Cooling by Flow Through Narrow Pores

W. J. Mullin[1] and Neal Kalechofsky[2]

[1]*Physics Department, Hasbrouck Laboratory, University of Massachusetts, Amherst, MA 01003, USA*
[2]*Oxford Instruments America Inc., 130A Baker Avenue, Concord, MA, 01742, USA*

Abstract. We consider the possibility of cooling ^{3}He atoms in dilute solutions with liquid ^{4}He by "filtering out" the hot atoms through a screen of small holes or channels. The proposed method is somewhat analogous to that employed to evaporatively cool trapped gases, and the specific heat of the ^{3}He-^{4}He mixture makes it feasible to use in a device to refrigerate other samples. Three methods are considered: 1) Effusion, where holes having diameters larger than a mean free path allow atoms to pass through easily; 2) Particle waveguide-like motion using very narrow channels that greatly restrict the quantum states of the atoms in a channel; and 3) Wall-limited diffusion through channels, in which the wall scattering is disordered so that local density equilibrium is established within a channel. The methods studied all require sufficiently low temperatures and holes or channels with sufficiently small dimensions that temperature equilibrium between the escaping gas and the original gas is avoided; that is, we assume that channel dimensions are smaller than the mean free path for atom-atom interactions. We find that the particle waveguide and the wall-limited diffusion methods using channels on order of the de Broglie wavelength give cooling. Recent advances in nano-filters give these methods some hope of being practical.

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We investigate here the possibility cooling a gas by "filtering out" hot atoms, perhaps as an adjunct to a dilution refrigerator with solutions of ^{3}He in liquid ^{4}He. We consider a situation in which a degenerate Fermi gas is passed through narrow constrictions formed by pores. Pores of two different sizes are investigated: those of order of a mean free path and those of diameter comparable to a de Broglie wavelength. We find the only later size can be successful; because the constriction is narrow the bands of states allowed in such a "particle waveguide" are widely separated, which means that not all energies are allowed through. Adjusting the states in the channel can allow selective passage of particles whose energy is at or above the Fermi energy, so that one removes hot atoms. For such a device to work involves having the constriction of order of the de Broglie wavelength of atoms at the top of the Fermi surface, which of order of the separation between fermions. Such a small size makes implementation the idea difficult, but perhaps not impossible considering recent advances in making nanometer size holes in polymers, carbon nanotubes, or glasses. Because of the limited states in the channels here we specify the channels of this size as "narrow" pores. Those having the larger size of a mean free path we designate as "wide."

Because fermion scattering can take place only at the Fermi surface—the rest of the particles being restricted by the Pauli principle—the mean free path can be much larger than the de Broglie wavelength, and is proportional to $1/T^2$, where T is the temperature. If we assume we are considering adding a cooling stage onto a dilution refrigerator we would work with a $< 6\%$ solution of ^{3}He in liquid ^{4}He at millikelvin temperature. A 3% solution then has a Fermi temperature of about 260 mK, a de Broglie wavelength of about 2.3 nm, and a degenerate mean free path at 10 mK of about 23 μm..

There are two methods that might allow draining off the hot atoms by using the wide pore criterion: effusion and wall-limited diffusion. Both of these approaches were considered by one of the authors some time ago for enhancing polarization.[1] In *effusion* one has holes less than the mean-free path in a thin membrane. The states in such channels are considered as three-dimensional. Some small fraction of the atoms can pass through the holes with their normal flux in one direction, and the atoms do not come into equilibrium with the channel walls as they pass through. The intensity of fast atoms is larger than that of the slow atoms and these, on average, carry more energy. Indeed it is a standard textbook exercise[2] to show that the average temperature of a gas of classical atoms effusing through a hole is twice the temperature of the original gas; the hot atoms are separated off.

An alternative situation for wide pores separating two volumes is *wall-limited diffusion*. The states in such channels are again three-dimensional and the flow down the channel is limited by collisions with the walls of the channel, by which local density equilibrium is established. The rate of diffusion down the channel from volume 1 to volume 2 depends on the diameter of the pore and the Fermi velocity. A density gradient down the channel maintains the flow. If container 1 has a higher density and Fermi energy than 2 we might expect that we are extracting high energy particles from the denser compartment, which one would intuitively guess cools the remaining gas. But intuition is wrong and this method does not work with wide channels.

In narrow pores we have limited energy and momentum states. The nature of the flow in such small channels depends on the nature of the collisions with the walls. We will consider two possibilities: specular reflection where the walls act only as boundary conditions for the quantum states, and only wall-limited diffusion, where, just as in the wide channel case the scattering is incoherent and local density equilibrium is established down the channel. Both cases lead to possible cooling.

The cooling power $C_V dT/dt$ at temperature T of any gas can be computed from

$$\frac{dE}{dt} = C_V \frac{dT}{dt} + \left(\frac{\partial E}{\partial N}\right)_{T,V} \frac{dN}{dt} \qquad (1)$$

where C_V is the heat capacity and where energy and particle rates dE/dt and dN/dt are computed from the particle and energy currents. The cooling power, $C_V dT/dt$, is a difference of two terms; whether it is positive or negative depends on whether the dE/dt term exceeds the dN/dt term or not.

Wide pores: In effusion of a degenerate Fermi gas effusing from container 1 through the pore to container 2, through wide holes is found to be $2\pi A\varepsilon_F m/(3h^3)$ where A is the total area of channel, ε_F is the Fermi energy in container 1, and m is the particle mass. The result is positive showing that the temperature of gas remaining in container 1 actually rises. Further, it turns out the gas gathering on the other side of the membrane in container 2 also increases in temperature, so no cooling is achieved by effusion. When we treat wall-limited diffusion in wide channels, where the states are again continuous in three dimensions, we need to solve a kinetic equation. In this case we also find no cooling, although we give no details here.

Narrow pores: The ”waveguide” method uses a smaller pore that has strongly restricted banded states with continuous momenta only along the channel axis and discrete transverse states. The particles travel through the pore with only specular scattering so the walls act just as boundaries for establishing the quantum states. In this case, if the lowest band starts far below the Fermi level of the gas, there is heating. On the other hand, if this band is in the neighborhood of, or just above, the Fermi level there is cooling with maximum power given by $-M\left(k_B T\right)^2/h$ where M is the number of nano-channels in the wall.

If we continue to assume the narrow channel case but with atoms to hitting the walls incoherently, perhaps even sticking to them momentarily, and backscattering, then they can reach a local equilibrium density along the tube, and we need to treat the gas under wall-limited diffusion conditions, i.e., Knudsen flow. But now the momentum states are restricted as in the waveguide method considered above. The result of solving a kinetic equation is that we still get cooling, with a cooling power achieved when the lowest band is quite near the Fermi energy and given by $-M\frac{d}{L}\frac{\varepsilon_F}{h}\left(\frac{k_B T}{\varepsilon_F}\right)^{3/2}$ where the pore diameter and length are d and L.

Classical gases cool under the effects of effusion; degenerate gases do not. However, we have seen that, if we use a filter with sufficiently small nano-scale channels so that the lowest energy band starts at roughly the Fermi energy, we can have cooling. Whether the waveguide description or the wall-limited diffusion analysis applies depends on the nature of scattering at the channel walls. If the scattering is an ideal specular reflection then the waveguide method should apply; if there is diffuse scattering involving a sticking time on the wall, the wall-limited diffusion approach should be applicable. For reasonable temperatures and channel parameters we find cooling powers in the range of 3 to 30 μW. Whether this can become a practical means of cooling remains to be tested experimentally.

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References

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