Added note on homomorphisms to 1-element algebras: Note by BHP extending an observation made on one of the homework papers for HW 4.

As noted by one student, "since a homomorphism doesn't have to be onto [true of our definition in the handout, not true of PtMW definition], any mapping from all elements in the carrier set of the first algebra to I in the second algebra is a kind of "trivial" homomorphism." E.g. map each of I, R', H, V in that 4-element algebra on to I in any of the other 4-element algebras [or any of the 2-element algebras, or in the 1-element algebra whose carrier is just {I}, or in the original 8-element algebra] and the result will be a homomorphism.

Good point. Not *all* kinds of algebras will necessarily have an "I", though.

That will always work for groups, which is what our algebras of the symmetries of the square are. All groups will have an "I" element, and all groups will contain a subgroup whose carrier is just that "I" element. Wherever you can find a proper subgroup, you can potentially find a homomorphism "into" by targeting just that subgroup. (It will in fact be a homomorphism *onto* the corresponding subgroup.)

It won't always work for lattices, because lattices don't have to have anything with the properties of a "1" or an I (an identity element for the given operation).

And for Boolean algebras, which always have to have a 0 and a 1, it's a little tricky. There will always be a subalgebra which has just the 0 and the 1, but to get a homomorphism from an arbitrary Boolean algebra onto just the {0,1} subalgebra of a second Boolean algebra, you have to figure out which elements to map onto 0 and which onto 1.

Another student found several of the "into" homomorphisms, including the one where everything is mapped onto I, but also noticed that the answer pages in the book didn't allow these. Again, that's because the book's definition of homomorphism requires "onto", and ours (and all current algebra books, apparently) does not.

Can there be 1-element Boolean algebras?

By the way, I may have written on some homework papers or said in class that the smallest Boolean algebra has 2 elements, a 0 and a 1. But something I was reading a couple of days ago pointed out that nothing requires the 0 and the 1 to be distinct, and therefore you can in principle get a one-element Boolean algebra. Can you find one?

I did say in class that all finite Boolean algebras are isomorphic to a power-set algebra. Does that help you find a one-element one? What do you know about how the cardinality of a power set $\wp$(S) is related to the cardinality of S?