Homework 7
Linguistics 726

one

$D = \{j, m\}$

$M_1 = \quad <D, I_1>$
$I_1 = \quad \{<j, John>, <m, Mary>, <love, \{<j, j>, <j, m>, <m, j>, <m, m>\}>, <happy, \{j, m\}>\}$

$M_2 = \quad <D, I_2>$
$I_2 = \quad \{<j, John>, <m, Mary>, <love, \{<j, j>, <m, j>\}>, <happy, \{m\}>\}$

Formulas that are false in M_1 and true in M_2 are, for example,

(1) $\forall x, y\ (love(x, y) \rightarrow \neg happy(x))$
"to be loved makes unhappy ..."
false in M_1: counterexample $love(m, j) \& \neg happy(j)$
true in M_2: $love(j, j) \& \neg happy(j)$ and $love(m, j) \& \neg happy(j)$

(2) $\exists x \forall y\ (happy(x) \& (happy(y) \rightarrow x = y))$
"exactly one person is happy ..."
false in M_1 because $happy(j) \& happy(m) \& j \neq m$
true in M_2 because $happy(m)$

two

I tried to find a general structure that describes "family relations". Then it should be possible to state the relations of parent and grandparent as intuitive as suggested on the handout, but assume within the restrictor that in addition this structural relation must hold.

What structure could this be? Family relations are often traced back in nicely drawn pictures of trees. But there is no need for a unique root. Also the linear order of the leaves does not really matter for this purpose. Therefore a 'treelike', but more general structure might do, namely directed acyclic graphs.

How is a general relation R constrained within this graphs?

Since ordered pairs are considered, and therefore not necessarily $R(a, b) = R(b, a)$, nothing more has to be said about directedness.

As for the ordering, the structure should be closed under transitivity:
(1) $\forall x, y, z(R(x, y) \& R(y, z) \rightarrow R(x, z))$

Now cyclicity can be excluded by demanding irreflexivity:
(2) $\forall x \neg R(x,x)$[1]

Finally we can refine our relations of parents and grandparents:[2]
(3) $\forall x, y(parent(x,y) \rightarrow R(x,y))$
(4) $\forall x, y, z(parent(x,y) \& parent(y,z) \rightarrow grandparent(x,z))$

For a possible distinction between grandmothers and grandfathers, I have to make an assumption about the basic facts that are stored in the knowledge base. Either we have every parent relationship and an additional gender information for at least each parent (e.g. $parent(x,y)$ and $female(x)$) or we have entries in terms of fathers and mothers and can then derive parenthood and gender.

Assuming the first, I can now say
(5a) $\forall x, y(grandparent(x,y) \& male(x) \rightarrow grandfather(x))$
(5b) $\forall x, y(grandparent(x,y) \& female(x) \rightarrow grandmother(x))$

V. Borschev:

1. You have no restriction on the amount of parents for a person.
2. Your formulas don't exclude configurations like this
   (a) parent(j,m) & grantparent(j,m)
   (b) grandparent(j,k) & parent(j,m) & parent(k,m)

three

ex. 1

(a)	$\forall x(black(x) \vee white(x))$
(f)	$\exists x \forall y(love(y,x))$
(m)	$\forall x(person(x) \& live\_in(x,ny) \rightarrow like(x,ny))$[3]

[1] I don't think that I have to demand asymmetry, since, if we have R(a,b) and R(b,a) we would necessarily have R(a,a), due to closure under transitivity. But R(a,a) is excluded by irreflexivity. Therefore we can never have R(a,b) and R(b,a).

[2] Assume that D is restricted to things that can have a parenthood relation

[3] This solution should cover the easiest case where 'it' refers to New York (ny). But it might be the case that 'it' refers to a larger part of that sentence. People who live in New York might for example like "living in New York". I think we wouldn't have a representation for that yet. If people who live in New York like that "they live in New York" the representation

$$\forall x(person(x) \& live\_in(x,ny) \rightarrow like(x, live\_in(x,ny)))$$

would not be a wff since $live\_in(x,ny)$ is a formula and not a term.
Maybe the assumption of a living event (or state) would help, a representation might look like

$$\forall e, x(living(e) \& person(x) \& where(e,ny) \& who(e,x) \rightarrow like(x,e))$$

ex. 3

		free occurrences of	bound occurrences of
(a)	$\underline{(\forall x)P(x)} \vee Q(x,y)$	x, y	x
(b)	$\underline{(\forall y)(Q(x) \to \underline{(\forall z)P(y,z)})}$	x	y, z
(c)	$\underline{(\forall x)\neg(P(x) \to \underline{(\exists y)\underline{(\forall z)Q(x,y,z)}}}$		x, y, z
(d)	$\underline{(\exists x)Q(x,y) \& P(y,x)}$	x, y	x
(e)	$\underline{(\forall x)(P(x) \to \underline{(\exists x)(Q(y) \to \underline{(\forall z)R(y,z)})})}$		x, y, z

ex. 4

fathers and integers
the following are equivalent:

$(\forall x)(\exists y)F(y,x) \& (\forall z)(O(z) \to I(z))$

(1) $(\forall z)(\forall x)(\exists y)(F(y,x) \& (O(z) \to I(z)))$
first Quantifier Movement (QM) for $(\forall z)$, then applying some other laws (Implication, Quantifier Negation (QN) and QM) to get the other quantifiers out

(3) $(\forall x)(\forall z)(\exists y)(F(y,x) \& (O(z) \to I(z)))$
from (1) by Quantifier Independence

bachelors vs. husbands
TFAE:

$B(a) \to \neg(\forall x)(M(x) \to H(x))$

(2) $(\exists x)\ B(a) \to \neg(M(x) \to H(x))$
QN, QM

(4) $B(a) \to (\exists x)(M(x) \& \neg H(x))$
QN, Implication, DeMorgan

go(o)d and evil
TFAE:

$(\exists x)\ E(x) \to \neg B(g)$

(1) $\neg((\exists x)\ E(x) \& B(g))$
Implication, DeMorgan

(2) $(\forall x)\ (E(x) \to \neg B(g))$
QM

four

$\|\forall x\,(love(Mary,x) \rightarrow happy(x))\|^{M_1,g} = 1$

holds iff

(S 7)

|

for each d in D,

$\|(love(Mary,x) \rightarrow happy(x))\|^{M_1,g[d/x]} = 1$

holds iff

(S 5)

|

for each d in D,

if $\|love(Mary,x)\|^{M_1,g[d/x]} = 1$

then $\|happy(x)\|^{M_1,g[d/x]} = 1$

holds iff

(S 1)

|

for each d in D,

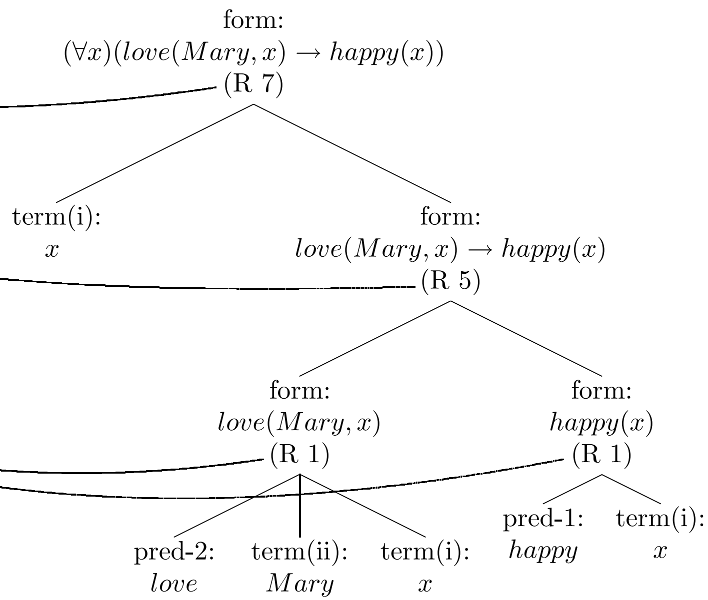
if $< \|Mary\|^{M_1,g[d/x]}, d > \in \|love\|^{M_1,g[d/x]}$

then $d \in \|happy\|^{M_1,g[d/x]}$

|

i.e., if $< I(Mary), d > \in I(love)$

then $d \in I(happy)$.



five

(a) Find a model for W consisting of the set P with four distinct objects, say a,b,c and d.

there are several ways of grouping them together (to lines):

a)	"1+1+1+1"	
	a,b,c,d	doesn't make much sense, since we now don't have any lines at all.
b)	"all together"	
	a,b,c,d	doesn't make much sense either, since there is no point not in this line.
c)	"3+1"	
	a,b,c,a,d,b,d,c,d	
d)	"2+2"	
	a,b,c,a,d,b,d,c,d	

c) and d) both satisfy the first four conditions.
In c) there are no disjoint sets since a,b or c occur everywhere.
But in b) we get

line m	point p (not in m) either	or	line n (containing p, but disjoint from m)
$\{a, b\}$	c	d	$\{c, d\}$
$\{a, c\}$	b	d	$\{b, d\}$
$\{a, d\}$	b	c	$\{b, c\}$
$\{b, c\}$	a	d	$\{a, d\}$
$\{b, d\}$	a	c	$\{a, c\}$
$\{c, d\}$	a	b	$\{a, b\}$

(b) From (2) and (3) we get that we have at least one line, say n. A point p can now be a) a member of n or b) not a member of n.
a) In the former case there is another point, q, not in n due to (4). From (3) we can now conclude that there must be a line containing p and q. Therefore there are at least two lines containing p.
b) In the latter case (b) p is not in n. Now there must, due to (3), be two lines, containing p and one of the points constituting n.

(c) If the empty set would be a member of L, i.e. a line, then there would be a point, say p, not in this line (due to (4)). We have shown above that p is at least in two lines. Since every set is disjoint from the empty set, there would be at least two lines containing p and disjoint from the line constituted by the empty set. This is not possible, due to (5).

(d) If p and q are the only point in P they would form a line (3). Then there must be a point not in this line (4). Therefore it must be distinct from p and q. This is not possible since we assumed that P has just two members.

(e) No. Assume that p and q are distinct points (2). Now, due to (3), there must be a line containing p and q. These line would contain at least two points, namely p and q, and can therefore never contain exactly one point.

six

Here I will try to define trees by a set of statements in first order logic. Some are not necessary, like (7), but I added them, because it was my first intuition, for example in this particular case that nodes which are not terminal should be called nonterminals.
I took some decisions about the structure that might as well be defined in another way. Mainly I excluded reflexivity in dominance and precedence relation and I tried to exclude crossing branches.

A tree is a structure on a nonempty set of nodes:

(1a) $\exists x(node(x))$
(1b) $\forall x(node(x))$

with a unique root:

(2) $\exists x \forall y(root(x) \& (root(y) \rightarrow y = x))$

Every two distinct nodes are either in dominance or precedence relation:

(3) $\forall x, y(x \neq y \rightarrow ((dom(x, y) \vee dom(y, x)) \text{ xor } (prec(x, y) \vee prec(y, x))))$

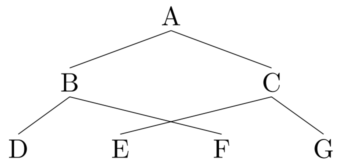
Transitive closure for precedence and dominance:

(4a) $\forall x, y, z(dom(x, y) \& dom(y, z) \rightarrow dom(x, z))$
(4b) $\forall x, y, z(prec(x, y) \& prec(y, z) \rightarrow prec(x, z))$

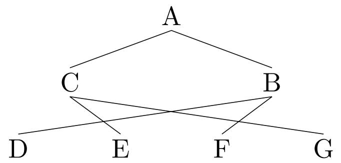
> V. Borschev:
> dom and prec should be order relations (strong or weak is a matter of choice)

Oh, ok, so I need
$\forall x \neg(dom(x, x))$
$\forall x \neg(prec(x, x))$
$\forall x, y \neg(dom(x, y) \& dom(y, x))$
$\forall x, y \neg(prec(x, y) \& prec(y, x))$
in addition

Things can get quirky if we allow crossing branches:



In this picture B seems to precede C, but isn't that just a display thing:



What I want to say is that, if B precedes C, every daughter of B must precede all daughters of C.[4] It would be nice to have a function selecting the mother node ($m : nodes \to nodes$) and to say something like $\forall x, y(prec(m(x), m(y)) \to prec(x, y))$. Since I don't have the ability to express that, I hope that the following will do:

(5) $\forall x, y, v, w(prec(x, y)\&dom(x, v)\&dom(y, w) \to prec(v, w))$

It would also be nice to have some other relations:
Some nodes are terminals:

(6) $\forall x \exists y(dom(x, y) \text{ xor } terminal(x))$

and the others are nonterminals:

(7) $\forall x(\neg terminal(x) \to nonterminal(x))$

A notion of immediate dominance and precedence is helpful:[5]

(8a) $\forall x, y, z(dom(x, z)\&(y \neq x\&dom(y, z) \to dom(y, x)) \to dom_{im}(x, z))$
(8b) $\forall x, y, z(prec(x, z)\&(y \neq x\&prec(y, z) \to prec(y, x)) \to prec_{im}(x, z))$

[4] This must be expressed in another way if nodes are allowed to precede/dominate themselves. I hope in this case it's ok.

[5] This also depends on the assumption of the basic relation. We could as well assume immediate dominance as basic notion and then derive general dominance.

> V. Borschev:
> What do you mean by "basic"? Formally, all relations are "basic".

Hmm, I probably thought about this too much in a constructing way. Especially, how a tree is described in the first place. My idea was that, if the first description would be entirely in terms of immediate precedence and immediate dominance, there would be no need to figure out this relation. Instead we could define every immediate dominance/precedence as general dominance/precedence and than "add" transitive closure.

And in addition notions of sisterhood and c-command:

(9a) $\forall x, y, z(x \neq y \& dom_{im}(z, x) \& dom_{im}(z, y) \leftrightarrow sister(x, y))$
(9b) $\forall x, y(sister(x, y) \leftrightarrow sister(y, x))$

(10) $\forall x, y((\forall z dom(z, x) \rightarrow dom(z, y)) \& \neg dom(x, y) \leftrightarrow c - command(x, y))$