ON THE CLOSURE OF THE NULL SET

3. $\emptyset$ IS TRIVIALLY CLOSED UNDER ALL OPERATIONS, EXCEPT FOR 0-ARY OPERATIONS. WHY?

Let g be an arbitrary n-ary operation operation : $A^n \rightarrow A$ and $B \subseteq A$. B is closed with respect to g iff $\forall x_1 \ldots x_n$ [$x \in B \rightarrow g|B\,(x_1 \ldots x_n) \in B$]. Notice that, in the case of $\emptyset$, the formula $\boldsymbol{\forall x_1 \ldots x_n [x \in B \rightarrow g|B\,(x_1 \ldots x_n) \in B]}$ is trivially true, since $\emptyset$ has no members whatsoever.

We need something else for 0-ary operations. Let f be any 0-ary operation: $\{\emptyset\}$ to A. $\emptyset$ is closed under g iff $\exists x$ [$g = \{<\emptyset,x>\}$ & $x \in \emptyset$]. Since $\sim\exists x$ [$x \in \emptyset$], $\emptyset$ is not closed under any 0-ary operation.