An interesting erroneous answer to the A^0 question.

Note from BHP: When I read this initially, I found it convincing. Only when later homeworks started coming in did I realize that the conclusion being argued for is not the one we wanted! We didn't want an argument that $\mathbf{A^0 = \varnothing}$, but that $\mathbf{A^0 = \{\varnothing\}}$.

So here's a new exercise: can you find any error in this proof? Or should we have decided that $A^0 = \varnothing$ (though that answer would mess up the nice cardinality generalization that $|A^n| = |A|^n$) ?

1. $\mathbf{A^0 = \varnothing}$. Why? Can you find any way of thinking about this to make an non-arbitrary decision?

For any $n \in \mathbf{N}$, A^n is the set of all ordered n-tuples that can be formed by combining elements of A (for instance, $A^2 = A \times A$, that is, the set of ordered pairs <x,y> such that $x \in A$ and $y \in A$)

Therefore, A^0 is the set of all ordered 0-tuples that can be formed by combining elements of A. But what is a 0-tuple? We know that a pair consists of two elements, a triple consists of three elements, and so on for any $n \in \mathbf{N}$. Hence, a 0-tuple will consist of zero elements.

For any set A, no possible combination of the elements of A will give us a 0-tuple. For instance, let A= {a,b}. Any possible combination of a with an element of A will, at the very least, contain a, and hence it will not be a 0-tuple. And the same goes for b.

Therefore, for any set A, A^0 will not have any elements. And the set that does not have any elements is the empty set. Hence, $A^0 = \varnothing$