HW7 Ex1 Answers (compiled from several student homeworks)

1. find one or two formulas which are false in M1 and true in M2

$\sim\forall x[\mathbf{happy}(x)]$[1]

$\sim\forall x[\mathbf{love}(x,x)]$

$\exists x\sim\forall y[\mathbf{love}(y,x)]$

$\sim\forall x[\mathbf{love}(x,\mathbf{John}) \rightarrow \mathbf{happy}(x)]$

For any terms τ,σ, let $[[\ \tau = \sigma\ ]]^{M,g} = 1$ iff $[[\tau]] = [[\sigma\ ]]$. We can then add formulas of the following type (I write its negation in infix notation):

$\exists x[x = \mathbf{Mary}\ \&\ \forall y[\mathbf{happy}(y) \leftrightarrow y = x]]$

$\forall x\forall y \sim [\ \mathbf{love}\ (x,y) \leftrightarrow \mathbf{love}\ (x,x)]]$

$\forall x[\ x \neq \mathbf{John} \rightarrow [\mathbf{happy}\ (x) \rightarrow \mathbf{love}\ (x,\mathbf{John})\ \&\ \sim\exists y\ [\ \mathbf{love}\ (y,x)]]$

$\forall x[\ x \neq \mathbf{Mary} \rightarrow \exists y\ [\mathbf{love}\ (x,y) \leftrightarrow x = y]]$

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Question 1

$\neg\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})$

In $\mathbf{M_1}$:

[1] $\|\mathbf{love}\|^{M1,g} = I_1(\mathbf{love}) = \{<j,j>,<j,m>,<m,m>,<m,j>\}$

[2] $\|\mathbf{Mary}\|^{M1,g} = I_1(\mathbf{Mary}) = m$

[3] $<m,m> \in \|\mathbf{love}\|^{M1,g}$

[4] $\|\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M1,g} = 1$

[5] $\|\neg\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M1,g} = \neg\|\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M1,g}$ S2

[6] $\|\neg\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M1,g} = 0$

In $\mathbf{M_2}$:

[1] $\|\mathbf{love}\|^{M2,g} = I_2(\mathbf{love}) = \{<j,j>,<m,j>\}$

[2] $\|\mathbf{Mary}\|^{M2,g} = I_2(\mathbf{Mary}) = m$

[3] $<m,m> \notin \|\mathbf{love}\|^{M2,g}$

[4] $\|\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M2,g} = 0$

[5] $\|\neg\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M2,g} = \neg\|\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M2,g}$ S2

[6] $\|\neg\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})\|^{M2,g} = 1$

And therefore we have $\neg\mathbf{love}(\mathbf{Mary},\ \mathbf{Mary})$ being true in $\mathbf{M_2}$ but false in $\mathbf{M_1}$.

$\neg\mathbf{happy}(\mathbf{John})$

In $\mathbf{M_1}$:

[1] $\|\mathbf{happy}\|^{M1,g} = I_1(\mathbf{happy}) = \{j,m\}$

[2] $\|\mathbf{John}\|^{M1,g} = I_1(\mathbf{John}) = j$

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[1] I will be a little liberal about the syntax. In the first two exercises I use quantifiers of the type '∀x' or '∃x', in the remainder, I stick to the PtMW notation '(∀x)' and '(∃x)'. I will use of '[', ']' and '(',')' interchangeably.

[3] $j \in \|\mathbf{happy}\|^{M1,g}$
[4] $\|\mathbf{happy}(\mathbf{John})\|^{M1,g} = 1$
[5] $\|\neg\mathbf{happy}(\mathbf{John})\|^{M1,g} = \neg\|\mathbf{happy}(\mathbf{John})\|^{M1,g}$ S2
[6] $\|\neg\mathbf{happy}(\mathbf{John})\|^{M1,g} = 0$

In $\mathbf{M_2}$:

[1] $\|\mathbf{happy}\|^{M2,g} = I_2(\mathbf{happy}) = \{m\}$
[2] $\|\mathbf{John}\|^{M2,g} = I_2(\mathbf{John}) = j$
[3] $j \notin \|\mathbf{happy}\|^{M2,g}$
[4] $\|\mathbf{happy}(\mathbf{John})\|^{M2,g} = 0$
[5] $\|\neg\mathbf{happy}(\mathbf{John})\|^{M2,g} = \neg\|\mathbf{happy}(\mathbf{John})\|^{M2,g}$ S2
[6] $\|\neg\mathbf{happy}(\mathbf{John})\|^{M2,g} = 1$

And therefore we have $\neg$**happy**(**John**) being true in $\mathbf{M_2}$ but false in $\mathbf{M_1}$.

[maybe more to be added if others will send us their electronic files]