Introduction to Mutual Funds
Basic Portfolio Mathematics

Week 3:
An Example of A Mutual Fund

• The largest actively managed mutual fund is the Fidelity Magellan Fund (FMAGX), with current assets of $55 B (compared with $76.885 billion on 31/1/2002). The fund has been in existence since May 1963. From 9/30/1997, it has been closed to new investors.

• What type of a fund is it? It invests in large caps, and blend of growth and value.

• Given its style, what should its benchmark be?
  – The appropriate benchmark, because of its emphasis on large caps, is the S&P 500.

• What kind of stocks would you buy if you were the manager of Magellan?
Magellan’s Stock Holding on 9/30/02

- 1. GENERAL ELECTRIC CO (3.93%)
- 2. CITIGROUP INC (3.54%)
- 3. VIACOM (3.52%)
- 4. AMER INTL GROUP INC (3.34%)
- 5. MICROSOFT (3.24%)
- 6. WAL MART STORES (2.83%)
- 7. PFIZER INC (2.59%)
- 8. EXXON MOBIL CORP (2.49%)
- 9. WELLS FARGO (2.19%)
- 10. BANK OF AMERICA (2.05%)
### Magellan vs. S&P 500 Top Ten

<table>
<thead>
<tr>
<th>Stock</th>
<th>S&amp;P 500</th>
<th>Magellan</th>
<th>P/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSFT</td>
<td>3.43%</td>
<td>3.24%</td>
<td>27.25</td>
</tr>
<tr>
<td>GE</td>
<td>3.03%</td>
<td>3.93%</td>
<td>15.16</td>
</tr>
<tr>
<td>XOM</td>
<td>2.85%</td>
<td>2.49%</td>
<td>15.35</td>
</tr>
<tr>
<td>WMT</td>
<td>2.72%</td>
<td>2.83%</td>
<td>27.05</td>
</tr>
<tr>
<td>PFE</td>
<td>2.32%</td>
<td>2.59%</td>
<td>15.25</td>
</tr>
<tr>
<td>C</td>
<td>2.23%</td>
<td>3.54%</td>
<td>12.99</td>
</tr>
<tr>
<td>JNJ</td>
<td>1.96%</td>
<td></td>
<td>23.91</td>
</tr>
<tr>
<td>AIG</td>
<td>1.88%</td>
<td>3.34%</td>
<td>17.03</td>
</tr>
<tr>
<td>IBM</td>
<td>1.88%</td>
<td></td>
<td>25.25</td>
</tr>
<tr>
<td>MRK</td>
<td>1.56%</td>
<td></td>
<td>17.33</td>
</tr>
<tr>
<td>VIA</td>
<td></td>
<td>3.52</td>
<td>44.12</td>
</tr>
<tr>
<td>BAC</td>
<td></td>
<td>2.83</td>
<td>11.59</td>
</tr>
<tr>
<td>WFC</td>
<td></td>
<td>2.19</td>
<td>14.51</td>
</tr>
</tbody>
</table>
Magellan vs. S&P 500 vs. Average Fund in Group (Year end Sep 30, 2002)

• Last 1 year:
  – Magellan = -27.84%.
  – S&P 500 = -28.36%.

• Last 5 years.
  – Magellan = -21.06%.
  – S&P 500 = -20.49%.

• Last 10 years.
  – Magellan = -6.06%.
  – S&P 500 = -7.88%.

• But Fidelity Magellan charges a “front-end load” – a fee of 3% for entering the fund. The 1, 5, 10 year returns after the load are: **-30.00%**, **-23.43%**, **-8.88%**. So, after accounting for the load, Magellan underperforms the S&P 500 over each of these periods.

What Magellan Charges for Managing Your Money (1/2)

• According to its annual report, 9/30/2002:
• Management fee
  Basic fee = $190,661,000.
  Performance adjustment = -$388,000.
• Besides the management fee, the fund will charge other operational expenses. Total expenses, including management fee, added up to: 251,105,000.
• But 2002 was a bad year, and the fund has shrunk in size compared to the previous year. Thus, the expenses were much lower compared to those in the previous year:
• From the annual report, 3/31/2001:
  – Management fee
    Basic fee = $571,113,000.
    Performance adjustment = $139,203,000.
    Total expenses = $872,538,000.
What Magellan Charges for Managing Your Money (2/2)

• According to the prospectus, the fund charges a management fee of 0.69%, and additional other expenses of 0.20%. The total expense ratio is 0.89%.

• If Magellan was open to new investment, it could have charged an additional fee, if necessary, called the 12B-1. This fee could be used for marketing purposes.

• Finally, Magellan can charge a “load” – either a front-end or a back-end load. Magellan has a 3% front-end load.
<table>
<thead>
<tr>
<th>Fidelity Magellan Year Ending 31/3/2002</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NAV (Beginning of Year)</strong></td>
<td>143.26</td>
</tr>
<tr>
<td><strong>Investment Operations</strong></td>
<td></td>
</tr>
<tr>
<td>Net Investment Income</td>
<td>0.37</td>
</tr>
<tr>
<td>Net Realized and Unrealized Gain on Investment</td>
<td>-34.17</td>
</tr>
<tr>
<td><strong>Distributions</strong></td>
<td></td>
</tr>
<tr>
<td>Dividends</td>
<td>-0.27</td>
</tr>
<tr>
<td>Distributions from Capital Gains</td>
<td>-4.69</td>
</tr>
<tr>
<td><strong>NAV (End of Year)</strong></td>
<td>$104.50</td>
</tr>
<tr>
<td><strong>Ratios/Supplemental Data</strong></td>
<td></td>
</tr>
<tr>
<td>Ratio of Total Expenses to Average Net Asset</td>
<td>0.89%</td>
</tr>
<tr>
<td>Ratio of Net Investment Income to Average Net Assets</td>
<td>0.59%</td>
</tr>
<tr>
<td>Portfolio Turnover Rate</td>
<td>24%</td>
</tr>
</tbody>
</table>
Understanding the Numbers (1/2)

• New NAV: Old NAV + investment income + net realized and unrealized gain - all distributions.

• **Distributions:** To avoid taxation at the fund level, the fund must pass on any dividend or capital gains directly to the investors. The investors will now pay tax at their personal rate on both the dividends and capital gains. Fidelity has distributed $4.96 per share.

• **Expense Ratio:** This summarizes the operating expenses of the fund as a fraction of its NAV. The Magellan Fund has an expense ratio of 0.89%. This is comparable with other funds, but appear high relative to its size. As we saw, this is equal to $251 and $872 million in 2002 and 2001, respectively.
The Types of Fees Charged by Funds: Loads and Fees

• Loads: front end or back end
• Fees:
  – Management Fee
  – 12B-1 Fees
  – Other expenses
Loads

- **Front End Load**: A commission or sales charge paid when the shares of the fund are purchased. For example, Oppenheimer Funds have a typical front end load of 5.75% for their Class A shares.

- **Back End Load**: This is a redemption or exit fee that is paid when the funds are withdrawn. For example, Oppenheimer charges a 5% back end fee for its Class B shares, that decrease by 1% each year, and is eliminated from 6th year onwards.
Fund Fees: Operating Expenses (2/2)

- **Annual Fund Operating Expenses**: Management Fee + 12b-1 + Other operating expenses.

- **12b-1 Charges**: The fund may charge a 12b-1 fee for marketing and advertising expenses, as well as commissions paid to brokers that sell the fund. This is in addition to front-end/back-end load. Oppenheimer charges a 12b-1 fee of 1% for both its Class B and C shares, and 0.23% for its Class A shares.

- **Management Fee**: This is a fee paid for management of funds. For Oppenheimer, it starts at 0.75% for the first $200 M, 0.69% for next $200M, declining to 0.54% on assets over $4.5B.

- **Other Expenses**: In addition, there are other expenses.

- Thus, for year ending 8/2002 (8/2001), the total operating expenses for this fund were 1.31% (1.06%) for its Class A shares, and 2.08% (1.84%) for B and C shares.
  - Oppenheimer converts Class B shares to Class A shares after 6 years.
Some Additional Notes on Calculation of Expenses and Loads

• *Back-End Load/Contingent Deferred Sales Load:*
  - (1) It is calculated as the *lesser* of the amount that represents a specified percentage of NAV at the time of purchase, or at the time of redemption.
  - (2) It is not applied on shares purchased through reinvestment of dividends or capital gains distributions.
  - (3) It is calculated as if shares that are not subject to a load are redeemed first.
  - (4) Shares are redeemed in the order purchased, unless some other order can result in a lower redemption fee.

• *Operating Expenses:* It is applied daily as fraction of NAV.
Fund Fees: Loads (1/2):

- **Oppenheimer Growth Fund.**

- **Front End Load:** A commission or sales charge paid when the shares of the fund are purchased. For example, Oppenheimer Funds have a typical front end load of 5.75% for their Class A shares.

- **Back End Load:** This is a redemption or exit fee that is paid when the funds are withdrawn. For example, Oppenheimer charges a 5% back end fee for its Class B shares, that decrease to 1% and is eliminated from 6th year onwards.
  - Oppenheimer’s Class B shares are converted automatically to Class A shares at the end of the 6th year.
Impact of Costs on Investment Performance (1/5)

• Let us calculate the impact of the fees on the investor’s return. We will assume the structure of the fees is similar to that of Oppenheimer.

• Consider an investor who starts with $10,000, and can choose between investing in either A, B or C class of shares. Suppose the investor expects that the fund will earn an average of 15% return every year, before expenses. Which class of shares should he invest in?

• Let us calculate the net return to the investor after costs for different investment horizons.
Impact of Costs on Investment Performance (2/5)

• Class A: 1-Year Horizon.
• Front End Load of 5.75%, total operating expenses 1.01% (12b-1 fee of 0.25%, management fee of 0.63%, other operating expenses of 0.13%).
• Original investment = $10,000.
• Amount invested into fund on 1/1/2002 after front-end load = 10,000(1 - 0.0575) = 9,425.
• Total return before expenses = 15%.
• Return after expenses of 1.05% = 15 - 1.01 = 13.99%.
• Value of investment on 12/31/2002 = 9425(1 + 0.1399) = 10,743.56.
• Net return over 1-year = 7.40%.
Impact of Costs on Investment Performance (3/5)

• Class B: 1-Year Horizon:

• Back End Load of 5.0%, total operating expenses 1.78%. Original investment = $10,000.

• Amount invested into fund on 1/1/2002 = $10,000.

• Total return before expenses = 15%.

• Return after expenses = 15-1.78=13.22%.

• Value of investment on 12/31/2002 before back-end load = 10000(1+0.1322)=11,322.

• If we assume that the backend load is applied to the initial amount of $10,000 –.
  – Value of investment after back-end load of 5% = 11322 - 0.05x10000 = 10,822.

• Net return over 1-year = 8.22%.

• *If we assume that the load applies to the final amount, then the value of the fund will be 11322x(1-0.05)=10756, or you will earn 7.56%.
Impact of Costs on Investment Performance (4/5)

• Class C: *1-Year Horizon:*
  • No front end load, total operating expenses 1.78%, back-end load of 1% in first year.
  • Original investment = $10,000.
  • Amount invested into fund on 1/1/2002 = $10,000.
  • Total return before expenses = 15%.
  • Return after expenses of 1.05% = 15-1.78=13.22%.
  • Value of investment at year-end before back-end load = 10000(1+0.1322)=11,322.
  • Value of investment after back end load of 1% applied to initial investment* = 11322 - 0.01x10,000 = 11222.
  • Net return over 1-year = 12.22%%.

* (*If the backend load is applied to ending amount, then the value of the investment is 11322x0.99 = $11,209, so that the net return is 12.09%).
Comparing Performance Across Share Classes

<table>
<thead>
<tr>
<th>Investment</th>
<th>Annual Returns</th>
<th>Operating Expenses</th>
<th>Front End</th>
<th>Back End</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>12.00%</td>
<td>Class A</td>
<td>Classes B&amp;C</td>
<td>5.75%</td>
</tr>
<tr>
<td></td>
<td>1.01%</td>
<td>1.78%</td>
<td></td>
<td>4.00%</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>3.00%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.00%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00%</td>
</tr>
</tbody>
</table>

Assumptions: Class B convert to Class A after 6th year.

<table>
<thead>
<tr>
<th>Horizon(Yr)</th>
<th>Class A</th>
<th>Class B</th>
<th>Class C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>104.608075</td>
<td>105.22</td>
<td>109.22</td>
</tr>
<tr>
<td>3</td>
<td>128.8643873</td>
<td>130.9001983</td>
<td>133.9001983</td>
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<td>5</td>
<td>158.7452049</td>
<td>161.6679649</td>
<td>162.6679649</td>
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<tr>
<td>6</td>
<td>176.191303</td>
<td>179.292631</td>
<td>179.292631</td>
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<tr>
<td>10</td>
<td>267.3744307</td>
<td>272.0807686</td>
<td>264.6086682</td>
</tr>
<tr>
<td>20</td>
<td>758.5048931</td>
<td>771.8561336</td>
<td>700.1774727</td>
</tr>
</tbody>
</table>
Yet Another Example

• Vanguard is a large fund family that is particularly known for its passive funds.
• However, it also has active funds - see the annual report on Vanguard’s large cap growth fund:
  – Its expenses are lower than average, but so are its returns!
    • Moral: Lower expenses by themselves are not a reason to buy active managed funds.
Passive Funds

• Passive funds have much lower expenses as they are simply trying to replicate an index, and thus do not require costly support staff.

• Moreover, fund returns relative to the benchmark are very sensitive to expenses, and thus there is additional pressure to keep expenses under control.

• As an example, let us consider the Vanguard Index Trust 500 Fund (VFINX).
  – Net Assets = $72.3B.
  – Management fee = 16 bps (0.16%).
  – Total expenses = 18 bps.
  – Return before taxes: -12.02% (1 yr), 1.06% (3 yrs), 10.66% (5 yrs), 12.84% (10 yrs).
  – The next slide provides a comparison with the S&P 500.
<table>
<thead>
<tr>
<th>Year Ended</th>
<th>Capital Return</th>
<th>Income Return</th>
<th>Total Return</th>
<th>S&amp;P 500 Total Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>-23.36%</td>
<td>1.22%</td>
<td>-22.15%</td>
<td>-22.10%</td>
</tr>
<tr>
<td>2001</td>
<td>-13.11%</td>
<td>1.08%</td>
<td>-12.02%</td>
<td>-11.89%</td>
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<tr>
<td>2000</td>
<td>-9.95%</td>
<td>0.90%</td>
<td>-9.06%</td>
<td>-9.10%</td>
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<tr>
<td>1999</td>
<td>19.70%</td>
<td>1.37%</td>
<td>21.07%</td>
<td>21.04%</td>
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<tr>
<td>1998</td>
<td>27.00%</td>
<td>1.61%</td>
<td>28.62%</td>
<td>28.58%</td>
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<tr>
<td>1997</td>
<td>31.11%</td>
<td>2.08%</td>
<td>33.19%</td>
<td>33.36%</td>
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<tr>
<td>1996</td>
<td>20.53%</td>
<td>2.35%</td>
<td>22.88%</td>
<td>22.96%</td>
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<tr>
<td>1995</td>
<td>34.35%</td>
<td>3.09%</td>
<td>37.45%</td>
<td>37.58%</td>
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<tr>
<td>1994</td>
<td>-1.51%</td>
<td>2.69%</td>
<td>1.18%</td>
<td>1.32%</td>
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<tr>
<td>1993</td>
<td>7.06%</td>
<td>2.84%</td>
<td>9.89%</td>
<td>10.08%</td>
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<td>4.45%</td>
<td>2.97%</td>
<td>7.42%</td>
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<td>1991</td>
<td>26.28%</td>
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<td>30.22%</td>
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<td>1990</td>
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<td>3.52%</td>
<td>-3.32%</td>
<td>-3.10%</td>
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<tr>
<td>1989</td>
<td>26.67%</td>
<td>4.70%</td>
<td>31.36%</td>
<td>31.69%</td>
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<td>1988</td>
<td>11.55%</td>
<td>4.67%</td>
<td>16.22%</td>
<td>16.61%</td>
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<tr>
<td>1987</td>
<td>2.27%</td>
<td>2.43%</td>
<td>4.71%</td>
<td>5.26%</td>
</tr>
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<td>1986</td>
<td>14.04%</td>
<td>4.02%</td>
<td>18.06%</td>
<td>18.68%</td>
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<tr>
<td>1985</td>
<td>26.09%</td>
<td>5.14%</td>
<td>31.23%</td>
<td>31.75%</td>
</tr>
<tr>
<td>1984</td>
<td>1.54%</td>
<td>4.68%</td>
<td>6.21%</td>
<td>6.27%</td>
</tr>
</tbody>
</table>
Exchange Traded Funds (ETF)

- Although passive funds can be bought directly from the fund family, a recent innovation is to list a passive fund as an “Exchange Traded Fund” – the fund’s shares trade continuously on an exchange. (In principle, ETF can be for both active as well as passive funds.).
  - The fund’s price tracks the NAV because the ETF allows for redemptions.

- Exchange traded funds also have low expenses – Barclay’s charges about 9 bps (0.09%)!
  - The fund saves on marketing costs, as ETF’s are listed on an exchange, and thus can be bought and sold like a regular stock. Mostly traded on the AMEX:

- **Examples:** Barclay’s Ishares, Vanguard’s VIPER, SPDRs (S&P’s Depository Receipts), WEBS (World Equity Benchmark Shares), QQQ (called “cubes,. track the NASDAQ 100).
Exercises

• Please attempt all numerical exercises from the back of Chapter 4.
Chapter 8 (See Also Chapters 5-7)

Basic Portfolio Mathematics
Road Map

1. Averaging: Geometric vs Arithmetic.
2. Calculation of Portfolio Returns and Variances.
3. Introduction to Asset Allocation.
Estimating the Mean Return (1/6)

- We can estimate the mean return in two ways: Arithmetic Mean and Geometric Mean.
- Suppose you want to estimate the mean return over the last three years, when the returns were $r_1$, $r_2$, and $r_3$.
- Arithmetic Average = $(r_1 + r_2 + r_3)/3$
- Geometric Average = $\left[(1+r_1)(1+r_2)(1+r_3)\right]^{1/3}-1$
- Note that the above method to calculate the geometric average is better than estimating is as $[(r_1)(r_2)(r_3)]^{1/3}$
Consider the following examples:

1. \( r_1 = r_2 = r_3 = 0.10 \)
   - Arithmetic average = \( \frac{0.1 + 0.1 + 0.1}{3} = 0.1 \)
   - Geometric average = \( \left[ (1.1) \times (1.1) \times (1.1) \right]^{1/3} - 1 = 0.1 \)
   - In this case, when all returns are identical, the arithmetic average is equal to the geometric average. In general, this is not true.
Arithmetic Vs Geometric (3/6)

2. \( r_1=0.10, \ r_2=0.15, \ r_3=0.05. \)

- Arithmetic average = \( (0.10+0.15+0.05)/3=0.10. \)
- Geometric Average = \( [(1.10)(1.15)(1.05)]^{1/3}-1=0.09924 \)

- The arithmetic average is greater than the geometric average.

- Qt: which average to use?
The Difference Between Geometric and Arithmetic Average (4/6)

• There are two points to note:
  • 1. The arithmetic average return will be always greater than or equal to the geometric average return.
  • 2. The difference between the arithmetic and geometric return will depend on the volatility of the return. The greater the volatility, the greater will be the difference in the return. If the volatility is zero (or the returns in every period are the same) then both averages will be the same.*.

• *Approximately, AA - GA = 0.5 (vol^2).
Choice Between Arithmetic and Geometric (5/6)

• 1. If you are simply trying to predict the next period’s return, then the arithmetic average will be, statistically, the better choice.

• 2. If you are trying to calculate the cumulative return over the past 3-year period, the geometric average is better. For example, the arithmetic average of 0.10 estimates the total 3-year return as \((1+0.10)^3-1=33.1\%\), while the geometric average estimates it as \((1+0.09924)^3-1=32.825\%\). In comparison, the exact three year return is \((1.1)(1.15)(1.05)-1=32.825\%\). Thus, if you know the geometric average, you can recover the cumulative return over the period. However, with the arithmetic average you will over-estimate the cumulative return.
Geometric v.s. Arithmetic: Past Historical Returns (6/6)

- The difference between estimates of geometric (GA) and arithmetic average (AA) are quite substantial.
- Here are some estimates over the period 1926-1996.
- 1. Large Cap: AA=12.5%/yr, GA=10.5%/yr.
Volatility and Correlations

- We have already seen that we can easily estimate the volatility and correlation using Excel functions STDEV and CORREL. The variance is defined as the square of the volatility (or standard deviation).

- Similar to the case of the returns, it is conventional to express the volatility in an annual basis.

- Annual Volatility = sqrt(12) [Monthly Volatility].

- Annual Volatility = sqrt(260)[Daily Volatility].

- For example, a daily volatility of 1% implies an annual volatility of about 16%.

- Recently, we have been observing daily fluctuations of about 1.5% - what does that imply about the annual volatility?
Portfolio Return and Variance

• Suppose we have two assets, with weights w1 and w2, respectively.

• The weight of w1 is defined as the ratio of the dollar invested in asset 1, divided by the total $ investment. Thus, if you invest $100 in asset 1 and $400 in asset 2, then w1=0.2 and w2=0.8.

• Portfolio return = w1*r1 + w2*r2.

• Portfolio variance = (w1*w1)*(var of asset 1) + (w2*w2)*(var of asset 2) + 2 (w1)(w2)(correlation)(vol of asset1)(vol of asset 2).

• To get the portfolio volatility, we take the square root of the portfolio variance.
A Digression: On Re-balancing a Portfolio (1/3)

• We have already observed that we can have different portfolios based on the way we choose the weights - in particular, we saw the equal weighted portfolio and the value weighted portfolio.

• Qt: How easy is it to maintain such portfolio weights as market prices change?
Re-Balancing Cap-Weighted Portfolio (2/3)

- Company A: # of shares=100, and Price = 20.
- Company B: # of shares=200, and Price = 40.
- Therefore: \[ w_a = \frac{100 \times 20}{100 \times 20 + 200 \times 40} = 0.20 \]
  \[ w_b = 1 - w_a = 0.80. \]
- Therefore, if you decide to invest a total of $1,000,000 you will buy $200,000 of A and $800,000 of B. Now what happens if the price of A increases by 25% to 25 and price of B remains constant?
- New weights: \[ w_a = 0.2381 \] and \[ w_b = 0.7619. \]
- Do you need to buy or sell new shares of A or B?
Cap vs Equal Weighted (3/3)

- Answer: No. To see why, calculate your dollar investment in each of the stocks, A and B. The 25% return has increased your $ invested in A from $200,000 to $250,000. The $ invested in B remains at $800,000. Therefore, \[ w_a = \frac{250}{250+800} = 0.2381. \]

- This is exactly the weight you wanted - so there is no need for you to buy/sell any additional shares. Thus, it is easy to maintain a cap-weighed portfolio.

- In contrast, you have to actively rebalance a portfolio that has fixed weights, like an equal weighted portfolio.

- Unfortunately, despite this huge drawback of using equal weighted portfolios, when we think of constructing optimal portfolios, we are forced to consider fixed-weights.
Preview of Week 4
Asset Allocation: The Fundamental Question

- How do you allocate your assets amongst different assets?
- Traditionally, we divide the discussion here into two parts:
  1. The allocation between riskfree and a portfolio of risky assets.
  2. The allocation between different risky asset within the portfolio of risky assets.
Asset Allocation: Risky vs Riskless Asset

• One of the first decisions that has to be made is the allocation between the risky and riskless asset.
• Rf = expected return on riskfree asset.
• Rp= expected return on risky asset portfolio.
• Volatility of riskfree asset = 0.
• w1 = proportion in riskfree asset.
• w2 = proportion in risky asset.
• The choice of w1 and w2 will depend on how risk averse you are.
Portfolio of Risky + Riskless Asset

- To calculate the portfolio return and portfolio variance when we combine the risky asset and riskless asset, we can use the usual formulas:
  - Portfolio Return = \( w_1 R_f + w_2 R_p \).
  - Portfolio Volatility = \( w_2 \times (\text{vol of risky asset}) \).
- If we draw a graph of the portfolio return v.s. portfolio volatility (for different weights), it is a straight line. The following graph shows this graph for the case when the mean return for the riskfree asset is 5%, the mean return for the risky asset is 12%, and the volatility of the risky asset is 15%. 
Riskfree Return=0.05, Risky Return=0.12, Vol of Risky Asset=0.15

<table>
<thead>
<tr>
<th>$w_1$ (weight of riskfree)</th>
<th>$w_2$ (weight of risky)</th>
<th>Portfolio Return</th>
<th>Portfolio Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.113</td>
<td>0.135</td>
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<tr>
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<td>0.05</td>
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</tr>
</tbody>
</table>
Portfolio Return v.s. Portfolio Vol

Series 1
Capital Allocation Line (CAL)

- The graph from the previous slide is called the Capital Allocation Line (CAL).

- It has a slope of \((Rp-Rf)/(Vol\ of\ Risky\ Asset)\), and equals the increase in return of the portfolio for a unit increase in volatility. Therefore, it is also called the reward-to-variability ratio.

- The greater the slope the greater the reward for taking risk. Ideally, you want to achieve the highest return per unit risk, so that you choose a risky portfolio that gives you the steepest slope.

- Note that this tradeoff will be essentially determined by the mean return and volatility of the risky portfolio.
The Decisions that an Investor Must Make

• Thus, there are two decisions that an investor must make:
  
• 1. Which is the risky stock portfolio that results in the best risk-return tradeoff?
  
• 2. After making the choice of the risky stock portfolio, how should you allocate your assets between this risky portfolio and the riskfree asset?
  
• Typically, the first objective of a financial advisor is to determine for her clients the appropriate allocation between the risky and riskless assets, and then to choose how the risky portfolio should be constructed.
How to allocate between the riskfree asset and the risky stock portfolio.

- There is no single answer here that is best for *all* investors. Your decision to allocate between the risky asset and the riskfree asset will be determined by your level of risk aversion. The more risk averse you are, the less you will invest in the risky asset.
- Thus, your decision will depend not only on your preferences, but also your age, wealth, etc.
- Although different investors may differ in the level of risk they take, they are also alike in that each investor faces exactly the same risk-return tradeoff.
Some Questions to Think About

• Suppose you are a financial advisor. What kind of asset allocations would you recommend?