Homework 5: pp.51-3, #4,5

Answers

(4) Faulty reasoning:
A relation $R$ is reflexive if for all $x \in \text{dom}(R)$, $<x,x> \in R$.
The reasoning does not allow for the case that there is some $x \in \text{dom}(R)$ s.t. for all $y \in \text{range}(R)$, $<x,y> \notin R$ (i.e. $x$ bears no relation in $R$).
For example,
- $S$ is the set of humans
- $R$ is a relation defined on $S$ such that $aRb$ iff $a$ and $b$ have the same parents and those parents have at least two children.
For an only child $x$, $x$ does not bear any relation to any other human, including him/herself. Therefore $R$ is not reflexive, though it is symmetric and transitive.

(5)
(a) $R = \{ <1,1>, <2,2>, <3,3>, <5,5>, <6,6>, <10,10>, <15,15>, <30,30>,$
    $<1,2>, <1,3>, <1,5>, <1,6>, <1,10>, <1,15>, <1,30>,$
    $<2,6>, <2,10>, <2,30>,$
    $<3,6>, <3,15>, <3,30>,$
    $<5,10>, <5,15>, <5,30>,$
    $<6,30>, <10,30>, <15,30> \}$
- $R$ is a weak order if it is reflexive and antisymmetric.
  (i) $R$ is reflexive: for all $x \in \text{dom}(R)$, $<x,y> \in R$
  (ii) $R$ is antisymmetric: for all $<x,y> \in R$ where $x \neq y$, $<y,x> \in R$.
- $R$ is a partial order if it is unconnected.
  (i) $R$ is connected if for every two distinct $x,y$, $<x,y> \in R$ or $<y,x> \in R$.
  $<3,10> \notin R$, $<10,3> \notin R$.
(b) 1 is minimal and least.
  - No element precedes 1.
  - 1 precedes all other elements.
30 is maximal and greatest.
  - No element follows 30
  - 30 follows all other elements.

PTO for diagrams…
Here is a predecessor diagram.

Note: the book actually asked for an immediate predecessor diagram. Such a diagram looks like this:

Immediate predecessors of \( x \) are all \( y \) s.t. there is no \( z \) s.t. \( y < z < x \).
For example, 1 is not an immediate predecessor of 30 because \( 1 < 10 < 30 \).
Note that no element can be an immediate predecessor of itself.

(c) Do the same for the set \( \mathcal{P}(\{a,b,c\}) \) and the relation “is a subset of”.

(i) \( \mathcal{P}(\{a,b,c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,c\}, \{a,b\}, \{b,c\}, \{a,b,c\}\} \)
(ii) \( R = \{\langle \emptyset, \emptyset \rangle, \langle \emptyset, \{a\}\rangle, \langle \emptyset, \{b\}\rangle, \langle \emptyset, \{c\}\rangle, \langle \emptyset, \{a,c\}\rangle, \langle \emptyset, \{a,b\}\rangle, \langle \emptyset, \{b,c\}\rangle, \langle \emptyset, \{a,b,c\}\rangle, \langle \{a\}, \{a\}\rangle, \langle \{a\}, \{a,c\}\rangle, \langle \{a\}, \{a,b\}\rangle, \langle \{a\}, \{a,b,c\}\rangle, \langle \{b\}, \{b\}\rangle, \langle \{b\}, \{b,c\}\rangle, \langle \{b\}, \{a,b\}\rangle, \langle \{b\}, \{a,b,c\}\rangle, \langle \{c\}, \{c\}\rangle, \langle \{c\}, \{a,c\}\rangle, \langle \{c\}, \{a,c\}\rangle, \langle \{c\}, \{a,b,c\}\rangle, \langle \{a,b\}, \{a,b\}\rangle, \langle \{a,b\}, \{a,b,c\}\rangle, \langle \{a,c\}, \{a,c\}\rangle, \langle \{a,c\}, \{a,b,c\}\rangle, \langle \{b,c\}, \{b,c\}\rangle, \langle \{b,c\}, \{a,b,c\}\rangle, \langle \{a,b,c\}, \{a,b,c\}\rangle \} \)

- \( R \) is a weak order if it is reflexive and antisymmetric.
  (i) \( R \) is reflexive: for all \( x \in \text{dom}(R) \), \( \langle x, y \rangle \in R \)
  (ii) \( R \) is antisymmetric: for all \( \langle x, y \rangle \in R \) where \( x \neq y \), \( \langle y, x \rangle \in R \).
- R is a partial order if it is unconnected.
  
  (i) R is connected if for every two distinct x, y, \( x,y \in R \) or \( y,x \in R \).
  
  \( \{b\}, \{a,c\} \not\in R \), \( \{a,c\}, \{b\} \not\in R \).