

Homework 6

II Show that the algebra **Form** is not a Boolean algebra.

In order to show that Form is not a Boolean algebra, we have to show that at least one of the properties (B1)-(B5) from the last handout does not hold for Form.

For simplicity, let's look at the commutative law. Does it hold for Form? Is $a \wedge b = b \wedge a$? It is not. These are different syntactic forms that have been derived in unique ways. They are not syntactically equivalent. Any two formulas that are not string-identical are not syntactically equivalent. Therefore, the laws (B1)-(B5) cannot hold of this algebra.

What we want to say, I think, is that the relevant formulas are identical with respect to their truth-values, i.e. the formulas on both sides of the equal symbol are equivalent in terms of truth values. The algebra T of truth values is a Boolean one.

What we need to make the syntax of statement logic a Boolean algebra is a set of axioms that allows us to form the equivalence classes of formulas that are equivalent in terms of truth conditions.

I suppose we could get the relevant equivalence classes by forming the inverse of the homomorphism σ ; the formulas in Form that end up in the same equivalence class for any arbitrary σ (which arbitrarily assigns $\{p, q, r, \dots\}$ to 1 or 0) will be the ones that are equivalent with respect to truth values.