1. (10 points total)

1a. (5 points)

A linear arrangement of three deoxyribonucleic acid (DNA) molecules is called a triplet. A nucleotide may contain any one of four possible bases: adenine (A), cytosine (C), guanine (G), and thymine (T). How many different ordered triplets are possible?

**Answer:** 64

**Solution:**
Ordered sampling with replacement
Number of possible triplets = n^k; n=4, k=3
4^3 = 64

1b. (5 points)

A menu lists three meats, four salads, and two desserts. In how many ways can a meal of one meat, one salad, and one dessert be selected?

**Answer:** 24

**Solution:**
(3)(4)(2) = 24
2. (10 points total)

The prevalence of a condition in a population is 12%. You take a random sample of 15 people from this population. Let X represent the number of individuals in the sample with the condition.

2a. (2 points)
Describe the distribution of X, being sure to define all terms.

Answer:
X is distributed Binomial (n, π) with n=15 and π = 0.12
Thus,
X = 0, 1, 2, …..15 with probabilities defined

\[ \text{Probability}[X=x] = \binom{n}{x} \pi^x (1-\pi)^{n-x} \]
\[ = \binom{15}{x} 0.12^x (0.88)^{15-x} \]
\[ = \frac{15!}{x!(15-x)!} 0.12^x (0.88)^{15-x} \text{ for } x=0,1,\ldots,15 \]

2b. (2 points)
What is the probability of seeing no cases in a sample?

Answer: 0.147

Solution: For “no cases”, use n=15 π=0.12 and x=0. The solution is then

\[ \text{Probability}[X=0] = \binom{15}{0} 0.12^0 (0.88)^{15} \]

\begin{tabular}{|c|c|}
\hline
Method 1. exact binomial calculation & 0.1469738539 \\
\hline
Method 2. approximation via normal & ------- \\
\hline
Method 3. approximation via Poisson & ------- \\
\hline
\end{tabular}
2c. (2 points)
What is the probability of seeing one case?

Answer: 0.3006

Solution: For “no cases”, use \( n=15 \), \( \pi=0.12 \) and \( x=1 \). The solution is then

\[
\text{Probability}[X=1] = \binom{15}{1} \cdot \frac{12^1}{14!} \cdot (0.88)^{14}
\]

2d. (2 points)
What is the probability of seeing one or fewer cases?

Answer: 0.4476

Solution: For “one or fewer”, use \( n=15 \), \( \pi=0.12 \) and \( x \leq 1 \). The solution is then

\[
\text{Probability}[X \leq 1] = \text{Probability } [X=1] + \text{Probability } [X=0]
\]

\[
= \sum_{x=0}^{1} \binom{15}{x} \cdot \frac{12^x}{x!} \cdot (0.88)^{15-x}
\]

\[
= \binom{15}{0} \cdot (0.12^0 \cdot 0.88^{15}) + \binom{15}{1} \cdot (0.12^1 \cdot 0.88^{14})
\]
2e. (2 points)
What is the probability of at least three cases?

Answer: 0.2654

Solution: For “at least 3”, use n=15 π=0.12 and x ≥ 3. The solution is then

\[
\text{Probability}[X \geq 3] = \text{Probability}[X=3] + \text{Probability}[X=4] + \ldots + \text{Probability}[X=15]
\]

\[
= \sum_{x=3}^{15} \frac{15!}{x!(15-x)!} \left( \frac{0.12^x}{0.88^{15-x}} \right)
\]

\[
= \binom{15}{3} \left( \frac{0.12^3}{0.88^{12}} \right) + \binom{15}{4} \left( \frac{0.12^4}{0.88^{11}} \right) + \ldots + \binom{15}{15} \left( \frac{0.12^{15}}{0.88^0} \right)
\]
3. (10 points total)

Seventy-nine firefighters were exposed to burning polyvinyl chloride (PVC) in a warehouse fire in Plainfield, New Jersey on March 20, 1985. A study was conducted in an attempt to determine whether or not there were short- and long-term respiratory effects of the PVC. At the long-term follow-up visit at 22 months after the exposure, 64 firefighters who had been exposed during the fire and 22 firefighters who were not exposed reported on the presence of various respiratory conditions. Eleven of the PVC exposed firefighters had moderate to severe shortness of breath compared to only 1 of the non-exposed firefighters.

What is the probability of finding 11 or more of the 64 exposed firefighters reporting moderate to severe shortness of breath if the rate of moderate to severe shortness of breath is 1 case per 22 persons?

Answer: 0.000136

Solution: This is a binomial distribution calculation with $n = 64$, $\pi = (1/22) = 0.04545$ and $x \geq 11$. The solution is then

$$\text{Probability}[X \geq 11] = \sum_{x=11}^{64} \binom{64}{x} \pi^x (1-\pi)^{64-x}$$

Method 1. exact binomial calculation

Method 2. approximation via normal

Method 3. approximation via Poisson
4. (10 points total)

Suppose it is known that, among persons 17 years of age and older, half the males and one-third of the females are smokers. A random sample of 10 males and 15 females is obtained.

4a. (5 points)

What is the probability that none are smokers?

Answer: .000002

Let X=# smokers in sample of size 10 males. X is distributed Binomial (n, π) with n=10 and π = 0.50

Let Y=# smokers in sample of size 15 females. Y is distributed Binomial (n, π) with n=15 and π = 0.33

X and Y are independent.

Solution:

The event that “none are smokers” means that there are 0 male smokers and 0 female smokers. This occurs with probability given by

\[
\text{Probability}[X=0 \text{ and } Y=0] = \left\{\binom{10}{0} 0.50^0 (1 - .50)^{10}\right\} \times \left\{\binom{15}{0} 0.33^0 (1 - .33)^{15}\right\}
\]

= {.00098} \times {.00229}

| Method 1. exact binomial calculation | 0.0009765625 |
| Method 2. approximation via normal  | 0.001937     |
| Method 3. approximation via Poisson | -------     |

| Method 1. exact binomial calculation | 0.002285371604 |
| Method 2. approximation via normal  | -------     |
| Method 3. approximation via Poisson | -------     |
4b. (5 points)
What is the probability that the sample contains 4 male smokers and 6 female smokers?

Answer: 0.0366

Solution:
Probability[X=4 and Y=6] = \{Pr [X = 4]\} * \{Pr [Y=6]\} by independence

\[
\begin{align*}
&= \binom{10}{4} 0.50^4 (1 - .50)^{10-4} \cdot \binom{15}{6} 0.33^6 (1 - .33)^{15-6} \\
&= \{0.2051\} \cdot \{0.1786\}
\end{align*}
\]

| Method 1. exact binomial calculation | 0.205078125 |
| Method 2. approximation via normal | 0.204051 |
| Method 3. approximation via Poisson | -------- |

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| Method 1. exact binomial calculation | 0.178562418666 |
| Method 2. approximation via normal | -------- |
| Method 3. approximation via Poisson | -------- |
5. (10 points total)  
Consider a random variable Z distributed standard normal ($\mu = 0, \sigma^2 = 1$)

5a. (3 points)  
Find the probability that Z lies between the values $Z = 0$ and $Z = 2.37$

Answer: 0.491

Solution:

5b. (3 points)  
What values of Z correspond to an area under the curve beyond $\pm Z$ equal to .01?

Answer: $-2.576$ and $+2.576$

Solution:
5c. (4 points)
Find $Z_{.80}$, the value of the 80th percentile

**Answer:** 0.8416

**Solution:**
6. (10 points total)

The figure below shows the assumed normal distribution of systolic blood pressure in a population of males. It has $\mu = 130$ mmHg and $\sigma = 17$ mmHg.
6a. (2 points)
What value of systolic blood pressure is the cut-off, below which a person is classified as **hypotensive**?

**Answer:** 102.04

**Solution:** “**Hypotensive**” are systolic blood pressure values at or below the 5th percentile.

6b. (2 points)
What value of systolic blood pressure is the cut-off, above which a person is classified as **hypertensive**?

**Answer:** 157.96

**Solution:** “**Hypertensive**” are systolic blood pressure values in the upper 5 percent, or beyond the 95th percentile.
6c. (2 points)
What two values of systolic blood pressure are the boundary values, between which a person is classified as *borderline hypotensive*?

**Answer:** 102.04 and 108.21

**Solution:** Values of systolic blood pressure that are *borderline hypotensive* are between the 5th and 10th percentile. From the solution to #6a, we already have the 5th percentile value = 102.04. Thus, need only solve for the 10th percentile.

6d. (2 points)
What two values of systolic blood pressure are the boundary values, between which a person is classified as *borderline hypertensive*?

**Answer:** 151.79 and 157.96

**Solution:** Values of systolic blood pressure that are *borderline hypertensive* are between the 90th and 95th percentile. From the solution to #6b, we already have the 95th percentile value = 157.96. Thus, need only solve for the 90th percentile.
6e. (2 points)
What two values of systolic blood pressure are the boundary values, between which a person is classified as normal?

Answer: 108.21 and 151.79

Solution: Systolic blood pressure values that are normal are between the 10th and 90th percentile. This is an area under the curve, symmetric about the mean, that covers the remaining 80% of the distribution.
7. (10 points total)

Suppose the instructor decides to give a grade of “A” to the 10% of the students who score highest on this exam. Suppose further that, in the past, the mean grade has been a $\mu = 80$ with $\sigma = 8$. Under the assumption that the distribution of grades is normal, what grade would a student need in order to earn an “A”?

**Answer:** At least 90.25

**Solution:**

![Normal Distribution Diagram](image)
8. (10 points total)
The duration of gestation in healthy humans is approximately 280 days with a standard deviation of 10 days.

8a. (5 points)
Under the assumption of normality, what proportion of (healthy) pregnant women will be overdue by 2 weeks or more?

Answer: 0.08 or 8%

Solution: Overdue by 2 weeks or more corresponds to a duration of gestation $\geq (280 + 14)$ days. This is $\geq 294$ days.

8b. (5 points)
Suppose now that, typically, there are 200 births per week at the UMass Memorial Hospital. How many of these births would you expect to be premature by 4 weeks or more?

Answer: 0.511

Solution: Premature by 4 weeks or more corresponds to a duration of gestation $\leq (280 - 28)$ days. This is $\leq 252$ days

Expected # = (Number of births)*(probability premature) = (200)*(0.002555) = 0.511
9. (10 points total)

9a. (5 points)
Eighty six students are taking the 540 exam 2 in Fall 2010. Assume that the scores will be distributed normal. Suppose further that your score is in the 60th percentile. How many people will have scored at or below your score?

Answer: 51.6

Solution: The 60th percentile separates the lower 60% of the distribution from the rest.
60% of 86 = 0.6*86 = 51.6

9b. (5 points)
Suppose the weights for 18-24 year old women are distributed normal with a mean of 132 pounds and a standard deviation of 27.4 pounds. In a random sample of 150 such women, how many of them would be expected to weigh between 100 and 115 pounds?

Answer: 21.9

Solution: Expected # = (Number of women)*(probability event of 100 to 115 pounds) = (150)*(0.146) = 21.9
10. (10 points total)

The Air Force uses ACES-II ejection seats that are designed for men who weigh between 140 lb and 211 lb. Suppose it is known that women’s weights are distributed Normal with mean 143 lb and standard deviation 29 lb. What proportion of women have weights that are outside the ACES-II ejection seat acceptable range?

**Answer:** 46.8%

**Solution:**