# A Taste of the Standard Model

Lagrangian formalism
simple relation with Feynman rules

## Two Big Ideas:

★ Local gauge invariance

- Invariance of $\mathcal{L}_{fermion}$ under transformations "requires" existence of massless vector bosons with specified interactions.
- Gauge invariant theories are renormalizable

★ Spontaneous symmetry breaking of scalar fields

The Higgs mechanism combines gauge invariance with spontaneous symmetry breaking to describe gauge invariant massive vector particles.

# Classical Lagrangians (review)

$$m \frac{d\bar{v}}{dt} = -\bar{\nabla} U$$ equation of motion

## Lagrangian formulation

$$L = T - U \qquad T = \frac{1}{2} m v^2$$

## Euler-Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i} \qquad q = x, y, z$$

so that $\frac{\partial L}{\partial \dot{q}_i} = m v_i \qquad \frac{\partial L}{\partial q_i} = -\frac{\partial U}{\partial q_i}$

$$\Rightarrow \quad m \frac{dv_i}{dt} = -\frac{dU}{dq_i}$$ as above

# Field Theory Lagrangians

Instead of coordinates of an object we use fields that are functions of space time $\phi_i(x^\mu)$

Examples

- $\phi$ could represent temperature as a function of position (scalar field).
- $\phi_{i=1,2,3}$ could represent 3 components of an electric field $\vec{E}$ (vector field).

generalized equ. of motion

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i}$$

Note

- space & time on equal footing
- prefer to work with Lagrangian density $\mathcal{L}$ such that $L = \int d^3x \, \mathcal{L}$

In our case, the fields will be particle fields (i.e. wave functions)

# Lagrangian Densities for Particle Fields

Scalar (spin zero) field:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

Check this gives the correct equ. of motion...

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi \quad ; \quad \frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi$$

Euler-Lagrange: $\partial_\mu \partial^\mu \phi + m^2 \phi = 0$ ★

★ This is the Klein-Gordon equation for a spin zero relativistic particle.

Spin ½ field:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Check:

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})} = 0 \qquad \frac{\partial\mathcal{L}}{\partial\bar{\psi}} = i\gamma^\mu\partial_\mu\psi - m\psi$$

$$i\gamma^\mu\partial_\mu\psi - m\psi = 0 \qquad \text{Dirac Equ.}$$

(satisfied by particle spinors)

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} = i\bar{\psi}\gamma^\mu \qquad \frac{\partial\mathcal{L}}{\partial\psi} = -m\bar{\psi}$$

$$i\partial_\mu\bar{\psi}\gamma^\mu + m\bar{\psi} = 0$$

(satisfied by antiparticle spinors)

photons (massless spin 1 fields):

First a reminder concerning the equations of motion. (section 7.4).

<u>Maxwell</u>:

(i) $\nabla \cdot \bar{E} = 4\pi \rho$ (iii) $\nabla \cdot \bar{B} = 0$

(ii) $\nabla \times \bar{E} + \frac{1}{c} \frac{\partial \bar{B}}{\partial t} = 0$ (iv) $\nabla \times \bar{B} - \frac{1}{c} \frac{\partial \bar{E}}{\partial t} = \frac{4\pi}{c} \bar{J}$

define the field strength tensor:

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

i and iv imply . . .

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$$

where $J^\nu = (c\rho, \bar{J})$

iii implies . . .

1) $$\bar{B} = \nabla \times \bar{A}$$ where $\bar{A}$ is called the vector potential

and, using ii

$$\nabla \times \left( \bar{E} + \frac{1}{c} \frac{\partial \bar{A}}{\partial t} \right) = 0$$

2) $$\Rightarrow \bar{E} = -\nabla V - \frac{1}{c} \frac{\partial \bar{A}}{\partial t}$$

where $V$ is the scalar potential

1) and 2) lead to

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

where $A^\mu = (V, \bar{A})$

Finally: $$\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = \frac{4\pi}{c} J^\nu$$

photon Lagrangian:

$$\mathcal{L} = \frac{-1}{16\pi}\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) - J^\nu A_\nu$$

$\leftarrow$ convention

check:

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = \frac{-1}{4\pi}\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right); \quad \frac{\partial \mathcal{L}}{\partial A_\nu} = -J^\nu$$

$$\Longrightarrow \quad \partial_\mu\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) = 4\pi J^\nu$$

also written $\partial_\mu F^{\mu\nu} = 4\pi J^\nu$

(Maxwell's eqns., as we expected)

A massive spin 1 free field gives

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} m^2 A^\nu A_\nu$$

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0 \qquad \text{Proca equation}$$

Incidently, $A^\mu$ is not uniquely determined.

Let $A^{\mu\prime} = A^\mu + \partial^\mu \lambda$

Then

$$\begin{aligned}\partial^\mu A^{\nu\prime} - \partial^\nu A^{\mu\prime} &= \partial^\mu (A^\nu + \partial^\nu \lambda) - \partial^\nu (A^\mu + \partial^\mu \lambda) \\ &= \partial^\mu A^\nu - \partial^\nu A^\mu\end{aligned}$$

$\Rightarrow$ $F^{\mu\nu}$ and the equs. of motion are unchanged.

This is called a <u>gauge</u> <u>transformation</u>

# Gauge Invariance

The Dirac Lagrangian:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

is invariant under the transformation

$$\psi \rightarrow e^{i\theta}\psi \quad (\text{and } \bar{\psi} \rightarrow e^{-i\theta}\bar{\psi})$$

This is called a "global gauge transformation"

Question:

What if the transformation were a function of spacetime?

$$\psi \rightarrow e^{i\theta(x)}\psi \qquad x \text{ stands for } t, x, y, z$$

Then

$$\partial_\mu (e^{i\theta(x)}\psi) = \underbrace{i(\partial_\mu \theta(x))e^{i\theta(x)}\psi}_{\text{extra term}} + e^{i\theta(x)}\partial_\mu \psi$$

$$\mathcal{L} \rightarrow \mathcal{L} - (\partial_\mu \theta(x))\bar{\psi}\gamma^\mu \psi$$

$\mathcal{L}$ is no longer invariant.

We want to try to restore the invariance by adding something to $\mathcal{L}$.

first, a notation change:

$$\lambda(x) \equiv \frac{-1}{g}\,\theta(x)$$

so $$\mathcal{L} \rightarrow \mathcal{L} + g(\partial_\mu \lambda)\bar{\psi}\gamma^\mu\psi$$

now, the fix:

try $$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - g\bar{\psi}\gamma^\mu\psi A_\mu$$

new gauge field

where the transformation rule for $A_\mu$ is $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$ as it was in Maxwell's equations.

The full Lagrangian is now invariant.

$\psi \rightarrow e^{-ig\lambda}\psi$ together with $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$ works!

This is called a "local gauge transformation"

BUT now we have introduced a new gauge field that couples to the fermions. We should also include some terms for the gauge particle itself.

$$\mathcal{L}_{gauge} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \frac{1}{8\pi} m_A^2 A^\nu A^\mu$$

this part is invariant (as we saw for Maxwell's equ.s.)

this part is not invariant

conclude: only massless vector gauge particles have local gauge invariant Lagrangians. ?

$$\mathcal{L}_{total} = [i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi] + [\tfrac{-1}{16\pi} F^{\mu\nu}F_{\mu\nu}] - [q\bar{\psi}\gamma^\mu\psi A_\mu]$$

free fermion

interaction

free gauge particle

The existence of the photon, coupling to fermions in the usual way, guarrantees local gauge invariance.

Another (conventional) way of describing this ...

$$\psi \to e^{-ig\lambda(x)}\psi$$

implies

$$\partial_\mu \psi \to e^{-ig\lambda(x)}\left[\partial_\mu - ig\,\partial_\mu \lambda\right]\psi$$

this term destroys the invariance of the derivative term in $\mathcal{L}$

solution:

In $\mathcal{L}$, replace $\partial_\mu$ by

$$D_\mu \equiv \partial_\mu + igA_\mu$$

"covariant derivative"

then

$$D_\mu \psi \to e^{-ig\lambda(x)} D_\mu \psi$$

This is equivalent to adding $-g\bar{\psi}\gamma^\mu\psi A_\mu$ to $\mathcal{L}$.

# Yang Mills Theory (Weak Interactions)

W couples 2 different types of fermions together.
$\Rightarrow$ Start with a two fermion theory
(for simplicity, use the same mass for each)

$$\mathcal{L} = i\bar{\psi}_1 \gamma^\mu \partial_\mu \psi_1 - m\bar{\psi}_1\psi_1 + i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 - m\bar{\psi}_2\psi_2$$

rewrite as $\psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$ $\quad \bar{\psi} = (\bar{\psi}_1 \;\; \bar{\psi}_2)$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \leftarrow \text{2 vector}$$

Previously we used $\psi \rightarrow e^{-ig\lambda}\psi$

phase transformation
(a.k.a. 1×1 unitary)

For a 2 fermion theory use:

$$\psi \rightarrow U\psi \qquad UU^\dagger = 1$$

where $U$ is a 2×2 unitary matrix ($\in SU(2)$)

This corresponds to rotations btwn $\psi_1$ & $\psi_2$
(rotations in "weak isospin" space)

A unitary matrix can be written as the exponential of some Hermitian matrix.

$$U = e^{iH} \quad \text{with} \quad H^\dagger = H$$

and a 2×2 Hermitian matrix can be written

$$H = \theta I + a \cdot \sigma$$ ← Pauli matrices

Thus,

$$U = e^{i\theta} \; e^{i\sigma \cdot a}$$

We already looked at this. It leads to the photon.

Now look at this.

$$\psi \rightarrow e^{-ig\,\sigma \cdot \lambda(x)}\,\psi \equiv S\psi$$

$\lambda$ is now a 3 vector

The derivative $\partial_\mu \psi \rightarrow S\,\partial_\mu \psi + (\partial_\mu S)\psi$ is not invariant. Fix it by replacing with the covariant derivative.

$$\mathcal{D}_\mu \equiv \partial_\mu + ig\,\sigma \cdot A_\mu$$

3 new fields

where the $A_\mu$ transform in such a way that $\mathcal{L}$ is invariant.

In the case of small $\lambda$ (see Griffiths 11.58 - 11.61)

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda + 2g(\lambda \times A_\mu)$$

The case of finite $\lambda$ is no more illuminating so we will stick to this for simplicity.

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi - m\bar{\psi}\psi$$

$$= i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi - g\bar{\psi}\gamma^\mu \sigma \psi \cdot A_\mu$$

is now invariant.

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Since we introduced 3 new gauge fields we should include their own free Lagrangian.

$$\mathcal{L}_A = \frac{-1}{16\pi} F_1^{\mu\nu} F_{\mu\nu 1} - \frac{1}{16\pi} F_2^{\mu\nu} F_{\mu\nu 2} - \frac{1}{16\pi} F_3^{\mu\nu} F_{\mu\nu 3}$$

$$= \frac{-1}{16\pi} F^{\mu\nu} \cdot F_{\mu\nu} \leftarrow$$ $F_{\mu\nu}$ is now a 3 vector

PROBLEM

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is not invariant!

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda + 2g\,(\lambda \times A_\mu)$$

leads to

$$F^{\mu\nu} \rightarrow F^{\mu\nu} + 2g\,[\lambda \times F^{\mu\nu} + A^\mu \times \partial^\nu \lambda - A^\nu \times \partial^\mu \lambda]$$

and

$$F^{\mu\nu} \cdot F_{\mu\nu} \rightarrow F^{\mu\nu} \cdot F_{\mu\nu} + 8g\,(A_\mu \times F^{\mu\nu}) \cdot \partial_\mu \lambda$$

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To fix this

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - 2g\,(A^\mu \times A^\nu)$$

then

$$F^{\mu\nu} \rightarrow F^{\mu\nu} + 2g\,(\lambda \times F^{\mu\nu})$$

and

$F^{\mu\nu} \cdot F_{\mu\nu} \rightarrow F^{\mu\nu} \cdot F_{\mu\nu}$ to 1st order in $\lambda$

## Conclusion

$$\mathcal{L} = \underbrace{i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi}_{\text{2 fermions}} - \underbrace{\frac{1}{16\pi} F^{\mu\nu} \cdot F_{\mu\nu}}_{\text{3 massless vector fields}} - \underbrace{g\bar{\psi}\gamma^\mu \sigma \psi \cdot A_\mu}_{\text{fermion gauge interactions}}$$

# Chromodynamics (short form)

start with:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \quad \text{where} \quad \psi = \begin{pmatrix}\psi_r\\ \psi_b\\ \psi_g\end{pmatrix}$$

$\psi \to U\psi$ with $U$ $3\times 3$ unitary $(\in SU(3))$

$U = e^{iH}$ with $H = \theta I + a\cdot\lambda \leftarrow$ 8 Gell-Mann matrices

we consider $\psi \to e^{-ig\lambda\cdot\phi(x)}\psi$

$$\mathcal{D}_\mu \equiv \partial_\mu + ig\,\lambda\cdot A_\mu \leftarrow \text{8 new fields}$$

$$A_\mu \to A_\mu + \partial_\mu\phi + 2g\,(\phi\times A_\mu)$$

where $(\phi\times A_\mu)_k \equiv \sum_{i,j} f_{ijk}\,\phi_i\,A_{\mu j}$

$\uparrow$ structure consts.

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - 2g\,(A^\mu\times A^\nu)$$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - \frac{1}{16\pi}F^{\mu\nu}\cdot F_{\mu\nu} - g\bar{\psi}\gamma^\mu\lambda\psi\cdot A_\mu$$

# From $\mathcal{L}$ to Feynman Rules

## Propagators

Euler-Lagrange yields free field equations

KG $\quad [p^2 - m^2]\,\phi = 0$

Dirac $\quad [\not{p} - m]\,\psi = 0$

Proca $\quad [(-p^2 + m^2) g_{\mu\nu} + p_\mu p_\nu]\,A^\nu = 0$

where we used $i\partial_\mu = p_\mu$

The propagator is $i$ times the inverse of the term in [ ] brackets.

$$\frac{i}{p^2 - m^2} \qquad \text{spin } 0$$

$$\frac{i}{\not{p} - m} = \frac{i(\not{p} + m)}{p^2 - m^2} \qquad \text{spin } \tfrac{1}{2}$$

$$\frac{-i}{p^2 - m^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right] \qquad \text{spin } 1$$

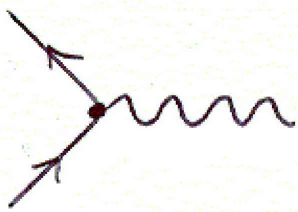
To understand <u>why</u> this is the case requires a knowledge of Green's functions.

# Vertex factors

$$i\,\mathcal{L}^{QED}_{INT} = -iq\,\bar{\psi}\gamma^\mu\psi A_\mu$$

interaction of 3 fields

describes



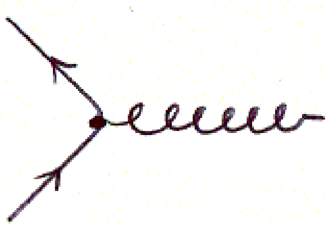
with vertex factor

$-iq\gamma^\mu$ $\qquad q \equiv -g_{e.m.}$

Similarly,

$$i\,\mathcal{L}^{QCD}_{INT} = -iq\,\bar{\psi}\gamma^\mu\lambda\psi\cdot A_\mu$$

describes



with vertex factor

$-iq\gamma^\mu\lambda$ $\qquad q \equiv \frac{g_s}{2}$

($g_s$ traditionally defined with an extra factor of 2)

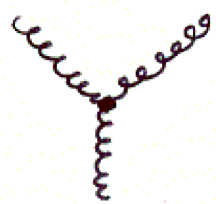
For QCD, there are also interaction terms in $F^{\mu\nu} \cdot F_{\mu\nu}$

$$F^{\mu\nu} \cdot F_{\mu\nu} = (\partial^\mu A^\nu - \partial^\nu A^\mu) \cdot (\partial_\mu A_\nu - \partial_\nu A_\mu)$$

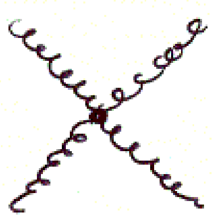
$$- 2g \left[ (\partial^\mu A^\nu - \partial^\nu A^\mu) \cdot (A_\mu \times A_\nu) + (A^\mu \times A^\nu) \cdot (\partial_\mu A_\nu - \partial_\nu A_\mu) \right]$$

$$+ 4g^2 \left[ (A^\mu \times A^\nu) \cdot (A_\mu \times A_\nu) \right]$$

Line 2 describes interaction of 3 fields



Line 3 describes interaction of 4 fields



So far, gauge invariant $\mathcal{L}$'s include only <u>massless</u> vector bosons.
<u>Massive</u> vector bosons can be made via the Higgs mechanism using scalar particles.

look at: $\mathcal{L}_{scalar} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2 \phi^2$

mass $\mu$

compare to: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$

Is this case imaginary mass?

Consider $\mathcal{L} = \mathcal{T} - \mathcal{U}$ with $\mathcal{U}(\phi) = \frac{-1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$

The ground state minimum is $\phi = \pm\mu/\lambda$

Perturbative Feynman calculus must be formulated in terms of deviations from the ground state.

Therefore, let $\eta \equiv \phi \pm \mu/\lambda$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \frac{1}{4}\lambda^2\eta^4 + \frac{1}{4}(\mu^4/\lambda^2)$$

a change of variable allows us to see the physical mass $m = \sqrt{2}\,\mu$.

Consider two REAL scalar fields ...

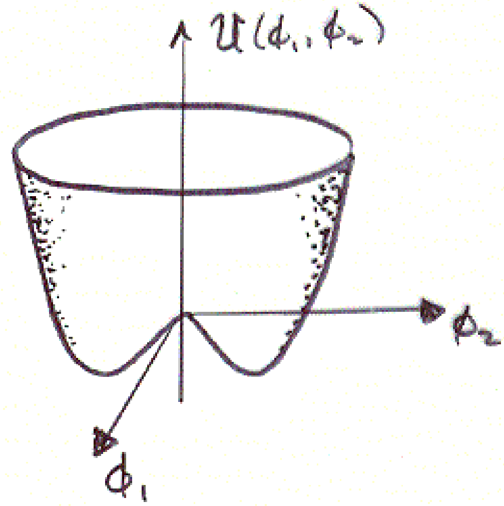
$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \tfrac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2)$$
$$+ \tfrac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \tfrac{1}{4}\lambda^2(\phi_1^2+\phi_2^2)^2$$

choose a minimum at

$$\phi_1 = \mu/\lambda \qquad \phi_2 = 0$$

and use change of variable

$$\eta = \phi_1 - \mu/\lambda \qquad \xi = \phi_2$$




massive scalar — massless scalar (Goldstone boson)

$$\mathcal{L} = \left[\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \left[\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right]$$
$$+ \mu\lambda(\eta^3 + \eta\xi^2) - \frac{\lambda^2}{4}(\eta^4 + \xi^4 + 2\eta^2\xi^2) + \mu^2/4\lambda^2$$

This is called spontaneously broken symmetry. The ground state $\phi$ does not preserve the symmetry of $\mathcal{L}$.

# The Higgs Mechanism

First, a change of notation

$$\phi = \phi_1 + i\phi_2 \qquad \phi^* = \phi_1 - i\phi_2$$

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \tfrac{1}{2}\mu^2 \phi^*\phi - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

$\mathcal{L}$ is invariant under a global phase shift

$$\phi \rightarrow e^{i\theta}\phi$$

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To make <u>local</u> invariance we require

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu + iq A_\mu \quad \text{(covariant derivative)}$$

The resulting Lagrangian:

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial_\mu + iqA_\mu)\phi$$
$$+ \tfrac{1}{2}\mu^2\phi^*\phi - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2 - \tfrac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

is invariant.

Since $\mathcal{L}$ is invariant under $\phi \to e^{i\theta(x)}\phi$

we can transform

$$\phi = \phi_1 + i\phi_2 \longrightarrow (\cos\theta + i\sin\theta)(\phi_1 + i\phi_2)$$

and choose $\theta$ $(-\tan^{-1}\phi_2/\phi_1)$ such that

$\phi' = \phi_1'$ is purely real and $\phi_2' = 0$.

Once again, change variables

$$\eta = \phi_1' - \mu/\lambda \qquad \xi = \phi_2' = 0$$

Result:

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 \quad \leftarrow \text{massive scalar}$$

$$+ \tfrac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}g^2\tfrac{\mu^2}{\lambda^2}A_\mu A^\mu \quad \leftarrow \text{massive vector!}$$

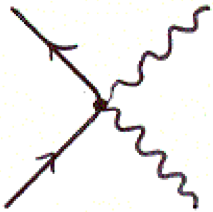
$$+ \tfrac{\mu}{\lambda}g^2\eta A_\mu A^\mu + \tfrac{1}{2}g^2\eta^2 A_\mu A^\mu - \lambda\mu\eta^3 - \tfrac{1}{4}\lambda^2\eta^4$$

$$+ \tfrac{1}{4}\mu^4/\lambda^2 \quad \leftarrow \text{constant}$$

interactions

Where did the mass for $A_\mu$ come from?

In the original $\mathcal{L}$ we had the term:



$$g^2 A_\mu A^\mu \phi^* \phi$$

Under SSB $\phi \rightarrow \eta + \mu/\lambda$,
giving rise to the term $g^2 A_\mu A^\mu \frac{\mu^2}{\lambda^2}$,
which describes a vector mass.

SSB of a scalar that couples to a vector field can produce a mass term for the vector field.

In the SM we start with a weak isospin doublet of Higgs particles.

Under symmetry breaking, this gives one massive scalar field & 3 Goldstones

If these are coupled to the four electroweak vector bosons in a locally gauge invariant way under SU(2) & U(1) transformations ...

3 GBs $\Longrightarrow$ 3 massive vector bosons

By the way, it is also possible to start with massless fermions that couple to the Higgs via terms like $-g\bar{\psi}\psi\phi$. When SSB occurs these terms give mass terms $-g\bar{\psi}\psi\frac{\mu}{\lambda}$.

For full details see "Quarks & Leptons" Halzen & Martin