Phase Transitions in Parallel Computational Complexity in Statistical Physics

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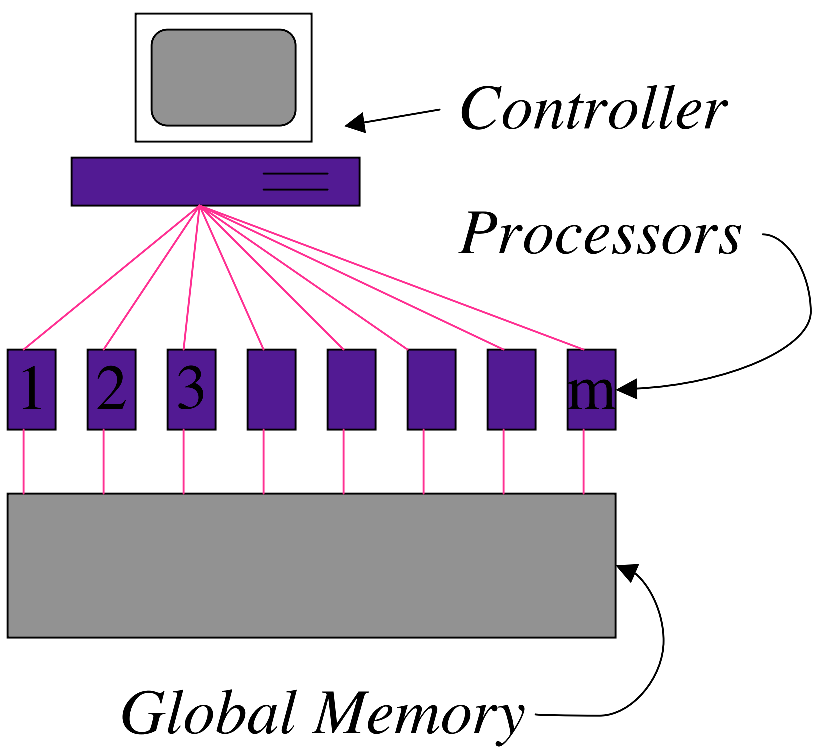
Outline

- Introduction
- Parallel computing and complexity
- Percolation
- Growing networks
- Conclusion

Parallel Random Access Machine

PRAM

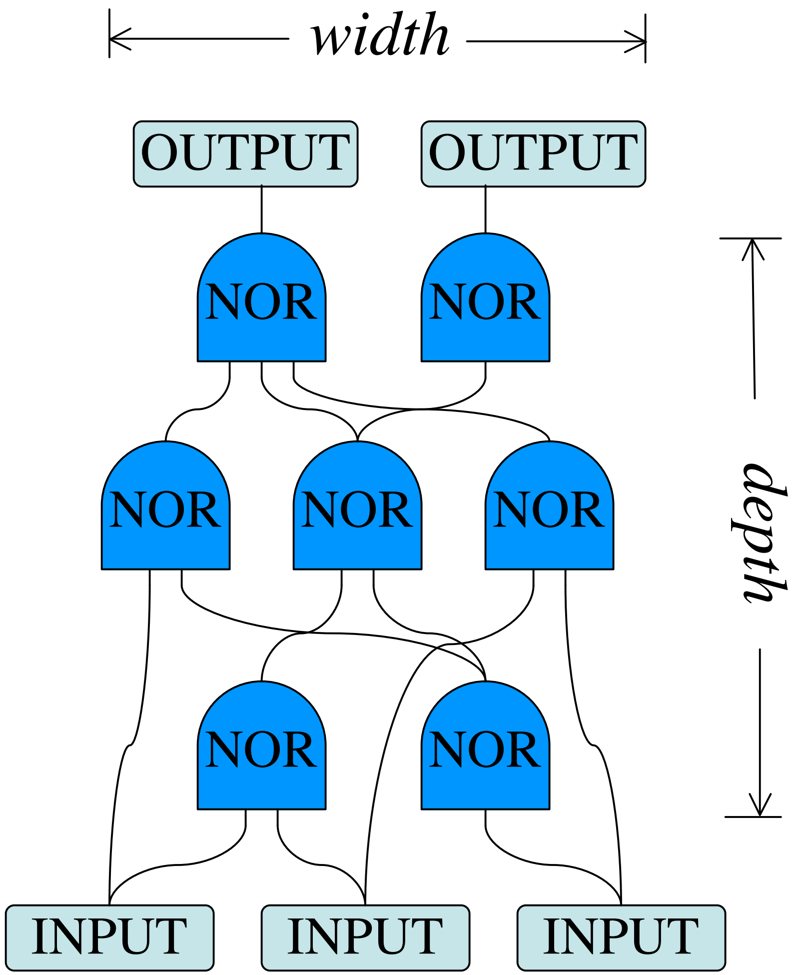


•Polynomially many processors connected to a global memory.

•Processors run synchronously.

•Any processor communicates with any memory cell in a *single time step*.

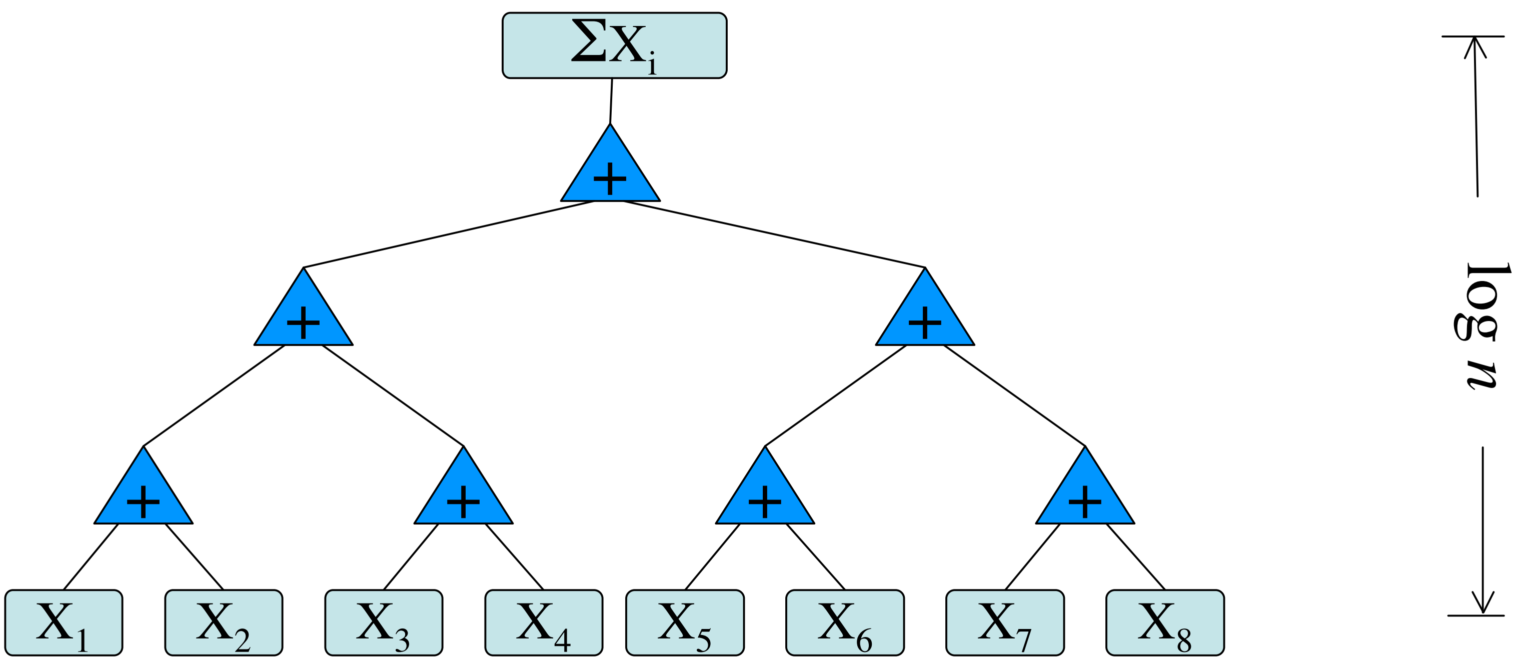
Boolean Circuit Family



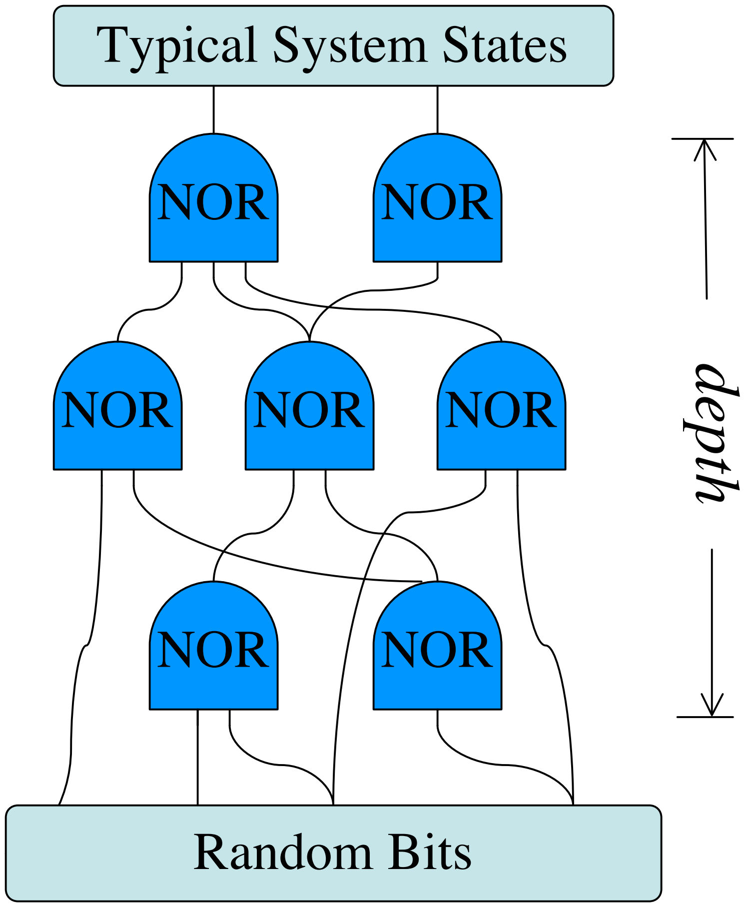
- Gates are evaluated one level at a time from input to output.
- One circuit in the family for each problem size.
- Equivalent to PRAM
  - ***Depth***=number of levels~parallel time
  - *Width*=maximum number of gates in a level~number of processors
  - *Work*=total number of gates

Parallel Computing

Adding n numbers can be carried out in $O(\log n)$ steps using $O(n)$ processors.



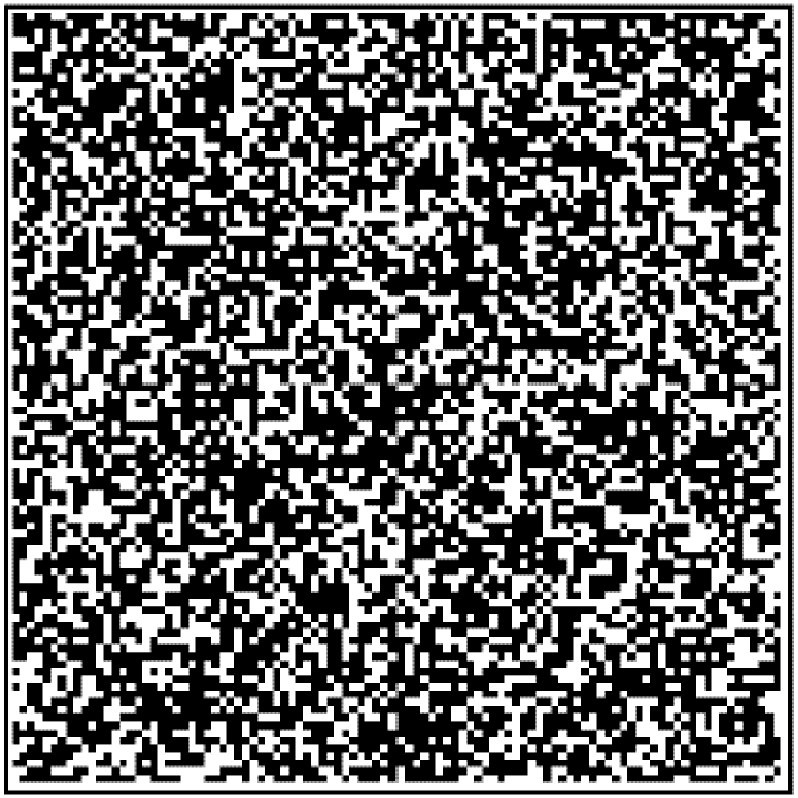
Monte Carlo Simulations and Depth



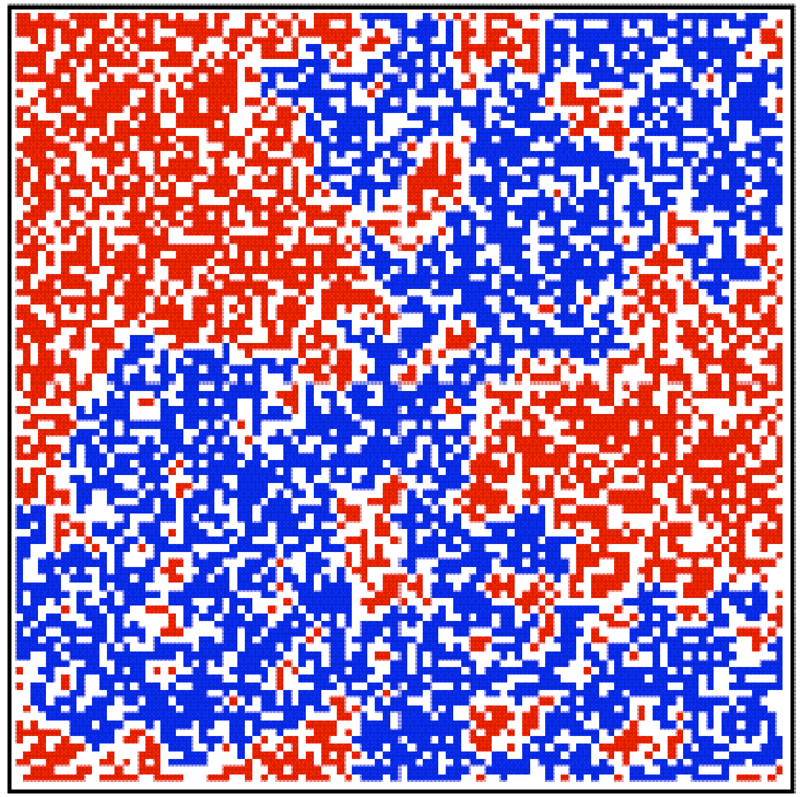
- MC simulations convert random bits into typical system states.
- Depth of a system is the depth of the shallowest circuit (running time of the fastest PRAM program) that generates typical states.
- Depth measures intrinsic history dependence.

Percolation

Occupy sites with probability *p*

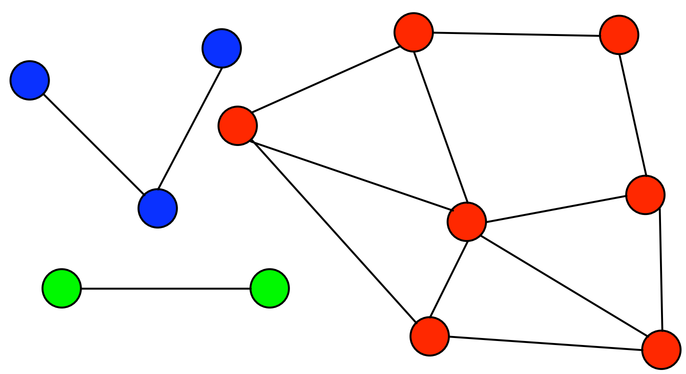


Identify clusters



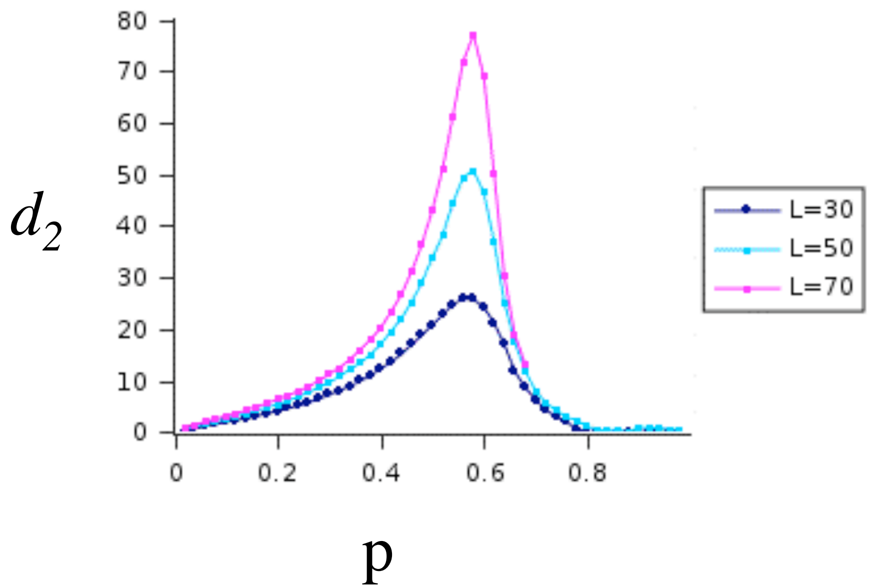
Connected Components

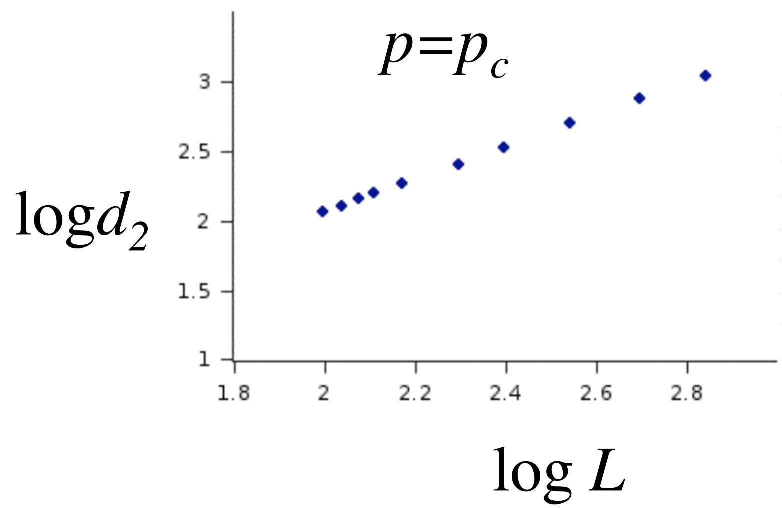
Find the connected components (clusters) of a graph.



Solution can be found in $O(\log d)$ parallel time where d is the diameter of the graph. The diameter is the longest shortest path.

Complexity of Percolation





depth~log L at $p=p_c$

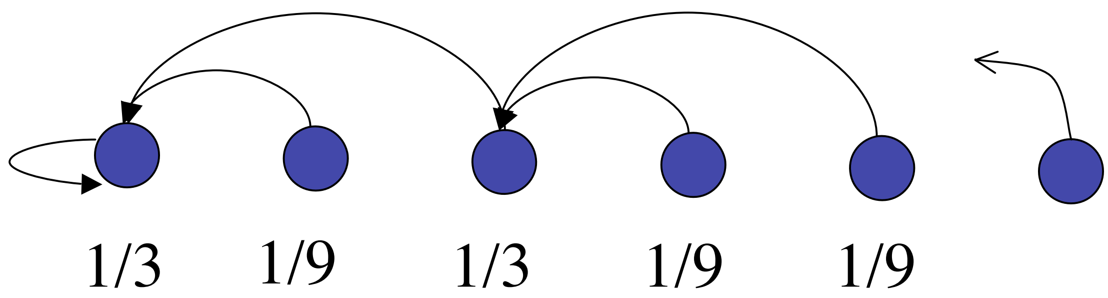
Growing Networks

Barabasi and Albert, Science 286, 509 (1999)
Krapivsky, Redner, Leyvraz, PRL 85, 4629 (2000)

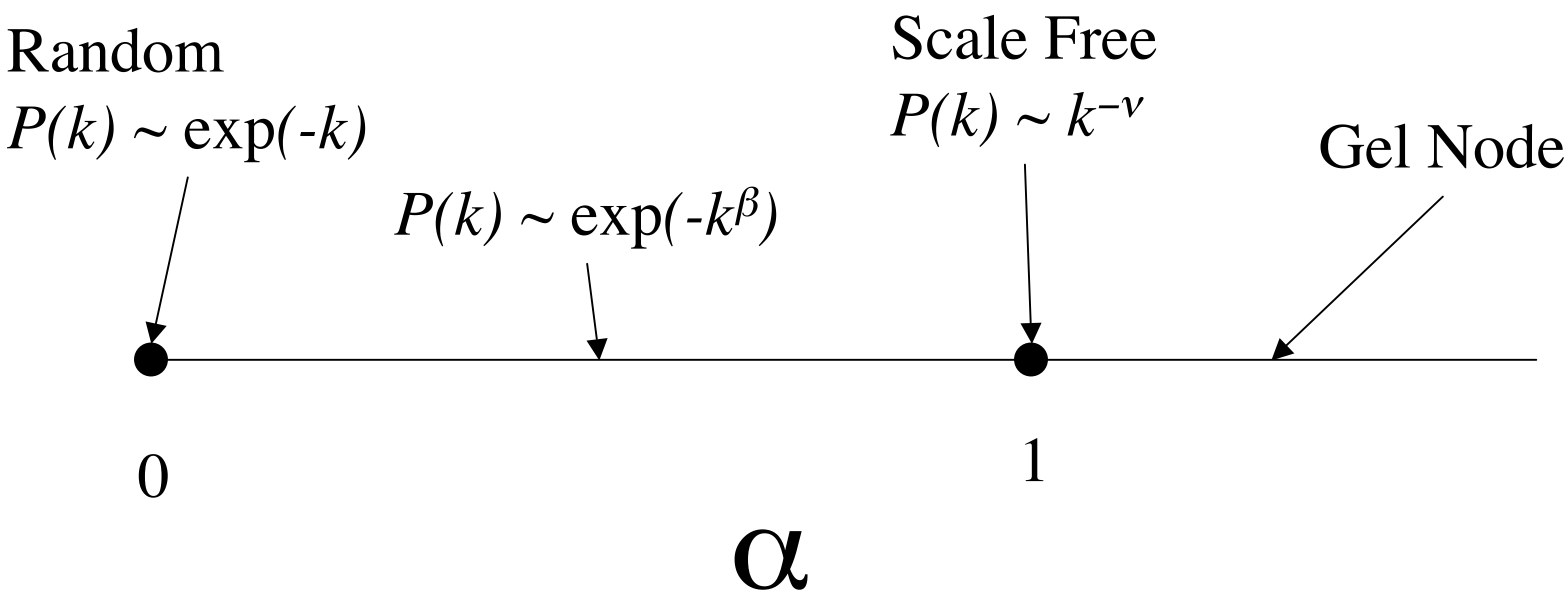
Add nodes one at a time, connecting new nodes to old nodes according to a “rich get richer” preferential attachment rule:

$$\text{Prob}\{\text{t connects to i<t}\} \propto k_i(t)^\alpha$$

where $k_i(t)$ is the number of connection to node i at time t.



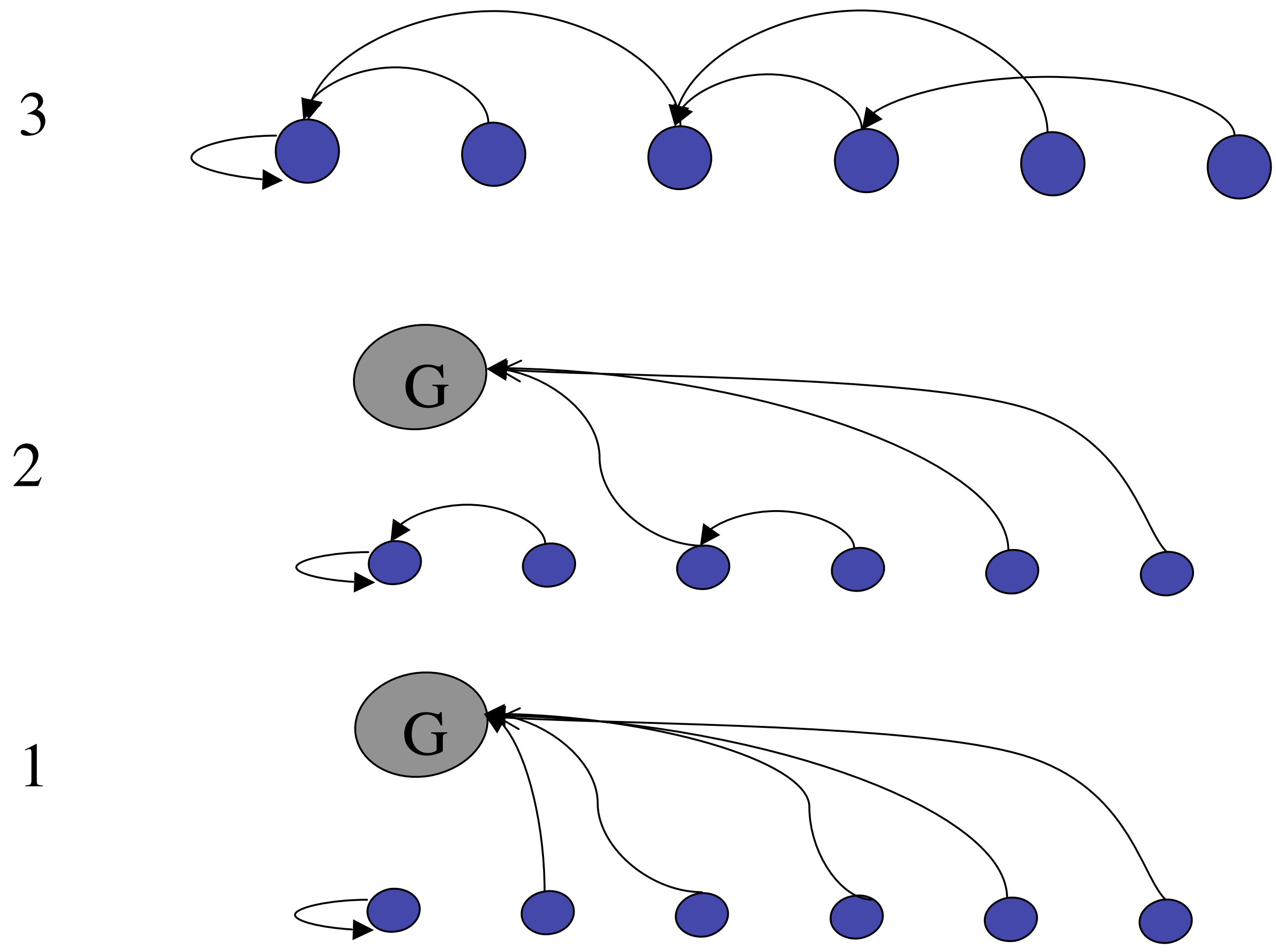
Behavior of Growing Networks



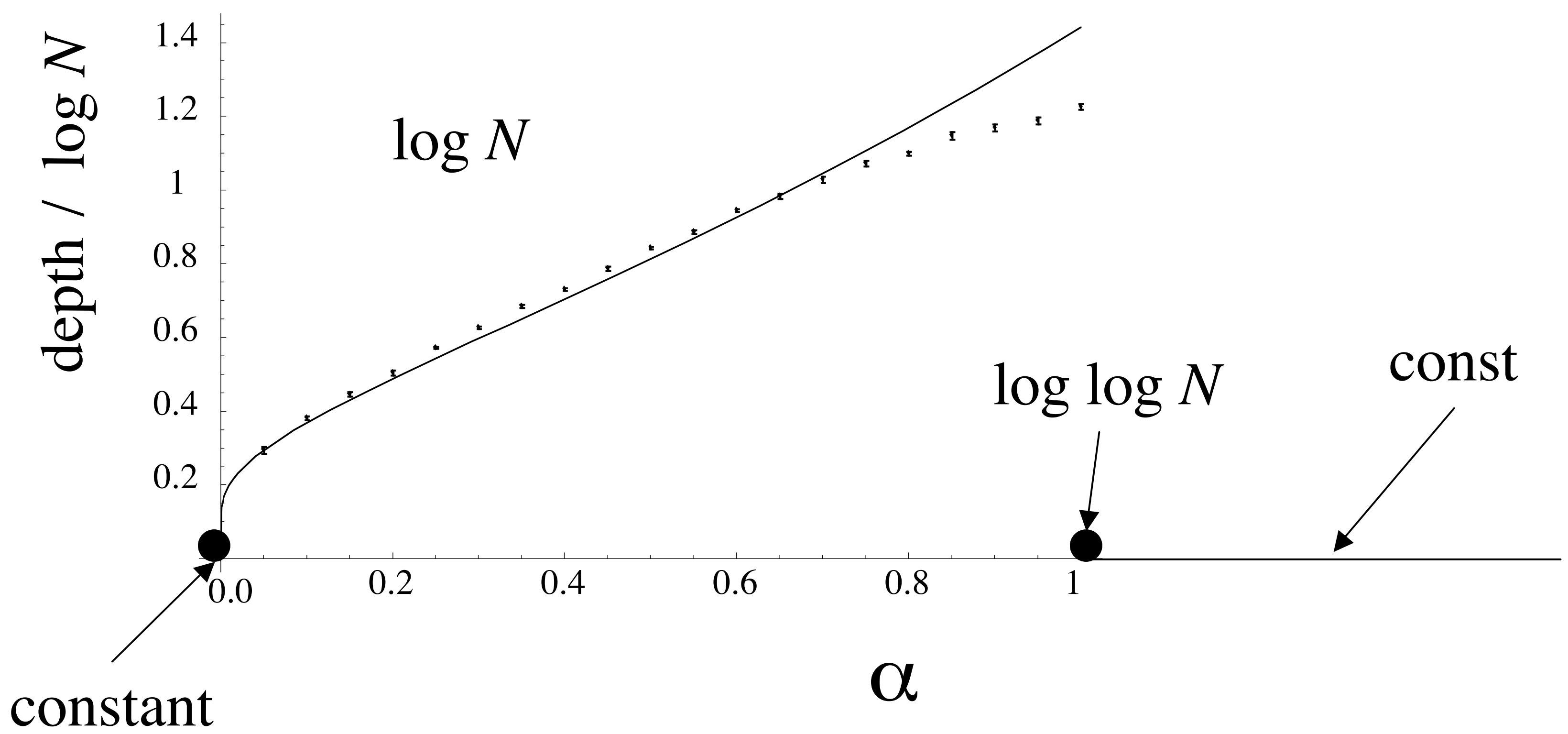
P(k) is the degree distribution

Fast Parallel Algorithm

For the “high temperature” phase $0 \leq \alpha \leq 1$



Complexity of Growing Networks



Conclusions

- Phase transitions in complexity often coincide with structural phase transitions.
- Depth or parallel time rather than computational work is the appropriate way to see many complexity transitions.