle arrives at point $(t_F, x_F)$. Therefore, the total travel time of this imaginary vehicle is $T_T = t_F - t_A$.

In the above process, we actually constructed a speed surface using the known traffic speeds observed at grid points and applying the smoothing scheme. As a consequence, one can read from this speed surface the instantaneous speed corresponding to any point in the space-time domain. Based on the instantaneous speed, one can easily reconstruct the trajectory of an imaginary vehicle given its boundary conditions, e.g. its entry point.
To facilitate real world implementation of the Trajectory Reconstruction Model in Intelligent Transportation Systems (ITS), we developed a computational algorithm which is listed below. Again, the algorithm uses Figure 2 as an illustrative example, but its generality is obvious.

1. Select a space increment, $\Delta x$, and a time increment, $\Delta t$, of sufficiently small.

2. If an imaginary vehicle starts its trip at point $A(t_A, x_A)$, the trajectory of the vehicle can be reconstructed by repeating the following steps:

3. Suppose the vehicle is currently at point $(t, x)$, determine the instantaneous speed of the vehicle at this point, $v(t, x)$, using the above smoothing scheme.

4. If $v(t, x) = 0$, advance time by a time increment, i.e., $t' = t + \Delta t$. update the current point to be $x' = x$ and $t' = t + \Delta t$. Go to Step 6.

5. Otherwise, compute the time needed to advance the vehicle by a space increment: $\Delta t' = \Delta x / v(t, x)$. Update the current point to be $x' = x + \Delta x$ and $t' = t + \Delta t'$. Note that, if $v(t, x)$ is extremely small, one may want to apply a smaller $\Delta x$ to get out of such a condition more quickly.

6. Check if $x' \geq x_F$. If false, go back to step 3.

7. Otherwise, interpolate to determine the exact time when the vehicle crosses $x_F$:

$$T_F = t + (t' - t) \times (x_F - x) / (x' - x)$$
8. The travel time of this imaginary vehicle is \( t_F - t_A \), during which the vehicle travels from point A to point F.

Note that a “divide by zero” error may occur at Step 5 when the instantaneous vehicle speed becomes zero, e.g. the observed traffic speeds at the four surrounding grid points are zero due to sensor error. One way to get out of the problem is to advance the time by a time increment and this gives rise to Step 4. The algorithm continues with the updated space-time point. It may take several iterations to move the vehicle out of the cell confined by the four zero-speed grid points. It is interesting to note that the zero speed problems apply not only to an algorithm driven by space increment but also that driven by time increment. A remedy is to advance the vehicle in the other direction (i.e. space or time) when the increment in one direction does not work.

The above formulation and algorithm implies a few appealing properties of the Trajectory Reconstruction Model. First, it is recognized that measuring travel time from vehicle trajectories is the ideal approach. However, when such measurement becomes impractical, estimating travel time from reconstructed vehicle trajectories, such as the proposed model, appears to be a natural choice. Second, the model accounts for continuous speed variation in both time and space. The computational algorithm implementing the model ensures that the reconstructed vehicle trajectory approximates the true trajectory. Third, the smoothing scheme of the model helps avoid abrupt vehicle speed change. In addition, the smoothing scheme greatly reduces the negative impact of zero speeds and observation errors. Moreover, the smoothing scheme can also effectively reduce the impact of high variation in raw data which is typical in ITS. Fourth, this model uses the space increment to drive the computation procedure for easy detection of the end of a trip. Meanwhile, this model is able to deal with the zero speed problems by alternating increment directions.

**EMPIRICAL RESULTS**

Empirical validation of the proposed model imposes a great challenge. The difficulty arises mainly because datasets containing point-based data (e.g. speed, flow, and occupancy) and route-based data (e.g. travel times) simultaneously are very rare. Without point-based data, we would be unable to construct the speed surface and reconstruct vehicle trajectories. Without route-based data, we would not know the Ground Truth travel times against which the estimated travel times are compared. Fortunately, the datasets collected by the Federal Highway
Administration’s (FHWA) Next Generation Simulation Program (NGSIM, http://www.ngsim.fhwa.dot.gov/) make the validation possible. The NGSIM datasets consist of detailed trajectories of all vehicles observed on a few highway segments within several time periods. With the vehicle trajectory data, we are able to generate point-based speed data using imaginary sensors. We are also able to measure Ground Truth travel times directly from vehicle trajectories.

Two sets of the NGSIM data were used in this validation. Dataset 1 was collected on a segment of Interstate 80 in Emeryville (San Francisco), California on Wednesday, April 13th, 2005 (Site 1), and Dataset 2 was on a segment of U.S. Highway 101 (Hollywood Freeway) in Los Angles, California on Wednesday, June 15th, 2005 (Site 2). Figure 4 provides a schematic illustration of both sites. Site 1 is approximately 1,650 feet (503 m) long, with an on-ramp at Powell Street and a downstream off-ramp at Ashby Avenue. Three separate 15-minute periods of data are available: 1) 4:00-4:15 p.m., 2) 5:00-5:15 p.m., and 3) 5:15-5:30 p.m. Complete vehicle trajectories were transcribed at a resolution of 10 frames per second. Site 2 is approximately 2,100 feet (640 m) long including a merge/weave section with an on-ramp at Ventura Boulevard and a downstream off-ramp at Cahuenga Boulevard connected by an auxiliary lane. Three separate 15-minute periods of data representing transitional and congested flow conditions are available: 1) 7:50-8:05 a.m., 2) 8:05-8:20 a.m., and 3) 8:20-8:35 a.m.

Figure 4 Sketches of NGSIM data collection sites (left-Site 1; right-Site 2)
To give a general taste of the NGSIM datasets, a speed surface was generated from Dataset 1, see Figure 5. The X axis is time of day, the Y axis is distance from the beginning of the site, and the Z axis is speed. The speed surface was generated using point-based speed data observed by imaginary sensors using traffic flow characteristics definition based on (Edie 1963), so the speed here is space mean speed. The spacing of the imaginary sensors was 100 ft (30.48m) and the aggregation interval of observations was 30 seconds. The speed surface exhibits high variation due to the fine granularity of data aggregation, in return this provides more details about speed variation in time and space domains.

![Figure 5 The speed surface generated from NGSIM Dataset 1 (I-80, 04/13/2005 5:15-5:30pm)](image)

The empirical validation focuses on comparing travel times estimated using the Trajectory Reconstruction Model against the Ground Truth travel times measured from true vehicle trajectories during the same trip starting time intervals. Meanwhile, it is also interesting to examine how the Trajectory Reconstruction Model, the Instantaneous Model and Linear Model perform differently. The reasons to choose the Instantaneous Model and
Linear Model are mainly because Instantaneous Model is the most extensively implemented model and Linear Model improved Instantaneous Model by smoothing speed variation using linear interpolation. Due to the well-known “drawbacks” of Instantaneous Model, the proposed Reconstructed Model shows its advantages by comparing with Linear and Instantaneous Model. In addition, their relative merits to other speed-based models such as the Time Slice Model and Dynamic Time Slice Model has been established in (Li et al. 2006).

Figure 6 shows the Ground Truth, Reconstructed, Linear and Instantaneous travel times in the same figure. The four curves were generated from Dataset 1 from 4:00pm to 4:15pm. Data in this period primarily represented transitional traffic conditions during the build-up to congestion, as can be seen from the figure that vehicles generally entailed increasing travel time over time. Also salient in the figure is that the Ground Truth, Linear and Instantaneous travel times exhibit much more variation than does the Reconstructed travel time. This confirms the ability of the Trajectory Reconstruction Model to deal with high variation in field data. Moreover, the astute reader may notice that the Linear travel time is a smoothed version of Instantaneous travel time and Reconstructed travel time represents a smoothed version of the Ground Truth travel time. In contrast, the Instantaneous and Linear travel time deviate from the Ground Truth travel time with varying degrees, but more deviation is observed toward the end of the curves when congestion began to build up. This result suggests that, except for variation, the difference among the four curves is not significant under normal traffic conditions as indicated in the left half of the figure, but the Instantaneous and Linear travel time differ noticeably from the remaining two when traffic condition deteriorates as in the right half of the figure.
To examine more closely how the three models defer from the Ground Truth, Figure 7 shows the residuals after the Ground Truth travel time was subtracted from the travel times estimated from the three models. The figure confirms the above observations, particularly regarding how the models perform under different traffic conditions.
To verify the results, the above comparison was repeated for other data available in NGSIM datasets. Figure 8 shows 6 figures (Figure 6 is repeated here for completeness) in two columns. The figures in the left column are comparison results from Site 1 (I-80) and the figures in the right column are from Site 2 (US101). These figures confirm our findings, i.e., (1) the Reconstructed travel time fits well the Ground Truth travel time; (2) the Reconstructed travel time is more smooth than the Ground Truth, Linear and Instantaneous travel times; (3) the Linear and Instantaneous travel time does not exhibit much difference from the Ground Truth when traffic condition is normal (e.g. low travel time for the same stretch of road); the difference is noticeable when traffic condition deteriorates and reaches its peak under severe congestion, e.g. the dotted spikes in most of the figures.
Figure 8 More comparison based on NGSIM datasets
Considering that the above travel times are time series data, statistical tests were conducted based on the Batch Means technique (Ni et al. 2004) to account for autocorrelation. The tests based on overall travel times showed the following results. If we compare the Reconstructed travel time with the Ground Truth travel time, the difference between the two will be zero approximately 95 out of every 100 trials, i.e., the mean of their residuals is not statistically different than 0. However, this result does not hold when comparing the Linear and Instantaneous travel time with Ground Truth travel time. In addition, mean absolute percentage errors (MAPEs) (Ni et al. 2004) are computed using the Ground Truth travel time as a benchmark. The MAPE of the Instantaneous and Linear travel time is 14.0% and 11.7% respectively and that of the Reconstructed travel time is 6.3%.

In addition to validating the Trajectory Reconstruction Model using NGSIM datasets, it is interesting to further examine the performance of the model using an independent dataset. Since the NGSIM datasets are confined within short stretches of road and short periods of time, datasets covering a larger space-time region may reveal more information about the model performance. The dataset collected by Georgia NaviGAtor, the ITS of Georgia, at GA 400 serves this purpose. Unfortunately, this set of data contains point-based traffic data only, so the Ground Truth travel time is absent. Our emphasis here is to show how the Trajectory Reconstruction Model and the Instantaneous Model perform differently.

Traffic conditions at the site were monitored by video cameras deployed approximately every one third mile of the road in each direction. Each camera is an observation station and watches all lanes at this location. An image processing software program was running in the background to extract traffic data from video images. Traffic conditions were sampled every 20 seconds and each sample contains values of a variety of variables, among which traffic speed is of major interest. The upstream end of the study site is station 4001101 and the downstream end is station 4001139. The study site covers a stretch of road of 20 km long. Excluding stations that do not report data, there were 36 stations in the study site. The mainline starts with 2 lanes and gradually increases to 4 lanes. The study site is schematically shown in Figure 9.
Figure 10 graphically shows the difference between the two models based on data collected on Friday, September 20, 2002. The morning peak was significant and lasted roughly from 6:30:00 to 9:30:00 and the afternoon peak mainly affected both ends of the site and spanned roughly from 16:00:00 to 18:00:00. During most of the off-peak hours, there is only slight difference between the two models, but during peak periods the difference is remarkable, especially in the morning peak. In addition, the Instantaneous travel time lags the Reconstructed travel time. This is probably because of the Instantaneous Model’s inability to stay current with speed in time and space.
To verify the results, the above comparison was repeated for two weeks. 10 weekdays were randomly drawn from the available database from September 2002 to January 2003. Knowing that the major difference between the two models is primarily in peak periods, we sampled travel times during 6:00:00-10:00:00AM and each sample contained a value of the Instantaneous travel time and a value of the Reconstructed travel time. A total of 2400 samples were resulted, among which 68 (2.83%) samples contained infinite values of Instantaneous travel time which were due primarily to extremely low speeds typically observed during severe congestion with stop-and-go traffic. These infinite values were then removed from consideration. Figure 11 plots the Instantaneous travel time against the Reconstructed travel time. If the two times match perfectly, all the dots should align on the 45 degree line. Since most dots appear above the line, it appears that the Instantaneous Model tends to overestimate travel time and exhibits large dispersion. This result is consistent with our findings in NGSIM data, especially during
congestion. The means of instantaneous and Reconstructed travel times are 1672.94 and 1390.13 seconds respectively (this translates to a difference of 4.7 minutes on a route of 20 km) and their standard deviations are 964.84 and 584.57 seconds respectively. A statistical test based on the Batch Means technique (Ni et al. 2004) showed that travel times of the two methods are statistically different and their mean absolute percent difference (which is calculated in the same way as MAPE, but we refrain from calling it MAPE just to avoid claiming which model is faulty) is 22.2%. Note that the above results have excluded samples with infinite values of Instantaneous travel time which, if included, will make the difference even larger.

Figure 11 Comparison of Instantaneous travel times against Reconstructed travel times
CONCLUSION

In this paper, we proposed the Trajectory Reconstruction Model as an improvement to existing speed-based travel time estimation models. The proposed model utilizes point-based speed data collected by existing ITS. Using the smoothing scheme proposed in this paper, it is possible to construct a speed surface as a function of space and time. Then, one can reconstruct the trajectory of an imaginary vehicle by allowing it to adopt the local speed determined by the speed surface wherever the vehicle travels. Therefore, the travel time of this vehicle can be readily read from its trajectory. This paper developed an analytical formulation of the model. Meanwhile, a discrete version of the formulation is also provided as a computational algorithm to facilitate real world implementation.

In comparison with existing models, the proposed model accounts for continuous speed variation in both time and space. This ensures that the model preserves vehicle trajectories and provides sound estimates of travel time. In addition, the smoothing scheme of the model helps avoid abrupt vehicle speed change, reduce the impact of erroneous observations, and cope with high variation in raw data. Empirical studies were conducted based on the NGSIM datasets and the GA400 dataset collected in California and Georgia, respectively. The NGSIM datasets consist of detailed vehicle trajectories observed within limited space-time regions. Empirical comparison was made among the Ground Truth travel time and the travel times estimated by the Linear and Instantaneous Model and the Trajectory Reconstruction Model. The comparison showed good agreement between the Ground Truth and the Reconstructed travel times and the latter appeared to be a smoothed version of the former. In contrast, the Linear and Instantaneous travel time deviate from the Ground Truth travel time with varying degrees, with more deviation being observed as traffic condition deteriorates. Statistical analysis suggested that the Reconstructed travel time is not statistically different from the Ground Truth travel time and the corresponding MAPE is 6.3 percent. In contrast, the Linear and Instantaneous travel time is statistically different from the Ground Truth travel time and the MAPE is 11.7 and 14.0 percent respectively. The GA400 dataset consists of point-based traffic flow data over a larger space-time region. Without the Ground Truth, this dataset is mainly used to understand the relative performance between the Instantaneous Model and the Trajectory Reconstruction Model. The comparison showed slight difference between the two during off-peak hours, but their difference during peak hours is remarkable. The mean absolute percentage difference between the two during peak hours can be as large as 22 percent.
It is interesting to note that travel time, as a personal indicator of traffic flow, is viewed differently under different traffic conditions. Under normal traffic conditions, travel time is frequently used as a means to keep schedules. However, under congested conditions, travel time is strongly associated with the perceived level of service. In addition, travelers are typically more concerned with travel time during congestion than during normal states. Therefore, the performance of a travel time estimation model during congestion deserves more weight.

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REFERENCES


