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Homework 4 from Lecture 4. Algebra. Section 1: Signature, algebra in a signature. Isomorphisms, homomorphisms, congruences and quotient algebras. Homework:

Ex. 1: an optional question of ours about kernel equivalence of a homomorphism, quotient algebra, and commutative diagram.
Ex 2: Problem 3a-f of PtMW pp 256-258: symmetries of the square, subalgebras, homomorphism, isomorphism, automorphism.
"Ex. 3": answer various questions posed throughout the handout.

In particular, what we will call 3.1, the question "Why does $A^0 = \{\varnothing\}$?" posed in fn. 1, p.2;

3.2. The question "Why is the empty set $\varnothing$ trivially closed under all operations except for 0-ary operations?", posed on p. 4; and

3.3. Show that the intersection of any two subalgebras of algebra **A** is a subalgebra of **A**.

3.4. p.5. [A question. Is it not enough to require only that $f^{-1}: A \rightarrow B$ be a function?].

3.5. p.5, end of example 2): Question: Why isn't the following mapping a homomorphism? *f*': Mod4 → Mod2 : *f*'(0) = 0, *f*'(1) = 1, *f*'(2) = 1, *f*'(3) = 0.

HW4, Solution 1. Good answers, with generally exhaustive reasons, to all parts of **Ex 2, i.e. 3a-f** of PtMW. There are also notes there by BHP on some possibly time-saving strategies for showing that algebras are or are not isomorphic or homomorphic, partly in response to this homework paper and partly in response to questions by others. (And such strategies become essential when dealing with large algebras.)

HW4, Solution 2. Has an almost-correct answer to **Ex. 1**, about the kernel equivalence etc. (corrections noted) Has mostly correct answers to **Ex. 2,** with good indication of reasoning. (I've added commentary where needed.) Has brief but I think on-the-right-track or correct answer to the handout question **3.1**, and definitely correct answers to **3.2, 3.3, 3.4, 3.5**.

HW4, Solution 3: Ex. 1 An even nicer, and fully correct, answer to question 1. Use this one to check your answer to that problem.
Note: This one is fully explicit in the argument that **nat (ker f)** is the natural homomorphism from **Mod4** to the quotient algebra **Mod4/ker f** . It's ok to be a little less exhaustively explicit: after working out in detail why "plus" works ok when *x* is even and *y* is odd, you could then say "A similar argument holds for *x,y* both even or both odd, and similarly for the multiplication operation." There is nothing at all wrong with being fully explicit; we mention this just so you know it's ok to leave out steps which are "obvious", if they "really are obvious". (If you have any doubts, or if you haven't already put in enough to show us that you can see that they're obvious, then leave the steps in explicitly.)

On A^0: A nice, and correct, student answer to question 3.1, together with some notes by BHP about a good partial answer and another good but incorrect answer.

On A^0 – wrong. This is the good but presumably wrong answer. Can you find any error or should we change our mind about what the answer is?

On closure of $\varnothing$: Question 3.2. In addition to the answer in HW 4 Solution 2, here is another which I think is fully correct.

Added note on homomorphisms to 1-element algebras: Note by BHP extending an observation made on one of the homework papers.