The critical point of the random field Ising model

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Supported by NSF

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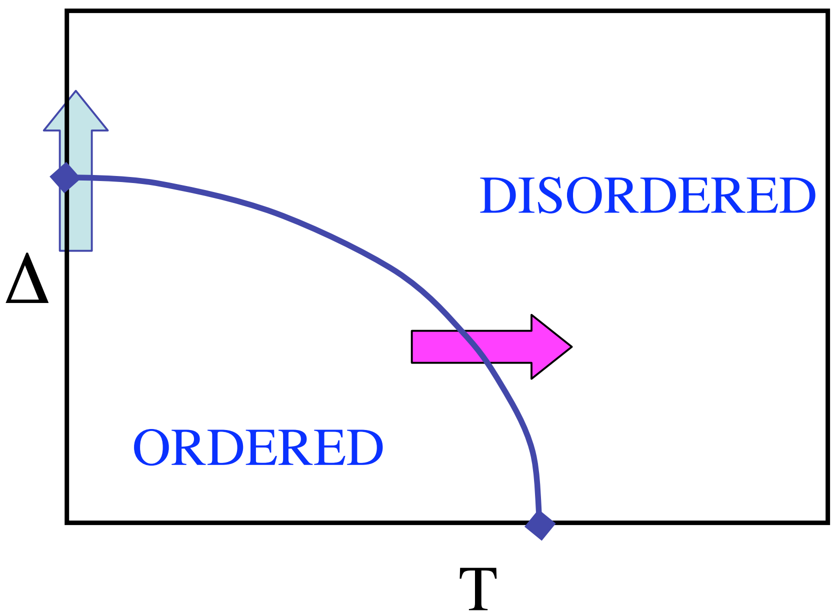
Random field Ising model

$$\mathcal{H} = -\sum_{<i,j>} s_i s_j - \Delta \sum_i h_i s_i - H \sum_i s_i$$

- $s_i = \pm 1$
- h_i quenched Gaussian variables, mean zero and variance one
- 3D cubic lattice

Phase transition



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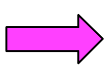
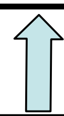
	
free energy	energy, $\mathcal{H}$
energy	bond energy, e
temperature	disorder, Δ

Zero Temperature Fixed Point

On large length scales thermal fluctuations are irrelevant. The transition results from a competition between random fields and couplings.

Specific heat exponent at T=0

Hartmann&Young PRB 64, 214419 (2001)

$$C = \frac{\delta e}{\delta \Delta}$$

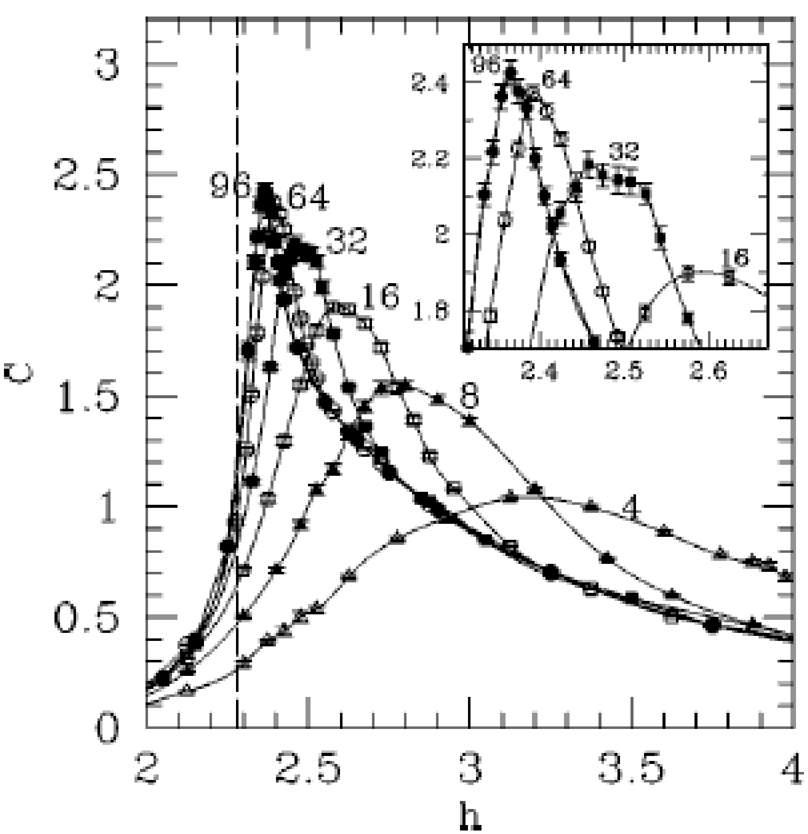
$$C_{\max} = a_1 - a_2 L^{\alpha/\nu}$$

$$\alpha = -0.63 \pm 0.07$$

but modifed hyperscaling predicts

$$\alpha = 2 - (d - \theta)\nu$$

$$\theta \approx 3/2, \nu \approx 4/3 \Rightarrow \alpha \approx 0$$



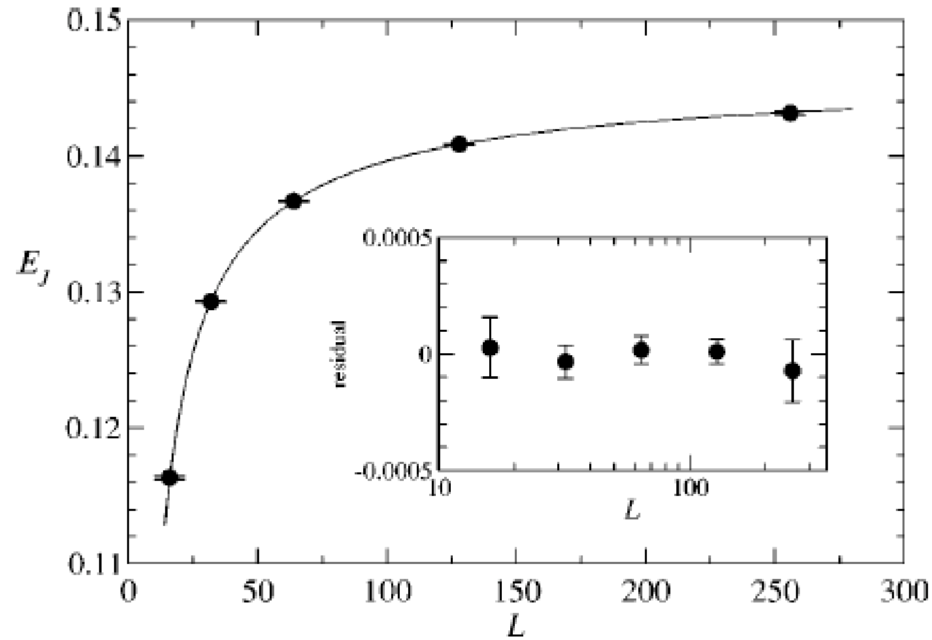
Specific heat exponent at T=0

Middleton&Fisher PRB **65** 134411 (2002)

$$e(\Delta_c) = a_1 - a_2 L^{(\alpha-1)/\nu}$$

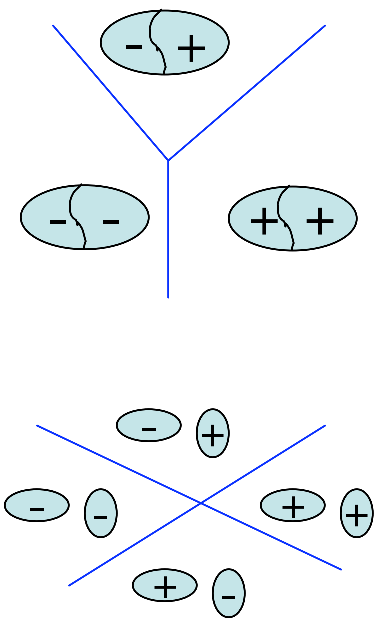
$$\alpha = -0.12 \pm 0.16$$

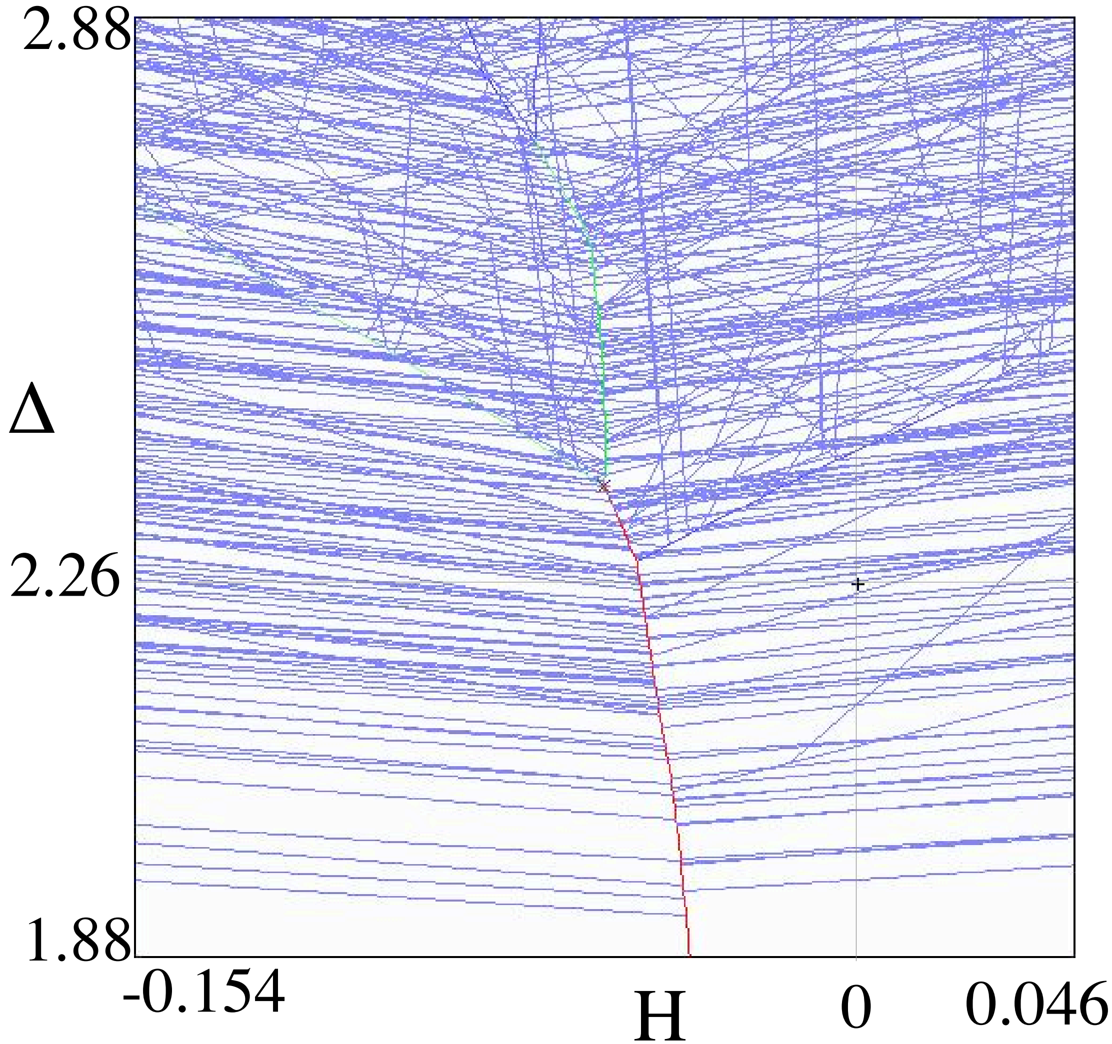
$$(1-\alpha)/\nu = 0.82 \pm 0.02$$



Ground states in the H-Δ plane

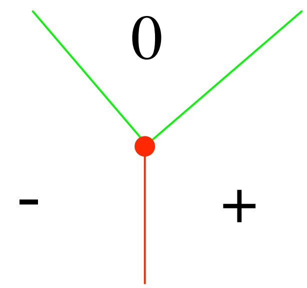
Degenerate ground states are separated by a line (point) across which one (two) domain(s) is (are) flipped.



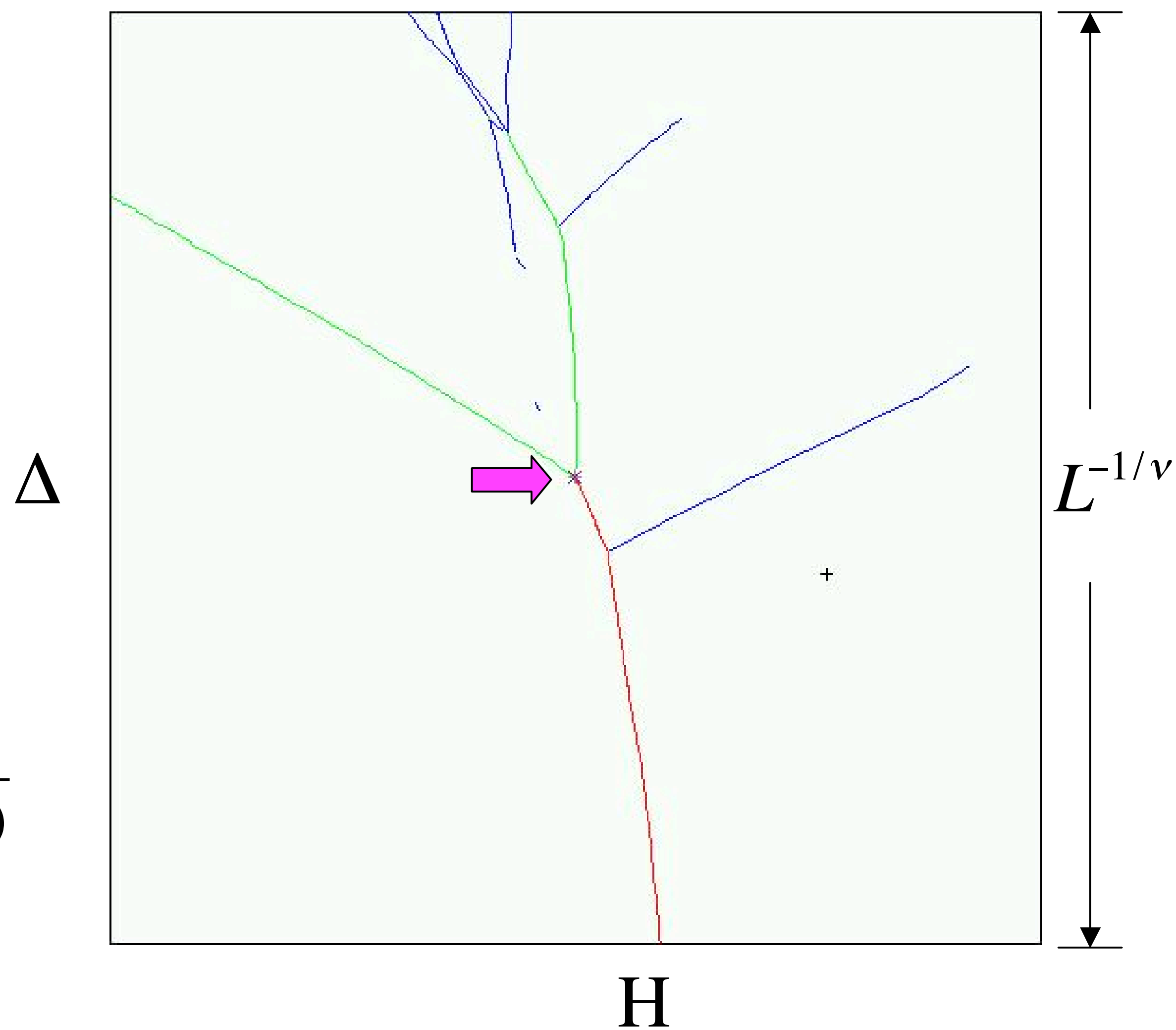


The finite size critical region

Hypothesis: The singularities in the finite size critical region are concentrated on a small number of "first-order" lines and points.



$$\delta e^* = \sqrt{(e_0 - e_-)(e_0 - e_+)}$$



Specific heat at T=0

Dukovski&Machta PRB 67, 0144XX (2003)

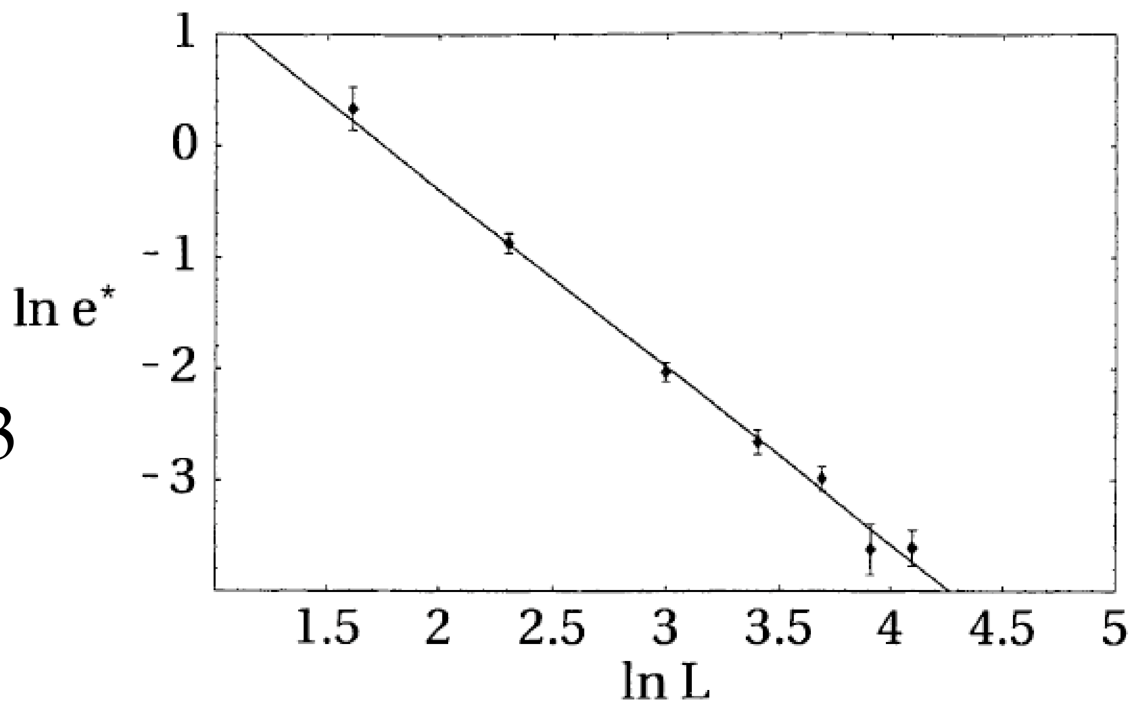
Scaling of the bond energy jump at the finite size transition:

$$\delta e^* \sim L^{(\alpha-1)/\nu}$$

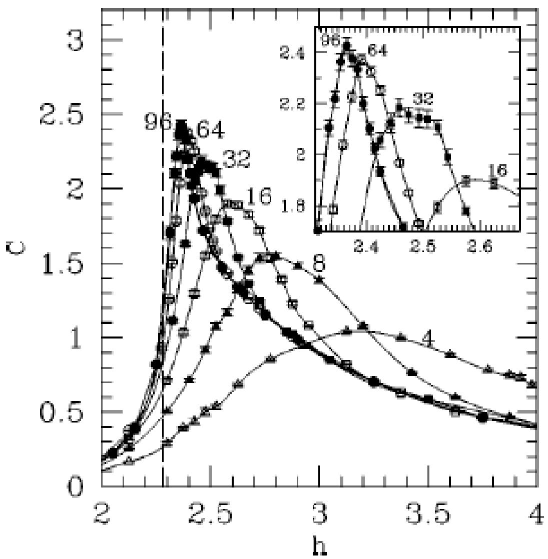
$$(1-\alpha)/\nu = 0.80 \pm 0.03$$

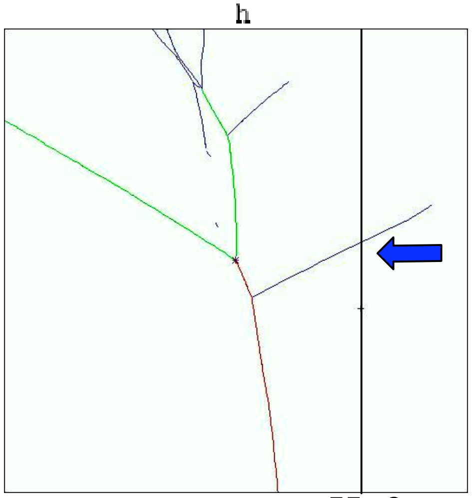
$$\Delta^* = \Delta_c + cL^{-1/\nu}$$

$$\nu = 1.1 \pm 0.1$$

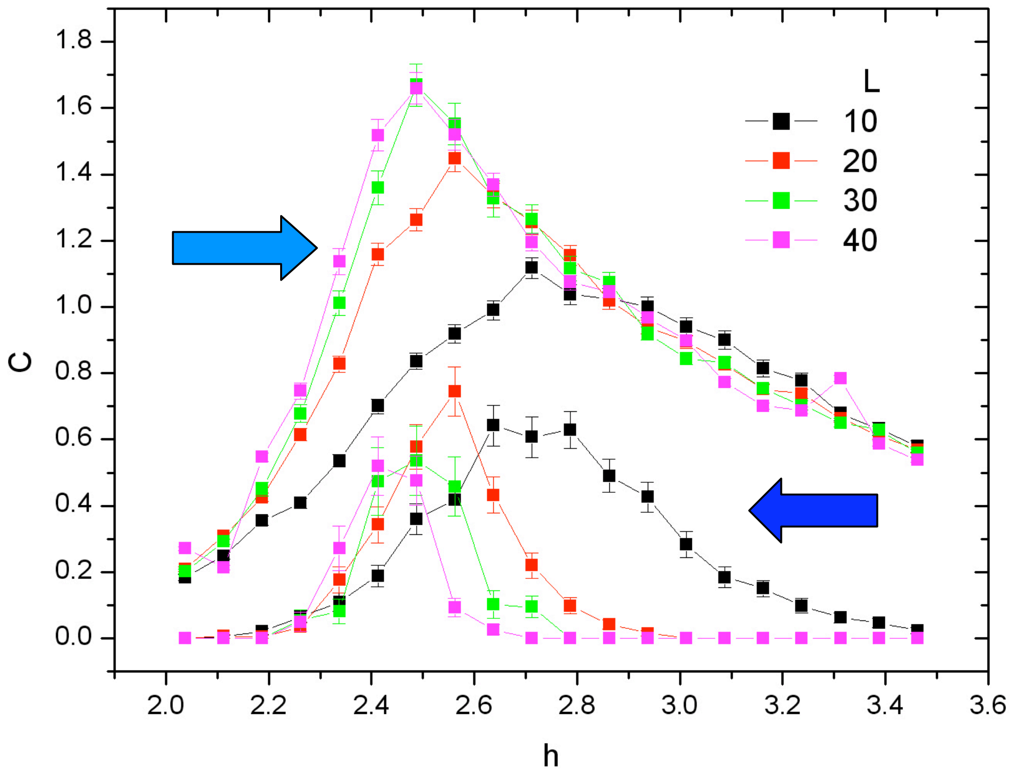


Specific heat at T=0





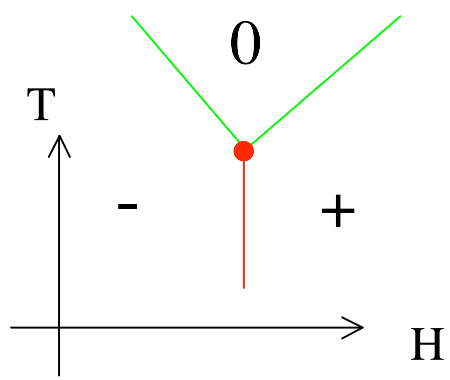
Contributions of biggest jump and all other jumps.

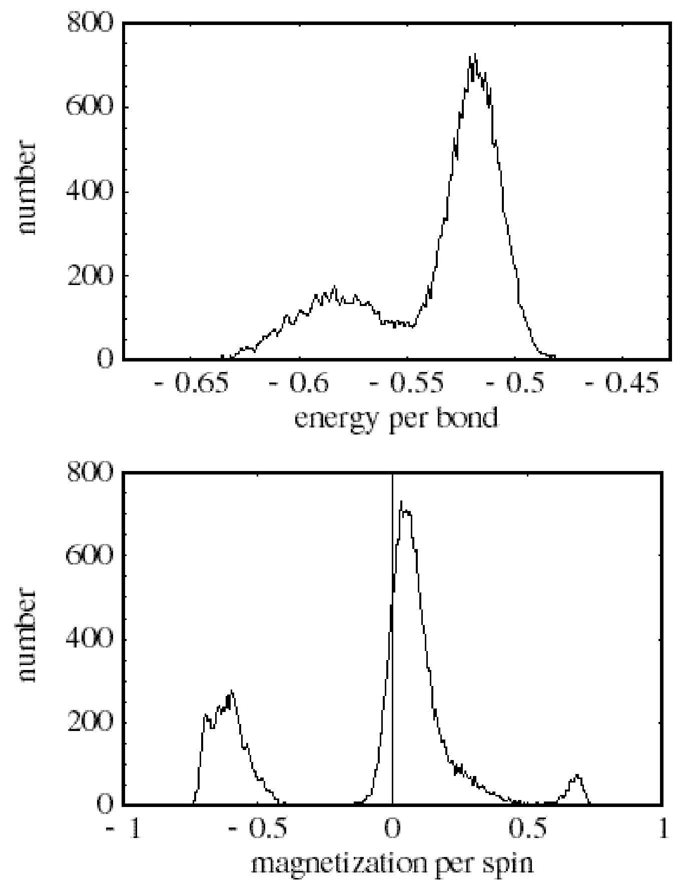


T > 0

Machta, Newman and Chayes PRE 62, 8782 (2000)

Qualitative features of the zero temperature transition should be seen in the T>0 transition for sufficiently large systems or strong disorder. For example, there should be points in the H-T plane for fixed Δ where three "phases" coexist.

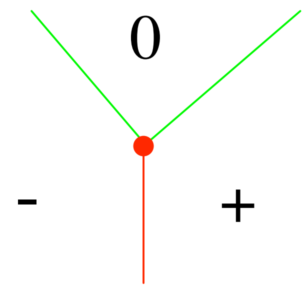




24^3 at Δ =1.31, T and H fine-tuned

Conclusion/Conjectures

- The finite size critical region of the 3D Gaussian RFIM is not self-averaging and is controlled by a few first-order like points (both a T=0 and T>0).
- The singularities at these points are described by exponents obeying modified hyperscaling.
- Measurements of these singularities provides a way to extract critical exponents from finite size scaling.



$$\delta e^* \sim L^{-(1-\alpha)/\nu}$$